

R.2.2 \Rightarrow Under what conditions can the dot product of 2 vectors be negative?

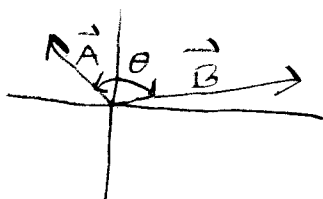
The definition of the dot product $\vec{A} \cdot \vec{B} = c$ (where c is a scalar) is

$$\vec{A} \cdot \vec{B} = c = |\vec{A}| |\vec{B}| \cos \theta$$

Since $|\vec{A}| \neq |\vec{B}| \geq 0$, c can be negative if $\cos \theta < 0$, or if

$$\boxed{\pi/2 < \theta < \pi}$$

Remember, θ is the smaller angle between the 2 vectors, so $0 \leq \theta < \pi$ for the definition of the dot product.



R 2.3 Write Down The Results of $A \cdot B$ & $A \times B$ if $A \parallel B$ & $A \perp B$.

a) For $A \parallel B$: $A \cdot B = |A||B|\cos\theta$ and in this case, $\theta = 0$ so $A \cdot B = |A||B|$

$$A \times B = |A||B|\sin\theta \cdot \hat{n} \text{ and } \theta = 0$$

and since $\sin\theta = 0$, $A \times B = 0$.

The cross product of 2 vectors always gives a vector perpendicular to the plane of the vectors. If the cross product of 2 parallel vectors is 0.

b) For $A \perp B$: $A \cdot B = |A||B|\cos\theta$ and here $\theta = \frac{\pi}{2} = 90^\circ$, so $A \cdot B = 0$. The dot

product of 2 vectors gives the projection of one vector on to another. For perpendicular vectors, neither vector has any component in the direction of the other.

$$A \times B = \hat{n} \cdot |A| \cdot |B| \cdot \sin\theta \text{ and } \theta = \pi/2$$

$$A \times B = \hat{n} \cdot |A| \cdot |B|$$

#3 R 2.4 which vector operation Below does NOT MAKE SENSE?

a.) $(\vec{A} \cdot \vec{B}) \times \vec{C} \Rightarrow$ Makes NO SENSE! The result $\vec{A} \cdot \vec{B}$ is a scalar, and you can cross a scalar $(\vec{A} \cdot \vec{B})$ with a vector (\vec{C}) . (Pg. 16)

d.) $A(\vec{B} \cdot \vec{C}) \Rightarrow$ Does Make SENSE! $\vec{B} \cdot \vec{C}$ is a scalar
So $A(\vec{B} \cdot \vec{C})$ is just \vec{A} scaled
The result $(\vec{B} \cdot \vec{C})$. (Pg. 14)

c.) $\vec{A} \times \vec{B} \times \vec{C} \Rightarrow$ NO PARENTHESES! WHICH OPERATION DO YOU PERFORM FIRST?

b.) $\vec{A} / \vec{B} \Rightarrow$ NO SENSE AT ALL! There is NO DEFINITION FOR THE DIVISION OF 2 VECTORS (Pg. 19)

e.) $\vec{A} / a_A \Rightarrow$ NO SENSE! \vec{A} is a vector; a_A is a unit vector in the \vec{A} direction. JUST like the last problem, you CANNOT DIVIDE 2 VECTORS

$(\vec{A} \times \vec{B}) \cdot \vec{C} \Rightarrow$ Does Make Sense! The result
of $\vec{A} \times \vec{B}$ is a vector, which is
simply dotted to \vec{B} .

R2.5

Is $(\vec{A} \cdot \vec{B})\vec{C} = \vec{A}(\vec{B} \cdot \vec{C})$? No!

$(\vec{A} \cdot \vec{B})$ is a scalar result, and it
changes the length of vector \vec{C}
in the operation $(\vec{A} \cdot \vec{B})\vec{C}$

$(\vec{B} \cdot \vec{C})$ is also scalar, and changes
the length of vector \vec{A} .

