

## Additional Exercises for Chapter 6

1. Consider the system

$$\dot{x}_1 = -x_1 + x_2, \quad \dot{x}_2 = -h(x_1) - x_2 + u, \quad y = x_2$$

where  $h \in [k, \infty]$  for some  $k > 0$ .

(a) Let  $u = 0$ . Using

$$V(x) = \int_0^{x_1} h(z) dz + \frac{1}{2}x_2^2$$

show that the origin is globally asymptotically stable.

(b) Show that the system is strictly passive and output strictly passive.

(c) Show that the system is finite-gain  $\mathcal{L}_2$  stable and find an upper bound on the  $\mathcal{L}_2$  gain.

(d) Consider the feedback connection of the dynamical system given above with a time-varying memoryless nonlinearity that belongs to the sector  $[-\frac{1}{2}, \frac{1}{2}]$ . Show that the origin of the unforced closed-loop system is globally uniformly asymptotically stable.