The Effect of Extreme Solutions Injection in Opposition-based Evolutionary Multi-objective Optimization

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ABSTRACT

For more than two decades, stand-alone evolutionary multi-objective optimization (EMO) methods have been adequately demonstrated to find a set of trade-off solutions near Pareto-front for various multi-objective optimization problems. Despite a long-standing existence of classical generative single-objective based methods, a very few EMO studies have combined the two approaches for a better gain. In this paper, we investigate the effect of seeding the initial population of an EMO algorithm with extreme solutions obtained using a single-objective method. Our proposed approach is further aided with an opposition based offspring creation mechanism which strategically places new solutions on the current Pareto frontier by a simple, yet a novel arbitration policy that utilizes the relative distances from the extreme solutions in the current population members. We conduct an extensive simulation of our proposed approach on a wide variety of two and three-objective benchmark MOP test problems. Results are shown to be remarkably better than the original EMO approach in terms of hyper-volume metric. The results are interesting and should motivate EMO researchers to integrate single-objective focused optimization and an opposition-based concept with diversity-preserving EMO procedures for an overall better performance.

Keywords

Single objective Search, Multiobjective Optimization, Opposition Based Learning (OBL)

1. INTRODUCTION

In the recent years, the idea of Opposition Based Learning (OBL) has been enjoying a noticeable attention among the AI and OR practitioners. The idea of OBL is to accelerate the learning rate (or convergence rate) by imposing an opposite estimate of the current solution, and deliberately introducing them to influence the search trajectory. This idea was first introduced in [10] and has been demonstrated its effectiveness in different scenarios – from Reinforcement Learning (RL) [11], Differential Evolution (DE) [8] to robotics [6].

In all of the cases, the OBL comes first into play during the initialization phase of the learning algorithm (or optimization algorithm), the argument behind this strategy is that a completely arbitrary (i.e. blind) initialization is no better than an informed boot-strap. Since a random sampling or selection of solutions from a given population has the possibility of visiting or even revisiting unpromising regions of the search space. In [8], it has also been demonstrated that the chance of such revisits are lower in the case of opposite than it is for purely random initialization. There are also several formal proofs that show that, in general, the opposite estimations are more likely to be closer to the optimal solution than the completely arbitrary ones, a more details of a formal probabilistic analysis of opposition based learning can also be found in [5].

Moreover, especially for the case of optimization algorithms, it has been demonstrated that keeping a small ratio of candidate solutions with the opposite estimate help to converge faster [8] [12] – and most of the recent studies are inspired by this approach. For example, in [7] similar strategy has been used in the case of MOEA/D [16]. Another relevant study can be found in [13], where the opposition based initialization has been tested with the Particle Swarm Optimization (PSO) scenario to induce a better convergence. Given all of these studies, still we could not find a good example of this idea applied to the Evolutionary Multi-objective Optimization (EMO) cases – more importantly – in a more meaningful and precise way. In this study we will try to fill this gap in an interesting, and more importantly – in a simpler way.

In our approach, we will address this idea of opposition in a different perspective, we will classify the solutions with respect to some desired traits that we will like to have. Then we will use this information to deterministically generate new solutions in a most viable locations in the search space – and this operation will be dictated by our special notion of opposition. In fact, our approach is somewhat similar in spirit to the previous studies done in [9] and [17]. Except the fact that our approach is extremely simple and does not assume any special property on the underlying search space, moreover, our approach does not build a computationally expensive models as done in [1].

This paper is organized as follows – first, we will discuss

1 not to be confused with the term “classification” as in the machine learning domain

2 Here, our method is not “deterministic” in a general sense, the stochasticity incurred by the candidate generation is not equivalent to the generic mutation operation.
why this idea of OBL needs to be reinterpreted for the Multi-objec-
tive Optimization (MOP) setting. Next we discuss one inter-
esting limitation that most EMO algorithms suffer from – the search trajectory bias, and also give some argument on why such hindrance becomes inevitable. Then we will dis-
cuss how the idea of OBL can come into rescue. In our model, we generate the opposite points in a strictly deter-
ministic way, by carrying out the arbitration of opposition in a different (and a more MOP relevant manner). To do this, we utilize the extreme solutions on the true Pareto-front (PF), and the next section discusses how to find them effi-
ciently. After that we will describe our main algorithm in details. Then we will show our experiments with different benchmark MOP problem sets.

