P.5-4 $I_1 = 0.1\,(A),\ P_{R1} = 3.33\,(mW);\quad I_2 = 0.02\,(A),\ P_{R2} = 8.00\,(mW);$
$I_3 = 0.0133\,(A),\ P_{R3} = 5.31\,(mW);\quad I_4 = 0.0333\,(A),\ P_{R4} = 8.87\,(mW);$
$I_5 = 0.0667\,(A),\ P_{R5} = 44.5\,(mW).\qquad \sum_n P_{Rn} = V_0 I_1 = 70\,(mW).$

P.5-5 $I_1 = 0.700\,(A),\ P_{R1} = 0.163\,(W);\quad I_2 = 0.140\,(A),\ P_{R2} = 0.392\,(W);$
$I_3 = 0.093\,(A),\ P_{R3} = 0.261\,(W);\quad I_4 = 0.233\,(A),\ P_{R4} = 0.436\,(W);$
$I_5 = 0.467\,(A),\ P_{R5} = 2.178\,(W).$

P.5-6 $\rho_0 = \dfrac{Q_0}{(4\pi/3)b^3} = \dfrac{10^{-3}}{(4\pi/3)(0.1)^3} = 0.239\,(C/m^3),\qquad \rho = \rho_0 e^{-(\sigma/\epsilon)t}$

a) $R < b:\ \bar{E}_i = \bar{a}_R \dfrac{(4\pi/3)R^3\rho}{4\pi\epsilon R^2} = \bar{a}_R \dfrac{\rho_0 R}{3\epsilon} e^{-(\sigma/\epsilon)t} = \bar{a}_R\, 7.5\times 10^9 R\, e^{-9.42\times 10^{11} t}\ (V/m)$

$R > b:\ \bar{E}_o = \bar{a}_R \dfrac{Q_0}{4\pi\epsilon_0 R^2} = \bar{a}_R \dfrac{9}{R^2}\times 10^6\ (V/m).$

b) $R < b:\ \bar{J}_i = \sigma \bar{E}_i = \bar{a}_R\, 7.5\times 10^{10} R\, e^{-9.42\times 10^{11} t}$

$R > b:\ \bar{J}_o = 0.$

P.5-7 a) $e^{-(\sigma/\epsilon)t} = \dfrac{\rho}{\rho_0} = 0.01 \longrightarrow t = \dfrac{\ln 100}{(\sigma/\epsilon)} = 4.88\times 10^{-12}\,(s) = 4.88\,(ps).$

b) $W_i = \dfrac{\epsilon}{2}\int_{V'} E_i^2\, dv' = \dfrac{2\pi \rho_0 b^3}{45\epsilon} e^{-2(\sigma/\epsilon)t} = (W_i)_0 \left[e^{-(\sigma/\epsilon)t}\right]^2.$

$\therefore\ \dfrac{W_i}{(W_i)_0} = \left[e^{-(\sigma/\epsilon)t}\right]^2 = 0.01^2 = 10^{-4}.$ Energy dissipated as heat loss.

c) Electrostatic energy stored outside the sphere $W_o = \dfrac{\epsilon_0}{2}\int_b^\infty E_o^2\, 4\pi R^2 dR = \dfrac{Q_0^2}{8\pi\epsilon_0 b} = 45\,(kJ)$
— constant.

P.5-8 a) $R = \dfrac{\ell}{\sigma S} = \dfrac{V}{I} \longrightarrow \sigma = \dfrac{\ell I}{SV} = 3.54\times 10^7\,(S/m).$

b) $E = \dfrac{V}{\ell} = 6\times 10^{-3}\,(V/m).$

c) $P = VI = 1\,(W).$

d) $\rho_e = -\dfrac{\sigma}{\mu_e}$ The given electron mobility $1.4\times 10^{-3}\,(m^2/V\cdot s)$ is that of a good conductor.

