

Additional Exercises for Chapter 2

1. Consider the system

$$\dot{x}_1 = x_2, \quad \dot{x}_2 = -x_1 + \mu x_2(1 - 3x_1^2 - 2x_2^2)$$

where μ is a constant that satisfies $|\mu| < 2$.

- Show that the origin $x = 0$ is the unique equilibrium point.
- Determine the type of the equilibrium point when $\mu < 0$ and $\mu > 0$.
- Using the Poincaré-Bendixson criterion, show that when $\mu > 0$ there is a periodic orbit in the set $\{x_1^2 + x_2^2 \leq \frac{1}{2}\}$.
- The phase portraits for $\mu = -0.2$ and $\mu = 0.2$ are shown in Figure 1. Mark the arrowheads and discuss the qualitative behavior in each case.
- Find bifurcations that occur as μ varies over the interval $(-2, 2)$. Sketch the bifurcation diagram.

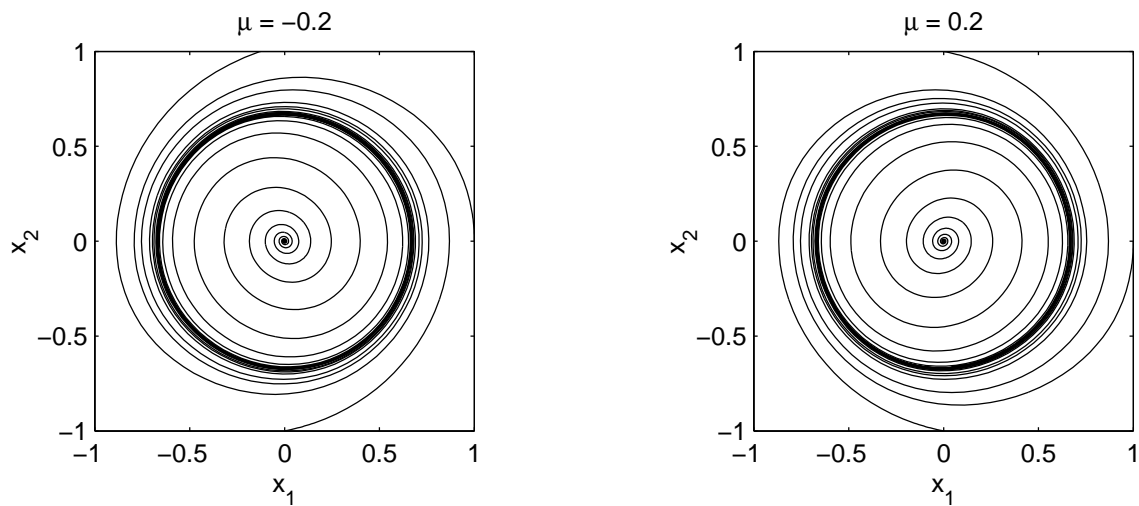


Figure 1: Exercise 1.