2. AN ALTERNATIVE INTERPRETATION OF OPPOSITION

As we have already discussed in the previous section, in most of the studies, the idea of opposition is employed as an incorporation of new candidate solutions with a certain kind of opposite traits into the existing population. Such traits could be interpreted in terms of different problem domain perspectives. For example, the opposite solutions could be – i) the ones with complete opposite representation from the current best individual, ii) the ones with the opposite estimates from the other spectrum of the variable bounds (i.e. in the case of real valued optimization). However, the injection of the opposite solutions could cause a re-route from the continuing search trajectory and thus could be a misleading operation – in a sense that the opposite candidate solutions might be useful given that the search space follows a desired pattern.

For example, if the task is to solve the N-queen problem, then the opposite representation of the current best can result into another valid global optima. One can easily verify this by computing the reverse assignment of queens from an existing optimal solution; and it will eventually take us to another global optima. This observation also demonstrates that the search space is multi-modal, following from the fact that the reverse representation of the current best solution needs to be an optimum for another peak of the search space. Therefore, most of the standard opposition based algorithms inject the opposite solutions during the optimization start-up; or maintain a constant (generally low) ratio of opposite points throughout the run. Therefore, the standard opposite injection scheme (most likely) to be effective given the two assumptions on the underlying search space are valid – multi-modality and the solution symmetry. For this reason, what we think – such approach is quite unwieldy to directly incorporate into a numerical optimization problems, e.g. multi-objective optimization problems (MOPs).

Moreover, in the case of black-box optimization scenarios, we hardly have a room to make any assumption about the multi-modality of the search space and/or the symmetry of the solution representation (i.e. unlike such assumptions could be made about the N-queen problem). Therefore, in this paper, we have revised the notion of opposition in terms of the preference criteria imposed on a solution. For example, most EMO algorithms aim to maximize two principal properties – i) the convergence and ii) the diversity, since the quality of a MOP solution depends on these two factors. Therefore, let us re-consider the opposite point gen-
eration/injection from a different perspective –

- **Opposite Convergence**: A solution far from the true Pareto-front is opposite to any solution that is closer to the true Pareto-front.

- **Opposite Diversity**: An isolated solution on the true Pareto-front is opposite to a crowded solution.

By taking the above two principles into account, we will deterministically generate opposite candidate solutions during the search. Obviously, the deterministic candidate generation scheme will only consider an opposite trait that is good. In the next section we will see, how the existing EMO algorithms shows the limitations in maintaining this two opposite traits during the search (i.e., solution generation) process.

3. LIMITATIONS OF CANONICAL EMOs: THE SEARCH TRAJECTORY BIAS

Most of the standard EMO algorithms (e.g. NSGA-II, SPEA-II [19] etc.), are elitist by design. They are also “op-
portunistic” in a sense that the population always try to converge to a particular portion of the Pareto-front (PF) which seems to be easier to solve at that moment. Therefore, they show preferences over a certain objective function which needs less exploration than the other. We can see such bias in the search when we try to solve the ZDT4 problem using NSGA-II. In this case, the first objective is easier to optimize than the second one, the readers can verify this fact from the Figure 1. Therefore, the search trajectory deliberately accumulates more points over the first objective to optimize one particular portion of the Pareto-front. Moreover, while putting more solutions to the vicinity of one particular objective axis, the search trajectory looses the uniformity by forming a crowded streak of points along that axis; on the other hand we can see that there is almost no solution on the other spectrum of the objective space. This kind of non-symmetric search behaviour is, we think, causes a hindrance to the optimization procedure. Therefore, it would be helpful if we could selectively inject points

![search trajectory bias](image)

Figure 1: The effect of the search trajectory bias could be seen when we try to solve ZDT4 problem with NSGA-II (at generation 46, population size 100). Here we can see a long streak of crowded solution near the objective $f_2$, where the distribution of solutions near the objective $f_1$ is extremely sparse.
during the search where the solution distribution is more sparse. In addition, we also believe that this biased nature of the search trajectory could degrade with the addition of more objective functions. Moreover, this could also lead to a stagnation on the local optima, given that the search space has many of them.