As long as vectors \vec{A} & \vec{C} are not
equal, the result $(\vec{A} \cdot \vec{B})\vec{C} \neq \vec{A}(\vec{B} \cdot \vec{C})$

P2.1

Given 3 vectors,

$$\vec{A} = \hat{x} + 2\hat{y} - 3\hat{z}$$

$$\vec{B} = -4\hat{y} + \hat{z}$$

$$\vec{C} = 5\hat{x} - 2\hat{z}$$

Find:

$\hat{a}_A \Rightarrow$ a unit vector in the \vec{A} direction.

$$\hat{a}_A = \frac{\vec{A}}{|\vec{A}|} \Rightarrow \text{This is NOT an illegal}$$

'Vector Division': We simply scale all components of \vec{A} by $\frac{1}{|\vec{A}|}$; where $|\vec{A}|$ is the

Magnitude of \vec{A} . So, $|\vec{A}| = \sqrt{(1)^2 + (2)^2 + (-3)^2} = \sqrt{14}$

$$\text{So, } \hat{a}_A = \frac{1}{\sqrt{14}} (\hat{x} + 2\hat{y} - 3\hat{z})$$

$$|\vec{A} - \vec{B}| = \left| \hat{x}(1-0) + \hat{y}(2-(-4)) + \hat{z}(-3-1) \right| = \left| \hat{x} + 6\hat{y} - 4\hat{z} \right|$$

$$= \sqrt{1^2 + 6^2 + (-4)^2} = \sqrt{53}$$

$$\vec{A} \cdot \vec{B} = (\hat{x} + 2\hat{y} - 3\hat{z}) \cdot (0\hat{x} - 4\hat{y} + \hat{z}) = 2(-4) + (-3)(1) = -8 - 3 = -11$$

$$\theta_{AB} \Rightarrow A \cdot B = |A||B| \cos \theta_{AB}$$

$$\cos \theta_{AB} = \frac{A \cdot B}{|A||B|} = \frac{-11}{\sqrt{14}\sqrt{17}} = -.713$$

← previous prob

$$\theta_{AB} = \cos^{-1}(-.713) = \boxed{135.48^\circ = 2.365}$$

The component of \vec{A} in the direction of \vec{c}

$$A \cdot C = |A| \cdot |C| \cdot \cos \theta_{Ac} = |C| \cdot \text{Proj}_A C = |A| \text{Proj}_C A$$

$$\text{So, } \text{Proj}_A C = \frac{A \cdot C}{|C|} = \boxed{\frac{11}{\sqrt{29}}}$$

$A \times C \Rightarrow$ set up a matrix & find the det

$$\begin{array}{l} A \rightarrow \\ C \rightarrow \end{array} \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(-4) - \hat{y}(-2+15) + \hat{z} \\ \boxed{= -4\hat{x} - 13\hat{y} - 10\hat{z}}$$

$(B \times C)$

$$B \times C = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & -4 & 1 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(8) - \hat{y}(-5) + \hat{z}(20) = 8\hat{x} + 5\hat{y} + 20\hat{z}$$

$$(B \times C) \cdot (A \times C) = (8\hat{x} + 5\hat{y} + 20\hat{z}) \cdot (-4\hat{x} - 13\hat{y} - 10\hat{z}) = 8 + 10 - 60$$

g cont'd)

$$(\vec{A} \times \vec{B}) \cdot \vec{C} \quad \vec{A} \times \vec{B} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 0 & -4 & 1 \end{vmatrix} = \hat{x}(14) - \hat{y}(1) + \hat{z}(-4) \\ = -10\hat{x} - \hat{y} - 4\hat{z}$$

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (-10\hat{x} - \hat{y} - 4\hat{z}) \cdot (5\hat{x} + 0\hat{y} - 2\hat{z}) \\ = -50 + 8 = \boxed{-42}$$

