$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln(ax+b)$$

$$\int \frac{xdx}{ax+b} = \frac{x}{a} - \frac{b}{a^2}\ln(ax+b)$$

$$\int \frac{x^2dx}{ax+b} = \frac{(ax+b)^2}{2a^3} - \frac{2b(ax+b)}{a^3} + \frac{b^2}{a^3}\ln(ax+b)$$

$$\int \frac{x^3dx}{ax+b} = \frac{(ax+b)^3}{3a^4} - \frac{3b(ax+b)^2}{2a^4} + \frac{3b^2(ax+b)}{a^4} - \frac{b^3}{a^4}\ln(ax+b)$$

$$\int \frac{dx}{x(ax+b)} = \frac{1}{b}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{dx}{x^2(ax+b)} = -\frac{1}{bx} - \frac{a}{b^2}\ln\left(\frac{x}{ax+b}\right)$$

$$\int \frac{xdx}{(ax+b)^2} = \frac{b}{a^2(ax+b)} + \frac{1}{a^2}\ln(ax+b)$$

$$\int \frac{x^2dx}{(ax+b)^2} = \frac{(ax+b)}{a^3} - \frac{b^2}{a^3(ax+b)} - \frac{2b}{a^3}\ln(ax+b)$$

$$\int \frac{x^3dx}{(ax+b)^2} = \frac{(ax+b)^2}{2a^4} - \frac{3b(ax+b)}{a^4} + \frac{b^3}{a^4(ax+b)} + \frac{3b^2}{a^4}\ln(ax+b)$$

$$\int \frac{xdx}{(ax+b)^3} = \frac{-1}{a^2(ax+b)} + \frac{b}{2a^2(ax+b)^2}$$

$$\int \frac{x^2dx}{(ax+b)^3} = \frac{2b}{a^3(ax+b)} - \frac{b^2}{2a^3(ax+b)^2} + \frac{1}{a^3}\ln(ax+b)$$

$$\int \frac{x^3dx}{(ax+b)^3} = \frac{x}{a^3} - \frac{3b^2}{a^4(ax+b)} + \frac{b^3}{2a^4(ax+b)^2} - \frac{3b}{a^4}\ln(ax+b)$$

$$\int \frac{dx}{a+bx+cx^2} = \frac{1}{\sqrt{-q}}\ln\frac{(2cx+b-\sqrt{-q})}{(2cx+b+\sqrt{-q})} \quad \text{where } q \equiv 4ac - b^2 \text{ and } q < 0.$$

Integration by parts $\int u\,dv = uv - \int v\,du$

Numerical integration by trapezoidal rule:

$$\int_{Xo}^{Xn} f(x)\,dx = \Delta x \sum_{i=0}^{n} f(x_i) - \Delta x\left[\frac{(f(x_0)+f(x_n))}{2}\right]$$

where Δx is a constant step size between discreet values of $f(x)$

See also Chapter 5 for additional mathematical relationships.

C-2 SOLUTIONS TO CUBIC EQUATIONS

A cubic equation of state may be solved by trial and error or analytically. Below are summarized two methods for solving analytically. These techniques are implemented for the Peng-Robinson equation in the spreadsheet PREOS.XLS. A cubic equation of the form

$$Z^3 + a_2Z^2 + a_1Z + a_0 = 0 \tag{C-1}$$

may be reduced to the form