Given this specific scenario, now the main problem is to devise a way to deterministically generate points where the distribution of the solution is sparse. Here we assume that the lesser the number of candidate solutions in a vicinity of the optimal solution distribution is sparse. Here we assume that we can know the exact mapping of the design variable to objective values – however such mapping is always unavailable and above all it is very expensive to infer. Another way could be to mutate the points where the solution is more sparse, but we think as the original algorithm already goes through such step, it is not going to be very effective.

In this paper, we are going to demonstrate a very effective approach to address the above mentioned issue, we will show how we can maintain a balanced distribution of solutions that is parallel to the true PF. The proposed approach is also extremely effective in fast converging to the true PF as well.

4. THE DETERMINISTIC OPPOSITE SOLUTION GENERATION SCHEME

As we have discussed in the Section 2, we will utilize the so-called notion of opposition to deterministically generate points into strategically useful places on the search space. In principle, we do not assume any exact mapping over the design variables to objective values, and we apply a linear translation to achieve our goal effectively\(^3\). In essence, instead of considering sparsity and crowd as two opposite traits, our method can impose any criteria like being ‘x’ and being ‘y’ as two opposites. Being said that, linear translation is a simplest way to deterministically generate ‘x’ like solutions from ‘y’ like ones.

So, the initial step is to infer the true PF before starting the actual optimization run. To do this, we depend on \(k\) extreme (or near extreme) points. However, for the DTLZ [4] problem set, these settings are found to be more useful:

- InitialMeshSize: 1/population size
- Search Method: \texttt{MADSPositiveBasis2N} to start searching with \(2N\) random directions, where \(N\) = number of variables.
- and keeping both \texttt{CompletePoll} and \texttt{CompleteSearch} to on

The actual extreme point computation algorithm was conducted in two steps – given a particular objective function.

\(^3\)As a matter of fact, we have also noticed that such opposite points are generally \(\sim 30\%\) useful in most cases and they are more effective during the initial generations – which is also a very interesting finding.

\(^4\)The readers might be aware that efficiently finding the extreme points on the true Pareto-front is itself a separate research problem [3].

\(^5\)We have used the \texttt{fmincon()} and the \texttt{patternsearch()} routine in MATLAB (v. R2014a) for IP method and MADS respectively.

\begin{algorithm}[h]
\caption{Find Extreme Points}
\label{alg:extreme_points}
\begin{algorithmic}[1]
\State \(k \leftarrow \text{no. of objectives}\)
\State \(N_p \leftarrow \text{population size}\)
\State \(N_{gen} \leftarrow \text{maximum generation}\)
\State \(T \leftarrow \frac{1}{10}N_pN_{gen}\)
\State \(E^* \leftarrow \emptyset\), an empty solution set
\For{\(i\) from 1 to \(k\)}
\State \(f_i \leftarrow \text{-th objective function}\)
\State \(x_i \leftarrow \text{random initial vector}\)
\Repeat
\State \(x_i \leftarrow \text{solve } f_i \text{ with IP method (or MADS)}\)
\Un\ Until \(\frac{\pi}{4}\text{ solution evaluation reached}\)
\State \(f_{aasf} \leftarrow \text{construct AASF function from } f_i\)
\Repeat
\State \(x_i \leftarrow \text{solve } f_{aasf} \text{ with IP method (or MADS)}\)
\Until \(\frac{\pi}{4}\text{ solution evaluation reached}\)
\State \(E^* \leftarrow \{E^* \cup x_i\}\)
\EndFor
\State \text{return } E^*
\end{algorithmic}
\end{algorithm}
$f_i$, first we try to solve it directly using either IPM or MADS (depending on the problem type); then after some $\frac{T}{2}$ iterations once we find a reference solution $z$ that is hopefully close to $f_i$’s optima, then we construct a so-called Augmented Achievement Scalarizing Function (AASF) \cite{15} from $f_i$ as $f_{aasf} = \max_{i=1}^k w_i (f_i(x) - z_i) + \rho \sum_{j=1}^k w_j (f_j(x) - z_j)$ and solve it again for $\frac{T}{2}$ iterations. Here, we set $w_i = 0.9$, $w_{j\neq i} = \frac{1}{10-\epsilon}$, and $\rho = 0.0001$.

