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COIN Report Number 2016022

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Design Optimization of an Artificial Lateral Line System Incorporating Flow and Sensor Uncertainties

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Abstract An artificial lateral line (ALL) consists of a set of flow sensors arranged around a fish-like body. An ALL aims at identifying surrounding moving objects, a common example of which is a vibrating sphere, called a dipole. Identification accuracy for an arbitrary dipole can be enhanced by a proper selection of design parameters of the ALL, including the number and location of the sensors as well as shape and size of the ALL. In this study, different sources of uncertainties are identified and simulated in the problem formulation. A parametric fitness function is defined that addresses computational and practical goals and encompasses the effect of different sources of uncertainty. A comprehensive parameter study is performed to minimize the uncertainty in evaluation of the fitness function. A specialized bi-level optimization tool is formed to optimize parameters of the ALL in different cases with different amount of uncertainties. The trade-off between localization accuracy and the number of sensors is analyzed. A comparison of results for different amount of uncertainty reveals that the optimized design highly depends on the amount of uncertainty as well as the number of available sensors. Consequently, these factors must be considered in design of the ALL. Another highlight of the proposed bi-level optimization methodology is that it is generic and can be extended to solve other noisy and nested optimization problems easily.

Keywords: Uncertainty formulation; Dipole source localization; Bi-level optimization; Sensor placement
1 Introduction

The lateral line system is an important flow-sensing organ, which is involved in various behaviors of fish (Bleckmann 1994), such as schooling (Pitcher et al. 1976), station holding (Bleckmann et al. 2012) and object detection (Von Campenhausen et al. 1981). A lateral line system consists of arrays of flow sensors called neuromasts (Hassan 1993). One of the two types of neuromasts is the superficial neuromasts, which stick out of the fish skin and respond to flow velocities (Engelmann et al. 2000). The response of the neuromasts, in the form of neuronal pulses, is further transmitted to the central nervous system for information processing (Bleckmann et al. 2008).

It is of great interest to develop an engineering equivalent of a biological lateral line for underwater applications. Such an artificial lateral line (ALL) system will introduce a novel and noiseless sensing modality for the navigation and control of underwater robots and vehicles, and provide complementary information to traditional underwater vision sensors and sonar (Fernandez et al. 2011).

In the past few decades, a number of bio-inspired flow sensors have been reported, exploiting micro-fabrication techniques (McConney et al. 2009; Qualtieri et al. 2011), optical transduction (Klein and Bleckmann 2011) and smart materials such as ionic polymer-metal composites (IPMCs) (Abdulsadda and Tan 2011, 2012; Lei et al. 2012). IPMCs, with built-in sensing and actuation capabilities (Shahinpoor and Kim, 2001), are one important class of electroactive polymers (EAPs). They are promising for versatile sensing applications (Lei, et al. 2013) due to their inherent polarity, high sensitivity and direct mechanosensory property. In particular, IPMCs have been used as bio-inspired flow sensors for different applications (Abdulsadda and Tan 2011, 2012).

Some theoretical work has also been conducted on flow modelling and information processing to extract information in ALLs (Hassan 1993; Ren and Mohseni 2012; DeVries and Paley 2013). Although some previous studies aimed at detecting moving objects and vortex (Venturelli et al. 2012; Chambers et al. 2014), most of them focused on localization of a vibrating source, called the dipole source. The dipole source emulates the rhythmic movement of fish body and fins, and has been commonly used as a biological stimulus such as counter-specific, predator, or prey (Coombs and Conley 1997a, 1997b). Dipole source localization has also played an important role in the development of ALLs, detection and estimation of nearby fish-like robots, and
coordination and control of underwater robots (Dagamseh, et al. 2010; Abdulsadda Tan 2012).

Although many studies have been conducted on localization of a dipole source using the ALL system (Dagamseh et al. 2010; Yang et al. 2010; Abdulsadda and Tan, 2011, 2012, 2013), very few have addressed the identification of an optimal ALL, the one that provides maximum localization accuracy for an arbitrary dipole. Once the ALL is identified, the next question is to locate a finitely specified number of sensors on the ALL. In a previous study (DeVries and Paley 2013), observability-based optimization of placement of flow sensors was discussed for the control purpose. The performed estimation and optimization were conducted in a uniform flow field, which was different from the localization of a dipole source. Furthermore, the employed flow model was commonly assumed to be accurate while like most other theoretically driven models, it depends on assumptions that may not be satisfied in practical situations.

In a recent study (Ahrari et al. 2015), a parametric fitness function was proposed for measuring accuracy of an ALL in presence of uncertainties. The ALL was to identify maximum velocity and direction of vibration in addition to coordinates of the dipole. A bi-level optimization method was developed to find the optimal ALL such that accuracy of identification for an arbitrary dipole source turns maximal. Unlike previous studies that either ignore uncertainties or consider only sensor uncertainty, both flow model and sensor uncertainty were formulated and simulated in the optimization process. Furthermore, design optimization of ALL was not limited to location of the sensors, it included the shape and the size of the ALL.

This paper is an extension to (Ahrari et al. 2015), where in-depth analysis of the algorithm parameters is rendered. Trade-off between computation time and quality of the lower loop is investigated and a reasonable parameter setting is proposed. A few extra practical limitation on the design of the ALL is imposed. Dependence of the optimal ALL on the amount of uncertainty is demonstrated as well. It is remarkable that the propose methodology can be utilized in other problems which require optimal sensor placement, such as structural health monitoring (Bruggi and Mariani 2013; Ranieri et al. 2014), although it is investigated for the ALL in this study.

The rest of this paper is organized as follows: In Section 2, the dipole identification problem is outlined. Different sources of uncertainties, including the flow model uncertainty and the sensor uncertainty are discussed. In Section 3, the proposed bi-level optimization method is explained including development of a robust parametric
fitness function for measuring accuracy of the ALL in presence of uncertainties. Numerical results are provided and discussed in Section 4. Finally, conclusions are drawn in Section 5 and prospective studies in the domain of this research are highlighted.

2 Problem Outline

For the ease of discussion, we focus on the setting of two-dimensional (2D) flows. In particular, an ALL system is considered to consist of a cylindrical body with fish-shaped cross-section and multiple sensors that measure local flow velocity around the body (see a prototype illustration in Figure 1(a)). In theory, we assume that the cylindrical body has infinite height and the flow is restricted to the plane of its cross-section. Consequently, the body configuration is fully captured by the size and shape of the cross-section.