$u = \left|\dfrac{J}{\rho_e}\right| = \left|\dfrac{\mu_e J}{\sigma}\right| = |\mu_e E| = 1.4\times 10^{-3}\times(6\times 10^{-3})$
$= 8.4\times 10^{-6}\,(m/s).$

P. 5-9 a) Eq. (3-118): $E_{1t}=E_{2t} \longrightarrow E_2 \sin\alpha_2 = E_1 \sin\alpha_1$.

Eq. (5-58): $J_{1n}=J_{2n} \longrightarrow \sigma_1 E_{1n} = \sigma_2 E_{2n} \longrightarrow \sigma_2 E_2 \cos\alpha_2 = \sigma_1 E_1 \cos\alpha_1$.

$$\therefore \quad E_2 = E_1 \sqrt{\sin^2\alpha_1 + \left(\frac{\sigma_1}{\sigma_2}\cos\alpha_1\right)^2}. \qquad ①$$

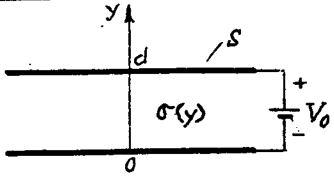
$$\tan\alpha_2 = \frac{\sigma_2}{\sigma_1}\tan\alpha_1 \longrightarrow \alpha_2 = \tan^{-1}\left(\frac{\sigma_2}{\sigma_1}\tan\alpha_1\right). \qquad ②$$

b) Eq. (3-121b): $D_{2n}-D_{1n}=\rho_s \longrightarrow \epsilon_2 E_{2n} - \epsilon_1 E_{1n} = \rho_s$.

$$\rho_s = \left(\frac{\sigma_1}{\sigma_2}\epsilon_2 - \epsilon_1\right)E_{1n} = \left(\frac{\sigma_1}{\sigma_2}\epsilon_2 - \epsilon_1\right)E_1\cos\alpha_1.$$

c) If both media are perfect dielectrics, $\sigma_1=\sigma_2=0$, Eqs. ① and ② revert to Eqs. (3-130) and (3-129) respectively and $\rho_s=0$.

P. 5-10



$$\sigma(y) = \sigma_1 + (\sigma_2-\sigma_1)\frac{y}{d}$$

a) Neglecting fringing effect and assuming a current density

$$\bar{J} = -\bar{a}_y J_0 \longrightarrow \bar{E} = \frac{\bar{J}}{\sigma} = -\bar{a}_y \frac{J_0}{\sigma(y)}.$$

$$V_0 = -\int_0^d \bar{E}\cdot\bar{a}_y\,dy = \int_0^d \frac{J_0\,dy}{\sigma_1+(\sigma_2-\sigma_1)\frac{y}{d}} = \frac{J_0 d}{\sigma_2-\sigma_1}\ln\frac{\sigma_2}{\sigma_1}.$$

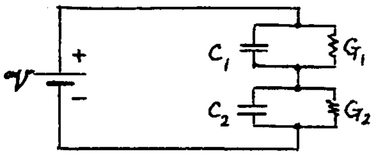
$$R = \frac{V_0}{I} = \frac{V_0}{J_0 S} = \frac{d}{(\sigma_2-\sigma_1)S}\ln\frac{\sigma_2}{\sigma_1}.$$

b) $(\rho_s)_u = \epsilon_0 E_y(d) = \frac{\epsilon_0 J_0}{\sigma_2} = \frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_2 d \ln(\sigma_2/\sigma_1)}$ on upper plate,

$(\rho_s)_\ell = -\epsilon_0 E_y(0) = -\frac{\epsilon_0 J_0}{\sigma_1} = -\frac{\epsilon_0(\sigma_2-\sigma_1)V_0}{\sigma_1 d \ln(\sigma_2/\sigma_1)}$ on lower plate.

c) $$\rho = \bar{\nabla}\cdot\bar{D} = \frac{d}{dy}(\epsilon_0 E) = -\epsilon_0 J_0 \frac{d}{dy}\left[\frac{1}{\sigma_1+(\sigma_2-\sigma_1)y/d}\right] = \epsilon_0 J_0 \frac{(\sigma_2-\sigma_1)/d}{[\sigma_1+(\sigma_2-\sigma_1)y/d]^2}.$$