To limit the solution evaluations, we kept $T$ to a constant value (as a budget). For all problems, we have fixed this maximum iteration count to the $\frac{1}{4}$-th of the total generation specified. More precisely, $T = \frac{1}{4} (\frac{1}{4} N_p N_{gen})$, where $k = \text{no. of objectives}$, $N_p = \text{population size}$, and $N_{gen} = \text{maximum generation}$. A basic listing for this routine is presented in Algorithm 1. The set of the extreme points $E^\star$ generated from this algorithm may not contain all the unique solution, and also they might not be the true extreme always, they can be weakly dominated solutions by the true PF extremes. However, our approach can utilize them efficiently to converge to the true PF extremes.

### 4.2 Generating the Opposite Solutions

Once the extreme points are discovered, now we utilize them to generate the so-called opposite points during the main evolutionary runs. On each generation, we select 25\% of the best individuals (front-wise) from the current population and deterministically change them to generate opposite solutions – in such a way that we can address the strategically preferable places. And to conduct this variation, we will utilize the points in the set $E^\star$ as pivot points. We call these points as “pivot” since we will selectively try to generate points around these pivots. However, before doing this, we will refine our pivot points $E^\star$ in a certain way.

The refinement starts by finding the current population extreme points $E_c$ and merging them with the set $E^\star$ such that $E = \{E_c \cup E^\star\}$. Next we apply the non-dominated sort on $E$ to find the Pareto-front within this set. We apply this sorting because we are not still sure if $E^\star$ contains true PF extremes. This sorting will keep the true extreme points if ones are found in the later generations. After this step, we select the points from $E$ that are on the best front and with $\infty$ crowding distances\footnote{By “crowding distance”, we mean the inter-solution distances computed in NSGA-II.}. Lets denote these selected points as $E'$. Now at this point, two situations are possible:

- The set $E'$ contains only the solutions $E^\star$ while we are in the initial generations, or
- The set $E'$ contains the solutions $E_c$ while we are in the later phase of the generations, where $E_c$ are the true PF extreme.

However, during the intermediate generations, it can also happen that we may include some solutions into $E$ that weakly dominate a subset of points already in $E$, this inclusion will reduce the expected spread of the pivot points – that may diminish the effect of maintaining the diversity. For example, if the actual true PF is a broken Pareto-front, and if $E$ contains the extreme points from one broken edge, then we need to expand the current edges so that the refinement procedure can include points from the further extreme ends. Therefore, if there exist a point in $E - E'$ that is on the best front and also weakly dominated by any point in $E'$, then we replace the weakly dominating point from $E'$ with the one from the set $E - E'$. The readers might have already noticed that $|E'| \leq |E|$.

Now, at this point, we can ensure that the set $E'$ contains either true PF extremes or points near them. Now if we can generate new points near $E'$, they will induce both better convergence and diversity. In section 3, we have discussed a scenario where we can see how the bias in the search trajectory is introduced. However, the difference in the relative difficulty of the objective functions may not be the only reason for a bias. The imbalance in the solution distribution could happen for other reasons as well. For example, a disconnected Pareto-front, a local optimal front or a specific portion of the Pareto-front being more difficult to solve than the rest. In such cases, we can see a gap forming over the Pareto-front during the search, we can see such a convergence pattern in many problems. As an example, we can see similar effect in solving ZDT4 problem as illustrated in Figure 2. To address these gaps, we also find the solutions with $k$-highest ($k = \text{no. of objectives}$) crowding distance from the best front that are not $\infty$, and call them as set $G$. Clearly, the $G$ solutions are those that reside on the edge of the broken front. Now we add the $G$ to the set $E'$, thus we make $E' (|E'| > k)$ as the final “pivot” solutions to generate the opposite points (see Algorithm 2).