2.1 Flow Model

With a given flow model, the flow velocity at any location \((x_k, y_k)\) can be computed if the parameters of the dipole source, including its maximum velocity of vibration \((A)\), orientation angle of vibration \((\beta)\), and location of the dipole \((x_s, y_s)\), are known (Figure 1(b)):

\[
v(x_k, y_k) = f(\theta, x_k, y_k), \theta = [x_s, y_s, \alpha_1, \alpha_2],
\]

where \(\alpha_1 = A\cos(\beta)\) and \(\alpha_2 = A\sin(\beta)\) and function \(f\) is provided by the flow model, which computes the local flow velocity at the place of the sensor caused by the dipole source \(\theta\).

The identification process, however, does the inverse calculation by computing parameters of the dipole, given the measured flow velocity at sensors \((M_k, k=1, 2, \ldots, N_{sensor})\). This inverse problem is solved by searching for a dipole \((\theta')\) whose flow field matches \(M_k\). In ideal situations, the solution to the inverse problem \((\theta^* = \theta)\) is the actual dipole \((\theta^* = \theta)\). In practice, several factors, which will be discussed later, result in an identification error, which is defined based on the difference between \(\theta\) and \(\theta^*\). The goal of design optimization is to find parameters of the ALL, including the shape and the size of the body as well as the number and locations of flow sensors, such that the identification accuracy is maximized for all possible dipoles within a certain area around the ALL.

We consider a potential flow generated by a dipole source, as assumed widely in the literature (Hassan 1993; Abdulsadda and Tan 2013). The potential flow model
provides an analytical formula for computing the flow distribution in terms of the dipole parameters, which is ideal for efficient computation in the optimization process. As to be discussed later, the error between the potential flow model and the actual dipole-induced flow will be accommodated in the design process. The potential at any spatial location around the dipole is expressed as:

\[ \varphi(r) = \frac{a^3(V_d \cdot r)}{2||r||^3}, \quad (2) \]

where \( r=[x-x_s, y-y_s] \) is the spatial point location relative to the dipole (Figure 1(b)), \( a \) is the sphere diameter and \( V_d \) is the instantaneous velocity of the vibrating sphere. The corresponding flow velocity is then given by:

\[ V(r) = -\nabla \varphi(r) = \frac{a^3(3(V_d \cdot r)r - ||r||^2V_d)}{2||r||^5}. \quad (3) \]

It is assumed that the ALL sensors, the dipole source, and the direction of the dipole vibration are all located in a same plane (xy-plane) parallel to the ALL body’s cross-section, and that the presence of body and sensors has negligible effect on the flow distribution (see more discussion in [Abdulsadda and Tan 2013]). With these assumptions, the flow velocity at the sensor site \((x_k, y_k)\) can be written as:

\[ v(x_k, y_k) = \frac{a^3v^d}{2||r_k||^5}((2(x_k - x_s)^2 - (y_k - y_s)^2)\cos\beta \]
\[ + 3(x_k - x_s)(y_k - y_s)\sin\beta), \quad (4) \]

where \( v^d=\text{Asin}(\omega t) \) represents the vibration of the dipole with maximum velocity of \( A \) and angular frequency \( \omega \). The value of \( \omega \) is assumed to be known. Typically, each sensor can only measure the flow velocity component along a specific direction, which is assumed to be the case in this work. We extract the signal amplitude for sensor \( k \) at frequency \( \omega \) as the measurement \( M_k \) through, for example, Fast Fourier Transform (FFT), which can be written as:

\[ M_k = f(\theta, x_k, y_k) = \left| \frac{(2(x_k - x_s)^2 - (y_k - y_s)^2)\alpha_1 + 3(x_k - x_s)(y_k - y_s)\alpha_2}{2||r_k||^5/a^3} \right|, \quad (5) \]
where \( \theta = [x_s, y_s, \alpha_1, \alpha_2] \) and \( \theta \) characterizes the dipole source, including its location and vibration amplitude/orientation. In Equation (5), it was assumed that the sensors measure the x-component of the flow velocity. By exploitation of the proper rotation matrix, Equation (5) can be generalized to the case of an arbitrary sensing direction.

Figure 1. a) A prototype of an ALL with sensors. b) Illustration of the dipole parameters.

### 2.2 Dipole Identification

Equation (5) demonstrates that the local flow speed can be determined if four parameters of the dipole are known. The ALL actually solves an inverse problem to identify the dipole, in which a dipole (\( \theta \)) is sought such that the difference between the sensor measurements and the flow field generated by \( \theta \) is minimal. This means that the inverse problem can be converted to a minimization problem:

\[
\text{Minimize } J(\theta) = \frac{\|M - |f(\theta, x, y)|\|}{\|M\|},
\]

\( M = [M_1, M_2, \ldots, M_{N_{\text{sensor}}}] \), \( f(\theta, x, y) = [f_1, f_2, f_3, \ldots, f_{N_{\text{sensor}}}] \), \( f_k = (\theta, x_k, y_k) \)

where \( M_k \) is the measured local flow velocity at the \( k \)-th sensor (given) and \( f_k(\theta, x_k, y_k) \) is the theoretically computed flow velocity at the \( k \)-th sensor using Equation (5). In the absence of uncertainties, the global minimum of the minimization problem is the actual dipole, at which the error function (\( J \)) is zero, i.e. \( \theta^* = \theta \) and \( J(\theta^*) = 0 \), unless \( N_{\text{sensor}} < 4 \). In the latter case, the number of equations is less than the number of unknown parameters,
therefore, there could be an infinite number of $\mathbf{\Theta}^*$ which result in $J=0$.

Due to the presence of uncertainty in the sensor measurements and the inaccuracy of the flow model, in general, there is a difference between $\mathbf{\Theta}^*$ and $\mathbf{\Theta}$, called the identification error ($e$):

$$e = \left\| \begin{bmatrix} x_s^* - x_s^* \frac{\text{cm}}{1}, y_s^* - y_s^* \frac{\text{cm}}{1}, \alpha_1 s - \alpha_1 s^* \frac{\text{cm/s}}{1}, \alpha_2 s - \alpha_2 s^* \frac{\text{cm/s}}{1} \end{bmatrix} \right\|,$$

(6)

where the underline denotes the parameters of the actual dipole. This equation means that the identification error is the Euclidean norm of $\mathbf{\Theta}^* - \mathbf{\Theta}$, which is made dimensionless to avoid dimension mismatch when calculating the norm. In general, $e \geq 0$ and besides, $0 \leq J(\mathbf{\Theta}^*) \leq J(\mathbf{\Theta})$. This means that the actual location of the dipole is not the global minimum anymore. For dipole identification, two sources of uncertainty can be identified: flow model uncertainty and sensor uncertainty. Most previous studies on dipole source localization/identification have not explicitly considered these factors or accommodated only the sensor uncertainty, such as (Abdulsadda and Tan 2011, 2013). In this study, the uncertainties introduced by both the flow model and the sensor measurement are incorporated into the ALL design systematically.