P. 5-11



a) $C_1 = \frac{\epsilon_1 S}{d_1}$, $\quad G_1 = \frac{\sigma_1 S}{d_1}$;

$C_2 = \frac{\epsilon_2 S}{d_2}$, $\quad G_2 = \frac{\sigma_2 S}{d_2}$.

b) $$P = \mathcal{V}^2 G = \mathcal{V}^2 \frac{G_1 G_2}{G_1+G_2} = \mathcal{V}^2 S \frac{\sigma_1\sigma_2}{\sigma_1 d_2 + \sigma_2 d_1}.$$

P. 5-12 Refer to Fig. 5-6. In the transient state, the equation of continuity must be satisfied at the interface.

$$-\frac{\partial\rho_{si}}{\partial t} = J_2 - J_1 = \sigma_2 E_2 - \sigma_1 E_1 \qquad ①$$

Now

$$E_1 d_1 + E_2 d_2 = \mathcal{V} \qquad ②$$

$$\epsilon_2 E_2 - \epsilon_1 E_1 = \rho_{si} \qquad ③$$

P.5-15 Assume a potential difference V_0 between the inner and outer spheres.

$$\nabla^2 V = 0 \longrightarrow \frac{1}{R^2}\frac{d}{dR}(R^2 V)=0 \longrightarrow V=\frac{k}{R} \longrightarrow E_R=\frac{k}{R^2}.$$

$$V_0 = -\int_{R_2}^{R_1} E_R\,dR = -k\int_{R_2}^{R_1}\frac{1}{R^2}dR = k\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

$$\longrightarrow k=\frac{V_0}{\frac{1}{R_1}-\frac{1}{R_2}}. \qquad J_R=\sigma E_R=\frac{\sigma V_0}{\frac{1}{R_1}-\frac{1}{R_2}}.$$

$$I=\int_0^{2\pi}\int_0^{\pi} J_R R^2\sin\theta\,d\theta\,d\phi=\frac{4\pi\sigma V_0}{\frac{1}{R_1}-\frac{1}{R_2}}.$$

$$R=\frac{V_0}{I}=\frac{1}{4\pi\sigma}\left(\frac{1}{R_1}-\frac{1}{R_2}\right),$$ which can be obtained by combining Eqs. (3-140) and (5-81).

P.5-16 Assume a current I between the spherical surfaces.

$$\bar{J}=\bar{a}_R\frac{I}{4\pi R^2}=\sigma\bar{E}.$$

$$V_0=-\int_{R_2}^{R_1}\bar{E}\cdot d\bar{R}=\int_{R_1}^{R_2}\frac{I\,dR}{4\pi\sigma R^2}=\frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2}\frac{dR}{R^2(1+k/R)}$$

$$=\frac{I}{4\pi\sigma_0}\int_{R_1}^{R_2}\frac{1}{k}\left(\frac{1}{R}-\frac{1}{R+k}\right)dR=\frac{I}{4\pi\sigma_0 k}\ln\frac{R_2(R_1+k)}{R_1(R_2+k)}.$$

$$R=\frac{V_0}{I}=\frac{1}{4\pi\sigma_0 k}\ln\frac{R_2(R_1+k)}{R_1(R_2+k)}.$$

P.5-17 Assume I. $\qquad \bar{J}(R)=\bar{a}_R\frac{I}{S(R)}.$

$$S(R)=\int_0^{2\pi}\int_0^{\theta_0}R^2\sin\theta\,d\theta\,d\phi=2\pi R^2(1-\cos\theta_0).$$

$$\bar{E}(R)=\frac{1}{\sigma}\bar{J}(R)=\bar{a}_R\frac{I}{2\pi\sigma R^2(1-\cos\theta_0)}.$$

$$V_0=-\int_{R_2}^{R_1}E(R)\,dR=\frac{I(R_2-R_1)}{2\pi\sigma R_1R_2(1-\cos\theta_0)}.$$

$$R=\frac{V_0}{I}=\frac{1}{2\pi(1-\cos\theta_0)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right).$$