$(\vec{A} \times \vec{B}) \cdot \vec{C} = \vec{A} \cdot (\vec{B} \times \vec{C})$, The expected result

h.) $(\vec{A} \times \vec{B}) \times \vec{C} \Rightarrow \vec{A} \times \vec{B} = -10\hat{x} - \hat{y} - 4\hat{z}$ (From part g)

$$(\vec{A} \times \vec{B}) \times \vec{C} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -10 & -1 & -4 \\ 5 & 0 & -2 \end{vmatrix} = \hat{x}(2) - \hat{y}(40) + \hat{z}(20) \\ = 2\hat{x} - 40\hat{y} + 20\hat{z}$$

$\vec{A} \times (\vec{B} \times \vec{C}) \quad \vec{B} \times \vec{C} = 8\hat{x} + 5\hat{y} + 20\hat{z}$ (part g)

$$\vec{A} \times (\vec{B} \times \vec{C}) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & 2 & -3 \\ 8 & 5 & 20 \end{vmatrix} = \hat{x}(55) - \hat{y}(44) + \hat{z}(10) \\ = \underline{\underline{55\hat{x} - 44\hat{y} + 10\hat{z}}}$$

The expected result: $(\vec{A} \times \vec{B}) \times \vec{C} \neq \vec{A} \times (\vec{B} \times \vec{C})$

#6 P.2.2 Given $\vec{A} = \hat{x} - 2\hat{y} + 3\hat{z}$
 $\vec{B} = \hat{x} + \hat{y} - 2\hat{z}$

Find a UNIT-VECTOR \vec{C} That is \perp TO \vec{A} & \vec{B}
 To Find a vector \perp TO \vec{A} & \vec{B} , we need to
 Take $\vec{A} \times \vec{B}$:

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 1 & -2 & 3 \\ 1 & 1 & -2 \end{vmatrix} = \hat{x}(4-3) - \hat{y}(-2-3) + \hat{z}(1+2)$$

$$= \hat{x} + 5\hat{y} + 3\hat{z}$$

$$\vec{C} = \frac{\vec{A} \times \vec{B}}{|\vec{A} \times \vec{B}|} \Rightarrow \frac{1}{\sqrt{1+25+9}} = \frac{1}{\sqrt{35}}$$

$$\vec{C} = \frac{1}{\sqrt{35}} (\hat{x} + 5\hat{y} + 3\hat{z})$$

#7 P.2.3 Two vector fields $\hat{x}A_x + \hat{y}A_y + \hat{z}A_z$
 $\{ \hat{x}B_x + \hat{y}B_y + \hat{z}B_z$ are parallel ev

What is The relationship BTWN. THE
 COMPONENTS?

IF 2 vector fields are parallel,

$$\vec{A} \times \vec{B} = 0 \quad \text{So, (next page)}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{x}(A_y B_z - A_z B_y) - \hat{y}(A_x B_z - A_z B_x) + \hat{z}(A_x B_y - A_y B_x)$$

The components of $\vec{A} \times \vec{B}$ must each be 0, so

$$A_y B_z - A_z B_y = 0 \quad A_y B_z = A_z B_y \quad (1)$$

$$A_x B_z - A_z B_x = 0 \quad A_x B_z = A_z B_x \quad (2)$$

$$A_x B_y - A_y B_x = 0 \quad A_x B_y = A_y B_x \quad (3)$$

$$\frac{A_y}{B_y} = \frac{A_z}{B_z} \quad \text{From 1}$$

$$\frac{A_x}{B_x} = \frac{A_z}{B_z} \quad \text{From 2}$$

$$\frac{A_x}{B_x} = \frac{A_y}{B_y} \quad \text{From 3}$$

$$\therefore \frac{A_x}{B_x} = \frac{A_y}{B_y} = \frac{A_z}{B_z} \quad \text{which means}$$

That for parallel vectors $\vec{A} \parallel \vec{B}$,

$$\vec{A} = k \cdot \vec{B} \quad \text{where } k \text{ is a scalar.}$$