2.2.1 Sensor Uncertainty

Limited precision of the sensors introduces another type of uncertainty. Since the measuring range of the sensor is fixed, the amount of uncertainty is assumed to be independent of the flow magnitude, therefore, it is simulated as an additive noise (Beyer 2006):

$$M_k(\mathbf{\Theta}) = [M_k(\mathbf{\Theta}) + \epsilon_{\text{sensor}}N(0,1)], k = 1,2,\ldots,N_{\text{sensor}},$$

(7)

where $M_k$ is the indicated flow velocity by the $k$-th sensor while the true flow velocity is $M_k$. The parameter $\epsilon_{\text{sensor}} \geq 0$ determines the noise strength of the sensor uncertainty.

2.2.2 Flow Model Uncertainty

Analytical methods, like the potential flow model adopted in this paper, are instrumental or even critical for design optimization. On the other hand, they typically depend on idealized assumptions or simplifications. Consequently, the actual velocity at a sensor
site might be different from the one computed using the flow model. Uncertainties caused by possible presence of other objects, interactions among the sensors, the ALL body, and the flow also contribute to the error of the flow model. Since the difference between the actual and the predicted flow—similar to most equations in engineering—is a fraction of the actual flow, the amount of uncertainty in the flow model is captured as follows:

\[
M_k(\theta) = |f_k(\theta)| \times \exp(\varepsilon_{\text{model}}N(0,1)), \quad k = 1, \ldots, N_{\text{sensor}}, \tag{8}
\]

where \( M_k(\theta) \) and \( f_k(\theta) \) are the actual flow velocity and the computed flow velocity according to the exploited flow model at the \( k \)-th sensor respectively. The parameter \( \varepsilon_{\text{model}} \geq 0 \) determines the magnitude of the uncertainty in the flow model, as commonly used for simulation of Gaussian noise (Hansen et al. 2009), which becomes equivalent to simple multiplicative noise for small values of \( \varepsilon_{\text{model}} \). \( N(0,1) \) is a random number sampled from the standard normal distribution.

Equations (7) and (8) enable simulation of the uncertainties in the identification process. Sensors indicate flow velocity of \( \mathbf{M} \), which differs from the true value (\( \mathbf{M} \)). On the other hand, an inaccurate flow model is utilized to interpret these inaccurate data. The identification error depends on many parameters including the values of \( \varepsilon_{\text{model}} \) and \( \varepsilon_{\text{sensor}} \), \( N_{\text{sensor}} \) and sensor locations and orientations.

3 Design Optimization

The ultimate goal of the ALL design optimization problem is to find the optimal shape and size of the body and the locations of sensors on its body such that a fitness measure \( g(X) \) of the design \( X \) is maximized. In this section, the design optimization problem is formulated and the design parameters, their ranges, constraints, the fitness function, and the optimization method to solve the problem are explained.

3.1 Design Parameters

The conformal mapping technique is used to describe a streamlined body (in our case, the cross-section profile of the cylindrical body) and the location of sensors on it. Consider the complex plan \( \mathbb{C} \) and a point \( \xi \in \mathbb{C} \). The following transformation maps \( \xi \) to \( z \) with respect to the transformation variable \( \lambda \in \mathbb{R} \) (Panton 1984):

\[
z = \xi + b^2 / \xi, \quad \xi = R \exp(i\beta) - \lambda, b = R - \lambda, \beta \in [-\pi, \pi). \tag{9}
\]
Equation (9) defines a disk with radius $R$, offset along the real axis by $\lambda \in \mathbb{R}$. By choosing $b$, we can map the disk to a symmetric, streamlined body. Therefore, $R$ and $\lambda$ specify the size and the shape of the body, and $\beta_k$ denotes location of the $k$-th sensor on the fish body. For $\lambda/R=1$, the shape of ALL is circular, while for $\lambda/R=0$, the shape turns to a line. Other values between these two extremes result in a fish-like ALL.

Symmetry about the real axis is exploited to reduce the number of design variables, and thus the set of design variables, $X=[X_1, X_2, \ldots, X_D]$ consists of:

- Size variable: $X_{1\text{min}} \leq X_1 = R \leq X_{1\text{max}}$
- Shape variable: $X_{2\text{min}} \leq X_2 = \lambda/R \leq X_{2\text{max}}$.
- Angular position of the first sensor on the fish body: $0 \leq X_3 = \beta_1 \leq \beta_{\text{max}}$.
- Angular position of the $k$-th sensor relative to the $(k-1)$-th sensor: $0 \leq X_k = \beta_{k-2} - \beta_{k-1} \leq \beta_{\text{max}}$, $k=4,5, \ldots, 0.5N_{\text{sensor}}$.

where $N_{\text{sensor}}$ specifies the total number of sensors in the ALL. Although $N_{\text{sensor}}$ can be optimized as well, it adds significant burden to the optimization algorithm by introducing integer variables. Therefore, it is treated as a fixed value; however, one may run the optimization problem for different values of $N_{\text{sensor}}$ independently to discover the contribution of an increased number of sensors. Because of the symmetry assumption about $y=0$, only locations of the sensors on the top part of the body are independent parameters, and thus there are $2+0.5N_{\text{sensor}}$ design parameters, where $N_{\text{sensor}}$ must be an even number. For the rest of this study, the following values for the range of design parameters are considered, unless mentioned otherwise: $X_{1\text{min}} = 0.5 \text{ cm}$, $X_{1\text{max}} = 4 \text{ cm}$, $X_{2\text{min}} = 0$, $X_{2\text{max}} = 1$, $\beta_{\text{max}} = 8\pi/N_{\text{sensor}}$. The optimization problem is then formulated as follows:

Maximize $g(X)$,

subject to $D_x \leq X \leq U_x$,

$X_3 + X_4 + X_5 + \ldots + X_D \leq \pi$,

where $D_x$ and $U_x$ are the lower and upper bounds for the design parameters and $g(X)$ is the fitness of the design $X$, which is directly related to the identification accuracy. The constraint ensures that all the independent sensors are placed on the top of the ALL. The sensing direction of a sensor is assumed to be tangent to the body at the place of the sensor.
3.2 Fitness Function

For each design, we need to compute its fitness function. The evaluation of the fitness function needs to incorporate that a dipole may lie anywhere in the predefined working area and vibrates along any direction. Since considering all possible cases is not possible, the identification problem is solved for a finite number ($N_{\text{dipole}}$) of dipole locations. For the problem at hand, the bounds for dipole parameters ($U_\theta$ and $D_\theta$) are defined such that a dipole may lie anywhere outside the ALL, inside a square, and may vibrate in any directions, while the maximum velocity of the dipole is larger than a minimum, ($A \geq A_{\text{min}}$). For the rest of this study, $A_{\text{min}}=3$ cm/s, $U_\theta=[10 \ 10 \ 10 \ 10]$ and $D_\theta=[-10, \ -10, \ -10, \ 0]$ (Figure 2(a)). Note that $[x, y, \alpha_1, \alpha_2]$ and $[x, y, -\alpha_1, -\alpha_2]$ are identical dipoles, therefore, the lower bound of $\alpha_2$ was set to 0.