P.5-18 $\bar{\nabla}\cdot\bar{J}=0=\bar{\nabla}\cdot(\sigma\bar{E})=\sigma\bar{\nabla}\cdot\bar{E}-(\bar{\nabla}\sigma)\cdot\bar{E}=0.$

$$\bar{E}=\bar{a}_R E,\quad \bar{\nabla}\cdot\bar{E}=\frac{1}{R^2}\frac{d}{dR}(R^2E);\qquad \bar{\nabla}\sigma=\bar{a}_R\frac{d\sigma}{dR}=-\bar{a}_R\frac{\sigma_0 R_1}{R^2}.$$

Substituting back: $R\dfrac{dE}{dR}=E \longrightarrow \bar{E}=\bar{a}_R\dfrac{c}{R}$

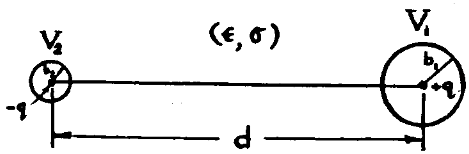
$$V=-\int_{R_2}^{R_1}\bar{E}\cdot d\bar{R}=c\ln\frac{R_2}{R_1}\longrightarrow c=\frac{V_0}{\ln(R_2/R_1)},\quad \bar{E}=\bar{a}_R\frac{V_0}{R\ln(R_2/R_1)}.$$

$$I=\int_S\bar{J}\cdot d\bar{s}=\int_S\sigma\bar{E}\cdot d\bar{s}=\int_0^{2\pi}\int_0^{\theta_0}\left(\frac{\sigma_0R_1}{R}\right)\left[\frac{V_0}{R\ln(R_2/R_1)}\right]R^2\sin\theta\,d\theta\,d\phi$$

$$=\frac{2\pi\sigma_0R_1V_0(1-\cos\theta_0)}{\ln(R_2/R_1)}.$$

$$R=\frac{V_0}{I}=\frac{\ln(R_2/R_1)}{2\pi\sigma_0R_1(1-\cos\theta_0)}.$$

P.5-19 Assume charges $+q$ and $-q$ to concentrate at the centers of spheres 1 and respectively. $d\gg b_1$, $d\gg b_2$.



$$V_1\cong\frac{q}{4\pi\epsilon}\left(\frac{1}{b_1}-\frac{1}{d-b_1}\right).$$

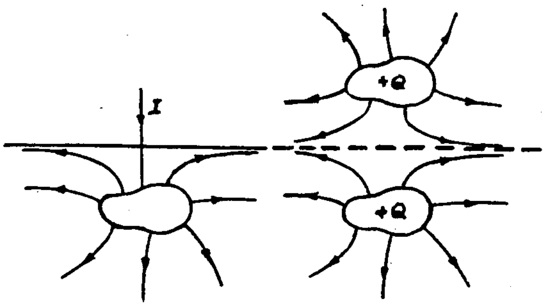
$$V_2\cong\frac{q}{4\pi\epsilon}\left(\frac{1}{d-b_2}-\frac{1}{b_2}\right).$$

$$C=\frac{q}{V_1-V_2}=\frac{4\pi\epsilon}{\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}}$$

$$=G\frac{\epsilon}{\sigma}=\frac{\epsilon}{R\sigma}.$$

$$R=\frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{1}{d-b_1}-\frac{1}{d-b_2}\right)\cong\frac{1}{4\pi\sigma}\left(\frac{1}{b_1}+\frac{1}{b_2}-\frac{2}{d}\right).$$

P.5-20



(a) (b)

The curved flow pattern of the lower half of Fig.(b), if both the conductor and its image are fed with the same current, is exactly the same as that of Fig.(a). All boundary conditions are satisfied.

$$\bar{\nabla}\times\left(\frac{\bar{J}}{\sigma}\right)=0\longrightarrow\bar{\nabla}\times\bar{J}=0.$$

We can write $\bar{J}=-\bar{\nabla}\psi$, where ψ and electrostatic potential V are simply related. The streamlines are similar to the $\bar{E}$-lines of a conductor and its image, both carrying a charge $+Q$ in the electrostatic case.