

Additional Exercises for Chapter 4

1. Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1^3(1 - x_1^2) - x_2$$

- (a) Show that $V(x) = \frac{1}{4}x_1^4 - \frac{1}{6}x_1^6 + \frac{1}{2}x_2^2$ is positive definite in some neighborhood of the origin.
- (b) Using $V(x)$ as a Lyapunov function candidate, show that the system has an asymptotically stable equilibrium point at the origin.
- (c) Is the origin globally asymptotically stable?

2. Consider the system

$$\dot{x}_1 = -x_1 + x_2, \quad \dot{x}_2 = -h(x_1) - x_2 + u$$

where $zh(z) \geq kz^2$ for all z , for some $k > 0$.

- (a) Let $u = 0$. Using

$$V(x) = \int_0^{x_1} h(z) dz + \frac{1}{2}x_2^2$$

show that the origin is globally asymptotically stable.

- (b) Show that the system is input-to-state stable.