![Diagram](image)

(a) Illustration of an ALL and a set of possible dipole locations; b) An example histogram of the identification errors.

An algorithm based on the heuristic employed in (Ahrari et al. 2009; Ahrari and Atai 2010) is employed to generate a set of uniformly distributed dipoles of size $N_{\text{dipole}}$ in the range defined by $U_\theta$ and $D_\theta$, so that the finite set of dipoles provides a sound approximation of all possible dipoles. The algorithm is based on generating dipoles one after another and rejecting those that lie closer than a threshold distance to other dipoles. The threshold distance is reduced if multiple successive dipoles are rejected. Since employing this algorithm is somewhat time-consuming, 50 sets of dipoles of the desired
size are generated and stored in a file. Each time a design is to be evaluated, a set is randomly chosen and used for fitness evaluation.

If a generated dipole lies inside the fish body, it is excluded from the subsequent evaluation. Similarly, a dipole too close to the fish body (closer than 1 cm in this paper) is also ignored considering the physical size of the dipole itself. The identification error is not calculated for these dipoles; therefore, the actual number of dipoles for which the identification error is computed is smaller than $N_{\text{dipole}}$. Furthermore, not all sensors can directly receive a signal from the dipole. In fact, the ALL body blocks wave propagation towards a sensor if some parts of it lie between the sensor and the dipole. Although these sensors may still provide some measurements, we exclude theirs from the identification process.

For each chosen dipole setting, the inverse problem is solved and the identification error is computed (see an example in Figure 2). Distribution of all identification errors ($e = [e_1, e_2, \ldots, e_{N_{\text{dipole}}}]$) is utilized to define the fitness function, $g(X)$. Algorithm 1 in Appendix explains how identification errors are calculated with uncertainties incorporated in the identification process.

A simple and reasonable fitness function can be defined by averaging all identification errors; however, the mean is not a robust statistics, since some outliers could significantly influence the calculated average (see Figure 2(b)). Considering different sources of uncertainties, a robust performance measure based on the statistical distribution of $e_i$ is strongly desired. The proposed fitness function follows this goal by giving higher weights (or credits) to more accurate identifications. The overall fitness is the mean of all the obtained credits:

$$ g(X) = \frac{1}{N_{\text{dipole}}} \sum_{i=1}^{N_{\text{dipole}}} \exp(-\zeta e_i^2). \quad (10) $$

In the above equation, parameter $\zeta$ determines how fast the obtained credit reduces when the identification error increases. For fixed distribution of the identification errors, the calculated fitness increases if $\zeta$ is reduced. Selection of an appropriate value for this parameter is discussed later.

It is notable that because of sensor and flow model uncertainties, random selection of a finite number of dipoles, and random initial solution for the inverse problem, there is uncertainty in evaluation of the design fitness, a common case in robust optimization.
The fitness function $g(X)$ is a random function, which means that independent evaluations of design $X$ leads to different values for $g(X)$, for which mean and standard deviation ($\bar{g}$, $s_g$) can be computed. This uncertainty in fitness evaluation results in selection noise, in which a bad solution might be preferred over a better one. Selection noise adversely affects reliability of the selection operator in the optimization process and as a consequence, the quality of the final design. Several factors including the way the fitness function is defined as well as $N_{dipole}$ can significantly moderate the selection noise, which are investigated subsequently.

Figure 3 a) distribution of the computed fitness of two designs $X_1$ and $X_2$ with $\bar{g}_1=0.4$ and $\bar{g}_2=0.6$. The fitness function is redefined to b) minimize the standard deviation of $g_1$ and $g_2$ or c) increase the gap between $\bar{g}_1$ and $\bar{g}_2$.

Figure 3(a) illustrates how the distributions of independent evaluations of two designs ($X_1$ and $X_2$) with $\bar{g}_1=0.4$ and $\bar{g}_2=0.6$ may look like. The area of the overlapping part of probability density functions of $g(X_1)$ and $g(X_2)$ is proportional to the selection noise, the probability that $X_1$ is wrongly preferred over $X_2$. The selection noise can be reduced either by redefining the fitness function such that the standard deviation is reduced (Figure 3(b)) or the difference between $\bar{g}_1$ and $\bar{g}_2$ is intensified (Figure 3(c)).

According to Figure 3, selection noise can be reduced if is variance among the true fitness of designs is maximized or when the variance of the estimated fitness under independent evaluations is minimized. Accordingly, Selection Reliability Index (SRI) can be defined as follows:

$$SRI = \frac{\text{StDev}(\bar{g})}{\text{mean}(s_g)}. \quad (11)$$
A larger SRI usually refers to a smaller selection noise. A parameter study is performed in the next section to monitor the effect of different parameters on SRI, which helps select a reasonable parameter setting.

### 3.3 A Bi-level Optimization Method

The fitness function formulated in the previous section is based on the values of the dipole identification errors. This means that for each design evaluation, the inverse problem, which is actually an optimization problem itself, must be solved many times. Consequently, the optimization process is bi-level, where at the upper level optimizes parameters of the ALL \((X)\), while at the lower level, the inverse solver is run many times to estimate the fitness of \(X\) by solving the inverse problem of each dipole (Minimizing \(J(\theta)\)).

\[
\text{Maximize } g(X) = \frac{1}{N_{\text{dipole}}} \sum_{i=1}^{N_{\text{dipole}}} \exp(-\xi e_i^2)
\]

where \(e_i = \left\| \begin{bmatrix} \frac{x_s - x_s^*}{1 \text{ cm}} & \frac{y_s - y_s^*}{1 \text{ cm}} & \frac{\alpha_1 - \alpha_{1s}^*}{1 \text{ cm}} & \frac{\alpha_2 - \alpha_{2s}^*}{1 \text{ cm}} \end{bmatrix} \right\|_i\),

Subject to \(\theta_i^* = \text{argmin}\{J(\theta_i)\}\)

\[D_x \leq X \leq U_x,\]

\[X_3 + X_4 + X_5 + \ldots + X_D \leq \pi,\]

Bi-level optimization has shown to be effective in many practical and even multi-objective optimization problems (Deb and Sinha 2009; Linnala et al. 2012), although it has been criticized for being complex and hard even when the problems in both levels are linear (Colson et al. 2007). The situation is more challenging for the problem at hand due to the presence of nonlinearities at both levels and the presence of uncertainties, which can be moderated by a proper choice of the inverse solver.

### 3.3.1 Inverse Solver (Lower Level)

The challenge of high computation of bi-level optimization can be moderated by selecting a fast optimization method for the lower level, especially for the problem at hand where the inverse solver should be run several hundred times per design evaluation in the upper level. Considering that the objective function at the lower level \(J(\theta)\) is explicitly defined, a gradient-based optimization method is the preferred choice. In this study, we select the
Newton-Raphson (N-R) method for the lower level, which was demonstrated to be a reliable tool for dipole source localization (Abdulsadda and Tan 2013); however, some changes are required since there are terms in \( J(\theta) \) involving absolute values, which means the gradient of \( J(\theta) \) is not defined at certain points.

The employed N-R based method in this study starts the search from a semi-random initial solution (\( \theta_{\text{ini}} \)):

\[
\theta_{\text{ini}} = \theta + \alpha_\theta (U_\theta - D_\theta) \otimes r,
\]

(12)

where \( r \) is vector of four uniformly distributed random numbers in \([-0.5, 0.5]\), \( U_\theta \) and \( D_\theta \) are the upper and lower bounds for the dipole parameters, \( \alpha_\theta \) specifies the relative size of the region around the true dipole parameters, and the notation \( \otimes \) refers to element-wise multiplication. For this study, \( \alpha_\theta = 0.5 \) is used.

Line search is performed along the best direction: \( d_{\text{best}} = -H^{-1} (J) \times \nabla J \), where \( H \) and \( \nabla \) denote the Hessian matrix and the gradient vector, respectively using the golden section search, with the initial step size of \( \Delta \). \( J(\theta_{\text{ini}}) \), \( J(\theta_{\text{ini}} + \Delta \times d_{\text{best}}) \) and \( J(\theta_{\text{ini}} - \Delta \times d_{\text{best}}) \) are computed. If the last term is the smallest, moving along \( d_{\text{best}} \) increases \( J(\theta) \), therefore, \( d_{\text{best}} \) is inverted. This might happen because of multimodality or absence of derivatives at some points. The line search is terminated after a maximum of \( N_J \) evaluations of \( J(\theta) \), or when the step size is too small (smaller than \( 10^{-4} \) in this work) to make considerable changes in \( J(\theta) \), and \( d_{\text{best}} \) is updated. This process continues until the predefined number of evaluation of \( J(\theta) \) for a single run of the inverse solver (\( \text{max}J\text{eval}) \) is consumed. The inverse solver is also terminated if the difference between the initial and final solutions in a line search loop is smaller than \( 10^{-4} \), therefore, the actual number of evaluations of \( J(\theta) \) (\( \text{used}J\text{eval}) \) can be smaller than \( \text{Max}J\text{eval}) \).

Based on our preliminary results, computation of \( d_{\text{best}} \) appeared to be about five times more costly than evaluation of \( J(\theta) \). Therefore, \( \text{used}J\text{eval} \) is added by five whenever \( d_{\text{best}} \) is updated. \( \text{Max}J\text{eval} \) and \( N_J \) should be large enough to allow the algorithm to converge, but not too large to result in unnecessary increase in computation cost. A parameter study is performed in Section 3.6 to decide on the values of these parameters.

It is noted that N-R is a local search method. Furthermore, there are some points where the function \( J(\theta) \) is not analytical. This means that there is no guarantee that the inverse solver can find the global optimum (\( \theta^* \)) especially in the presence of noise. One alternative, which is adopted in this paper, is to solve the inverse problem \( N_{\text{inverse}} \) times
with different initial solutions ($\theta_{ni}$) for a given set of measurements $M$, which leads to $N_{\text{inverse}}$ estimated values for the dipole source. Among them, the one with the minimum $J$ is selected as the localized dipole. The effect of $N_{\text{inverse}}$ is investigated in Section 3.6.

### 3.3.2 Upper-Level Optimization Method

Since the landscape of the $g(X)$ in unknown and derivatives of $g(X)$ are not explicitly available, a metaheuristic which may handle multimodality, correlation and especially noise in the fitness function is preferred. A variant of covariance matrix adaptation evolution strategy (CMA-ES) (Hansen and Ostermeier 2001; Hansen 2009) is employed to perform optimization at the upper level considering its promising results on BBOB2009 optimization workshop on noisy test problems (Auger et al. 2010). CMA-ES belongs to the category of evolution strategies that adapt the full covariance matrix, which makes it an efficient method for handling correlation among design variables. In CMA-ES, offspring are sampled from multivariate normal distribution, which is specified by the covariance matrix and the global step size. The best offspring are recombined and form the parent for the next generation. The covariance matrix and the global step size are updated as well.

In comparison with other meta-heuristics, CMA-ES requires a little tuning effort. The only problem-dependent control parameter is the population size. To avoid ad-hoc tuning effort, recent variants of CMA-ES (Auger and Hansen 2005; Hansen 2009) perform independent restarts. Starting from a small default value, the population size is increased in successive runs. On-the-fly population sizing was proposed in another study (Ahrari and Shariat-Panahi 2013), which enhanced efficiency and robustness of the original CMA-ES; however, it needs a parameter to be tuned as well. A dynamic update of population size is preferred in this study, which will be discussed in Section 3.6. The pseudo code of the proposed bi-level algorithm is provided in the Appendix.

### 3.4 Parameter Study

To use the proposed bi-level algorithm, one should decide on the values of $\zeta$ in the fitness function, the evaluation budget for one run of the inverse solver ($\text{MaxJeval}$), the maximum number of calls of $J(\theta)$ in one line search in the inverses solver ($N_J$), the number of dipoles over which $g(X)$ is evaluated ($N_{\text{dipole}}$) and the number of times the inverse problem is solved for a given dipole and a set of sensor measurements with different
initial solutions ($N_{inverse}$). The goal is to find the set of these parameters such that the identification accuracy and SRI (Equation (11)) are maximized, while the computation cost is minimized. For this purpose, 100 random designs ($X_1, X_2, ..., X_{100}$) are generated. Since there are many parameters, we study a few at a time while the rest are set to some default values unless mentioned otherwise. The default values, obtained using a preliminary parameter study, are as follows: $N_{dipole}=1,024, N_{inverse}=1, N_{inverse}=1$ and $\zeta=1$.

To demonstrate flexibility and robustness of the proposed optimization method, and to analyze the effect of the amount of uncertainty on the optimized design, three distinct cases are considered for the rest of this study:

- **Case I**: Ideal case with $\varepsilon_{model}=\varepsilon_{sensor}=0$.
- **Case II**: Low uncertainty with $\varepsilon_{model}=0.01$ and $\varepsilon_{sensor}=0.0015$ cm/s, and
- **Case III**: High uncertainty with $\varepsilon_{model}=0.20$ and $\varepsilon_{sensor}=0.0015$ cm/s.

Parameter study and optimization results are investigated for these 3 cases. Increasing $N_{inverse}$ predictably increases the identification accuracy as demonstrated in (Ahrari et al. 2015); however, the computation time grows proportionally. One interesting finding in (Ahrari et al. 2015) was that the effect of increasing $N_{inverse}$ is almost similar for all designs. This means that if $g(X_1) > g(X_2)$ for $N_{inverse}=1$, the probability that $g(X_1) < g(X_2)$ for other values of $N_{inverse}$ is small, i.e., the rank of solutions does not change if $N_{inverse}$ is changed. Since selection in CMA-ES is performed according to the rank of solutions, the optimization can be performed with $N_{inverse}=1$ to minimize the computational cost. The obtained optimum design is then used with $N_{inverse}>1$ in practice to increase the identification accuracy.

### 3.4.1 Effect of maxJeval and $N_J$

Parameters $\text{maxJeval}$ and $N_J$ control the inverse solver by limiting the overall computation budget and computation budget per line search step respectively. Improper selection of these parameters may lead to premature interruption of the inverse solver or excessive but unnecessary computation. To investigate the effect of $\text{maxJeval}$ and $N_J$ on the identification accuracy, we evaluate the randomly generated designs using different values for these parameters. Figure 4 illustrates the mean fitness of 100 random designs for Cases I, II and III. Note that the actual computation (usedJeval) can be smaller than $\text{maxJeval}$, because the inverse solver employs some statistical criteria to abort whenever
it is predicted that further search cannot result in significant change in the solution.

Figure 4. Average fitness of 100 random designs for different values of $maxJeval$ and $N_J$ in a) case I, b) case II, and c) case III. d) Average $usedJeval$ for different values of $maxJeval$ for different cases with $N_{dipole}=1,024$.

As it can be observed:

- The average fitness increases when $maxJeval$ is increased. This effect is spectacular in case I, significant in case II, but insignificant in case III.
- For case I and case II, increasing $N_J$ up to a certain point (about $N_J = 15$) improves the fitness of the designs. After that, increasing $N_J$ has almost no significant effect.
- A huge decline in the average fitness of the design is observed when the amount of uncertainty increases. For instance, the average fitness is much higher in case II (low uncertainty) than case III (high uncertainty).
• The growth of \textit{usedJeval} with \textit{maxJeval} is slower than linear. Since computation time is proportional to \textit{usedJeval}, the trade-off between \textit{maxJeval} and identification accuracy suggests not sacrificing the fitness for a smaller \textit{maxJeval}.

Based on the result from this section, we set \textit{maxJeval}=1,600 and \textit{N_J}=30 for the rest of this study.

3.4.2 Effects of \textit{N_dipole} and \textit{ζ}

Increasing \textit{N_dipole} provides a better representation of all possible situations of the dipole source, and at the same time, reduces the effect of random nature of the fitness function. This means that a greater value of \textit{N_dipole} provides a better estimation of the true fitness function. This reduces \( s_g \) (Equation (11)) which increases SRI. The drawback is that it increases the computation cost proportionally. Assuming that the distribution of independent realization of \( g(X) \) is normal, \( s_g \) is inversely proportional to the square root of \( N_dipole \), which matches the simulation results in (Ahrari et al. 2015), and the desirable accuracy in computation of \( g(X) \) can be achieved if \( N_dipole \) is sufficiently large. A dynamic scheme where \( N_dipole \) is increased with the iteration number might help, as it will be discussed later.

![Figure 5. Effect of \( ζ \) on SRI.](image)

Parameter \( ζ \) also affects the selection noise; a too small or a too large value might reduce variation of fitness among different solutions, however, practical requirements, e.g. the demanded identification accuracy, should also be considered as well while setting this parameter. For example, such requirements may specify how much credit should be given to identification error of 0.01, 0.1 or 0.5, from which the appropriate value of \( ζ \) is determined. To examine the effect of \( ζ \) on SRI, 100 randomly generated designs are
evaluated 20 times. For each design, the mean and standard deviation of the fitness function ($\mu_g$ and $s_g$) are computed. SRI is then computed for different values of $\zeta$ for each case, which is plotted in Figure 5. According to this plot, the optimum value of $\zeta$, considering the SRI metric, is smaller for a case with larger uncertainty. This is intuitive, since when uncertainty is high, our expectation of the identification accuracy should be less. Based on the results from this section, we set $\zeta=0.1$ for the rest of this study.

3.4.3 Population size

In this study, problem-specific population sizing for CMA-ES is adopted in which the population size is reduced dynamically while $N_{\text{dipole}}$ is increased. The justification is that in early iterations, a large population size enhances exploration of the algorithm; however, a small value for $N_{\text{dipole}}$ keeps the computational time in a reasonable range. While this strategy is useful for the initial stage of the algorithm, due to the presence of high uncertainty in $g(X)$, the progress rate of the search algorithm towards the global minimum becomes zero before the time the population converges to the exact location of the minimum (Beyer 2006). To address this problem, $N_{\text{dipole}}$ is dynamically increased to reduce the standard deviation of $g(X)$. At the same time, the population size is reduced. The evaluation budget of the whole optimization process is specified in terms of the number of times the inverse problem is solved ($\text{MaxInvSolve}$). When one-third of the evaluation budget is consumed, the population size is halved while $N_{\text{dipole}}$ is doubled. This is performed again when two-third of the evaluation budget is consumed. Based on a preliminary parameter study, we set the initial population size and $\text{MaxInvSolve}$ to 180 and $2 \times 10^6$, respectively.

4 Numerical Results

4.1 Estimation of uncertainties

To have a rough estimation of the values of $\hat{\epsilon}_{\text{sensor}}$ and $\hat{\epsilon}_{\text{model}}$, a CDF simulation and an experiment is performed in this section.

4.1.1 Estimation of sensor uncertainty

One can use various types of flow sensors for an ALL. Here we illustrate how to
characterize the flow sensor uncertainty with the example of an ionic polymer-metal composite (IPMC) sensor. Figure 6(a) shows the experimental setup, where an IPMC sensor (1 cm long) encapsulated with parylene (Lei and Tan 2014) was placed deep enough under the water surface. An aluminium ball with diameter of 1.9 cm was fixed to a metal stick which was excited by a mini-shaker (Type 4810, Brüel & Kjær) with a frequency of 5 Hz and varying amplitudes. The flow velocity field generated by this vibrating sphere was estimated by the aforementioned potential flow model, and the output signal (short-circuit current) of the IPMC sensor subjected to the flow was conditioned through an amplifying circuit and collected by a dSPACE data acquisition system (RTI 1104, dSPACE). One laser displacement sensors (OADM 20I6441/S14F, Baumer Electric) was mounted against the metal stick, measuring vibration amplitude of the dipole.

The experiments were conducted under different vibration amplitudes and at different times while the sensor position remained unchanged. For any configuration, sensor signals were recorded five times with intervals of 12 hours. Accordingly, for a fixed configuration, the difference between sensor measurements is solely because of the sensor uncertainty. Figure 6(b) plots the theoretical flow velocity versus the sensor output signal (I). The mean sensor measurement should be zero when there is no vibration and the relation between the current and the flow velocity is nonlinear. Based on the plot, a power curve passing through the origin is suggested, which is then utilized to calibrate the sensor as well. According to the obtained data and the fitted line, $\varepsilon_{\text{sensor}} = 0.0015 \text{ cm/s}$, which is used for the sensor uncertainty through the rest of this study.

\[ y = 0.1406x^{0.4905} \]

\[ R^2 = 0.9382 \]
Figure 6 a) Experimental setup for determination of sensor uncertainty. b) Computed flow velocity using the potential flow model ($V_{pot}$) versus the output current of the sensor (I) and the fitted curve.

4.1.2 Estimation of the flow model uncertainty

While it is impossible to capture the error between the actual flow and the flow predicted by the potential flow model in all situations, CFD computation was used as a surrogate of “ground truth” to quantify the error of the theoretical model. The CFD simulation was conducted using COMSOL Multiphysics (version 4.4) and FSI license (Fluid Structural Interaction). In the simulation setup, the tank (40×40 cm) is filled with still water (Figure 7(a)). An aluminium sphere (radius=1 cm) is placed in the middle of the tank, simulating a dipole. The sphere vibrates with frequency of 3 Hz and amplitude of 0.7 cm to simulate the mini-shaker in the experimental setup. After running the simulation for 2 seconds, velocity components at different locations of the tank are computed using FFT function. The computed velocity from the CFD model ($v_{cfd}$) was compared to the velocity from the potential flow model ($v_{pot}$), and relative error ($||v_{cfd} - v_{pot}||/||v_{pot}||$) was calculated. Figure 7(b) illustrates contours of the relative error in the tank. The average error is observed to be 18%.

Figure 7 a) CFD setup b) contours of relative error
Figure 8. Mean of the final designs from 10 independent runs for Cases I, II and III when dipoles are a) on the right b) on the left c) on the top and bottom of the ALL. The units of both horizontal and vertical axes are cm.

4.2 Case Studies

Three simulation studies are performed to analyze the effectiveness of the proposed optimization tool in these specific cases, where the final solution is checked with engineering intuition. For this purpose, the optimization algorithm is run for Cases I, II and III, while the dipoles generated by the algorithm mentioned in Section 3.2 are relocated so that they lie only on the left (mode a), right (mode b), or top and bottom (mode c) of the ALL. For these trials, $N_{\text{sensor}}=12$ is used. Figure 8 illustrates arithmetic mean of the final designs in 10 independent runs for each case.

- For Cases IIa and IIIa, most sensors have moved to the right side of the fish body to maximize the average number of sensors that can receive a signal from the dipoles. In contrast, for Cases IIb and IIIb, most sensors have moved to the left side for the same reason.
- The final solutions for Cases IIc and IIIc are totally different. The sensors are quite uniformly distributed on the ALL and besides, the ALL is highly stretched, probably in order to maximize the spread of sensors so that most of them can still receive a signal from the dipoles on the same side.
- The optimized design of Case I has comparatively a smaller size and does not show any significant variation with respect to the locations of the dipoles. This can be due to the fact that since there are four unknown parameters in the inverse
problem, only measurements from four sensors are required to solve the inverse problem. If the measurements are completely accurate, neither extra sensor measurements nor diversity in location of the sensors may provide more information on parameters of the dipole. One justification for the small size of the ALL might be maximization of the number of sensors that can receive a signal so that for all dipoles at least four sensors can provide a signal.

These results demonstrate that in all cases, there is good agreement between the final designs found by the optimization method and the one predicted by the engineering intuition. The optimal designs strongly depend on the locations of the dipoles. More importantly, the optimization method can reliably find the optimal design parameters with respect to the defined objective function for each case, although it turns harder when uncertainty, and thus the selection noise, exacerbates.

4.3 Bi-objective Optimization

In this section, it is assumed that dipoles may lie anywhere in the 20×20 cm tank, the ultimate case within the scope of this study. Furthermore, two constraints are imposed by practical requirements. First, the lower limit of $X_2 = \lambda / R$ is increased to 0.1 to prevent very flat designs that are not practical. Second, in contrast to the setting so far which excludes the dipoles that lie inside or very close to the fish body, we give zero credit for such dipoles. This implicitly penalizes a larger ALL, since it get more zero credits which reduces the fitness. This counteracts the undesirable advantage of larger ALLs which occupy a larger fraction of the search space and leave a small region for dipoles to lie in.

Predictably, increasing $N_{\text{sensor}}$ can increase the fitness of the final design, since it provides more data on the dipole; however, fabrication cost and integration complexity increase with the number of sensors. The trade-off between $N_{\text{sensor}}$ and $g(X)$ can be analyzed by performing a bi-objective optimization in which the second objective is minimization of $N_{\text{sensor}}$. Practically, a limited options for the number of sensors are worth considering, e.g., $N_{\text{sensor}} \leq 30$ and $N_{\text{sensor}}$ is an even number. This motivates running the single objective optimization problem with different values of $N_{\text{sensor}}$ instead of running a bi-objective optimization. For each $N_{\text{sensor}}$, the optimization problem is solved 10 times independently and the final solutions from the optimization runs are reevaluated again with $N_{\text{dipole}} \approx 10,000$ for different values of $N_{\text{inverse}}$. Figure 9 plots the obtained Pareto front for different values of $N_{\text{inverse}}$ for each case and some selected values of $N_{\text{sensor}}$. Figure 10
depicts 5 designs representing the distribution of the 10 final solutions. The parameters of these designs are \( p \)-th percentile of the corresponding parameters in the final solutions. For example, the parameters of 50\%-percentile design are the median of the corresponding parameters in the final solutions. This may show whether the algorithm has converged to similar solutions. Average values of the size variable \((X_1=R)\) and shape variable \((X_2=\lambda/R)\) in the final solutions are illustrated in Figure 11. Based on the obtained results, it can be concluded that:

- Figure 9 demonstrates that increasing \(N_{\text{sensor}}\) always improves the fitness, except for Case I after \(N_{\text{sensor}}=8\), and thus a knee in the Pareto front is observed. This knee emerges at \(N_{\text{sensor}}=12\) for Case II. For Case III, in contrast, no knee can be detected and the slope of the curve remains quite high everywhere. This implies that when the amount of uncertainty increases, the contribution of extra sensors becomes more significant.

- In Case I when \(N_{\text{sensor}}\geq8\) (Figure 9), the fitness of the optimized design is close to 1 when \(N_{\text{inverse}}\) is great. This means that the identification error for any arbitrary dipole is almost zero. Note that zero credit is assigned if a dipole is located inside or very close to the fish body, therefore, it is impossible to reach the fitness of one.

- Increasing \(N_{\text{inverse}}\) (Figure 9) has a significant positive effect on the identification accuracy. In practice, the inverse solver should be run multiple (e.g., greater than 4) times to maximize identification accuracy. This effect can be observed for all the cases.

- The similarity of the final designs for independent runs for a fixed \(N_{\text{sensor}}\) (Figure 10) confirms the reliability of the optimization results. Case II with \(N_{\text{sensor}}=16\) is an exception, in which a few runs have converged to a circular ALL. It seems that for this case there are two (near-) global minima with almost identical fitness.

- Figure 11 demonstrates that the optimal shape changes from circular with small \(R\) to flat with a great \(R\) as the amount of uncertainty increases. For Case III, the shape variable \((X_2=\lambda/r)\) has reached the lower limit while \(R\) has reached its upper limit.
Figure 9 Trade-off between fitness and $N_{sensor}$. a) Case I, b) Case II and c) Case III.
Figure 10. Representation of final solutions for Case II and Case III. a) $N_{\text{sensor}}=8$, b) $N_{\text{sensor}}=16$ and c) $N_{\text{sensor}}=24$. Parameters of each illustrated solution are $p$-th percentile of the corresponding parameters of the final solutions from 10 independent runs.

![Figure 10](image)

4.4 Importance of Uncertainty Amount

Optimized designs from each case are reevaluated in other cases to investigate fitness degradation caused by ignoring the effect of the amount of uncertainty in the optimization process. Figure 12 illustrates the average fitness of the optimized designs from different cases, reevaluated for conditions of all cases.

![Figure 12](image)

Figure 2. Average fitness of the optimized designs in different cases reevaluated ($N_{\text{inverse}}=4$) for a) Case I b) Case II and c) Case III.
According to Figure 12, the optimized solution of each case is the optimal one for that case only. For example, if the conditions of Case III are applied in practice, solution from Case III are the best choice, solutions of Case II have lower fitness, and solutions of Case I are far behind the other two cases. Similar conclusions can be drawn for other situations, with a small exception: if the condition of Case II is applied in practice, solutions from Cases II and III are almost equally fitted if $N_{\text{sensor}} \geq 12$, although their design parameters significantly differ.

5 Conclusions

Optimal design of an artificial lateral line (ALL) is a challenging task and it demands specialized algorithms to handle different sources of uncertainty. In this research, two different sources of uncertainty have been considered in solving the inverse problem of localizing a dipole source based on flow measurements at multiple sensor sites. A robust parametric fitness function has been proposed so that the uncertainty in fitness evaluation is made minimal. Thereafter, a bi-level optimization method has been proposed and employed to optimize parameters of the ALL including shape, size of the lateral line, and placement of the sensors over the body. Several numerical studies have been performed to investigate the effect of the parameters of the algorithm. The recommended parameter settings are then derived from the results of these studies.

To show the flexibility of the method and effect of uncertainty on the optimized design, three cases with different magnitudes of uncertainties have been considered. The developed optimization method has been first validated by a sensitivity analysis in which dipoles are placed at particular regions of the search space, so that an intuitive prediction of placement of sensors can be obtained. The lateral line has been subsequently optimized for different numbers of sensors to monitor trade-off between the number of sensors and the accuracy of identification. A comparison among the results of different such cases demonstrated that the amount of uncertainty not only significantly influences identification accuracy but also varies the optimal ALL. Increasing the number of sensors monotonically and predictably boosts the identification accuracy; however, the gain after a certain point, called the knee, turns insignificant. The recommended value of the number of sensors, based on the trade-off between the fitness and the number of sensors, increases as the amount of uncertainty increases, and for the case with the largest amount of
uncertainty, no knee-like point could be observed for the tested range of the number of sensors.

Dependency of the optimized design on the number of sensors and the amount of uncertainty highlights the importance of consideration of these factors in the design process, which, in most cases, can hardly be rendered by engineering intuition. It was also observed that the final solutions from each case can be the optimal ones only for that case, and significant fitness degradation is observed if these solutions are used in other conditions. Future research in the domain of this study includes extension of the proposed optimization method for the case in which the ALL should localize a dipole in 3D space, or when the dipole is moving, instead of vibrating. We also plan to prototype the obtained optimal design and conduct experiments to validate the numerical results.

7 Acknowledgment

Computational work in support of this research was performed at Michigan State University’s High Performance Computing (HPCC) Facility. This material is based in part upon work supported by the National Science Foundation under Cooperative Agreement No. DBI-0939454 and by the Office of Naval Research (N000141210149, N000141502246). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation or the Office of Naval Research.

Appendix

Algorithm 1. Evaluation of a design

Input: \(N_{\text{dipole}} \) dipoles, design \( X \)

Output: \( g(X) \)

For \( i=1:N_{\text{dipole}} \)

For a sensor that can receive a signal from the dipole, simulate sensor measurement for the \( i \)-th dipole \( (\theta_i) \) as follows: \( M_k(\theta_i) = \left| f_k(\theta_i) \times \exp(\epsilon_{\text{model}}N(0,1)) + \epsilon_{\text{sensor}}N(0,1) \right|, k = 1, 2, ..., N_{\text{sensor}} \)

Solve the inverse problem to localize this dipole \( (\theta_i^*) \).

Find the localization error: \( e = ||\theta_i^* - \theta_i|| \)

End

Compute fitness of the design, \( g(X) \) based on vector of localization errors, \( e \).
Algorithm 2: The proposed bi-level optimization tool

**Input:** Population size, $N_{\text{dipole}}$, $\text{MaxInvSolve}$, parameters of CMA-ES

**Output:** Optimized design

**Initialization:**

$\text{UsedInvSolve} \leftarrow 0$

While $\text{UsedInvSolve} < \text{MaxInvSolve}$

For $k = 1$ to $\lambda$

Generate $X_k$

Let $P$ be sum of squared constraint violation according to (9)

If $P = 0$

Calculate $g(X)$ using Algorithm 1

$\text{UsedInvSolve} = \text{UsedInvSolve} + N_{\text{dipole}}$

Else

$g(X) \leftarrow -10 \times (P + 1)$

End If

End For

Update parameters of CMA-ES

Update $\lambda$ and $N_{\text{dipole}}$ as explained in Section 6.2

End While

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**References**