![Figure 2: The effect of the gap in the trade-off, could be seen when we try to solve ZDT4 problem using NSGA-II.](image)
4.3 Integrating into an Elitist EMO Algorithm: NSGA-II

As we have mentioned at the beginning that we select front-wise best 25% of the current population for opposite point generation. We go through each of them randomly and every time we pick \( k \) number of random points from \( E' \) and pick the pivot point that is the furthest from it, and find the opposite vector using a linear translation. A straight-forward way – given a pivot vector \( \mathbf{v} \) and a parent vector \( \mathbf{x}_p \), we generate an opposite child \( \mathbf{x}_c \) as \( \mathbf{x}_c = \mathbf{x}_p + \mathbf{U}([0, \frac{3d}{4}], \frac{3d}{4}) \circ \mathbf{U}(d, u) \). Here, \( d = ||\mathbf{v} - \mathbf{x}_p|| \) and \( \mathbf{U}(d, u) \) is a uniform random vector where each element is within the range \([d, u]\). The overall procedure is presented in Algorithm 3 in line 9–12 and illustrated in the Figure 3. The lines 9–12 in Algorithm 3 can be recapped as follows: \( \mathbf{v} \) is the true PF extreme and \( \mathbf{x}_i \) is far from \( \mathbf{v} \), therefore, move \( \mathbf{x}_i \) closer to \( \mathbf{v} \) – opposite of \( \mathbf{x}_i \) is close. Similar interpretation can be made when the vector \( \mathbf{v} \) is an intermediary gap.

Moreover, upon generating the vector \( \mathbf{x}_c \), it may happen that one of the variable values go beyond the variable bounds (i.e. \( x_i > x_{ij} \) or \( x_i < x_{ij} \)), in that case, we replace the overshoot value from the corresponding variable value \( v_j \) from the pivot point \( \mathbf{v} \). Therefore, if a certain vector \( \mathbf{x}_c \) can’t make a successful translation, then \( \mathbf{x}_c \) is reverted back to the vicinity of \( \mathbf{v} \). Thus, we assure a local best estimated translation of the parent vector \( \mathbf{x}_p \). This process is done on the line 13 of the Algorithm 3.

When we apply this algorithm to NSGA-II, we follow the obvious way, the generated opposite population will be inserted into the child population \( Q_t \), the Algorithm 3 also shows how to integrate everything in NSGA-II. Moreover, this algorithm is “pluggable” in a sense that we can integrate it to any other elitist EMO algorithm. In the following section, we are going to see in details, how our opposite generation algorithm drastically improves the convergence rate. The readers should be aware that the 25% allocation was found by empirical experiments, and also to be better for most of the test problems.

5. EXPERIMENTS AND RESULTS

First we have tested the performance of Algorithm 3 on five 2-objective problems [18], namely ZDT1, ZDT2, ZDT3, ZDT4 and ZDT6, and we have set NSGA-II as the control. To maintain a fair comparison, we have compensated the extra solution evaluations by the Algorithm 1 for the NSGA-II runs, and compared NSGA-II and Algorithm 3.

Algorithm 3 NSGA-II with Opposition

**Require:** true PF extreme points \( E^* \) from Algorithm 1
1: \( \mathcal{N} \leftarrow \) population size \( |P_t| \)
2: \( N_{gen} \leftarrow \) maximum generation
3: \( t \leftarrow 1 \)
4: while \( t \leq N_{gen} \) do
5: \( P'_t \leftarrow \) front-wise best 25% from \( P_t \) and shuffle
6: \( E'_t \leftarrow \) construct pivot set \( E' \) using algorithm 2
7: \( Q_t \leftarrow \emptyset \)
8: for each solution \( \mathbf{x}_i \in P'_t \) do
9: \( S \leftarrow \) pick \( k \) random solutions from \( E'_t \)
10: \( \mathbf{v} \in S \) such that \( \mathbf{v} \) is the furthest point from \( \mathbf{x}_i \)
11: \( d \leftarrow ||\mathbf{v} - \mathbf{x}_i|| \)
12: \( \mathbf{x}_e \leftarrow \mathbf{x}_i + \mathbf{U}([\frac{3d}{4}, \frac{3d}{4}], \frac{3d}{4}) \circ \mathbf{U}(d, u) \)
13: \( \mathbf{x}_j \leftarrow \mathbf{v} \) if \( x_j \in \mathbf{x}_e \) and \( x_j \in \mathbf{x}_c \)
14: \( O_t \leftarrow \{O_t \cup \mathbf{x}_c\} \)
15: end for
16: \( P_t \leftarrow \{P_t \cup E^*\} \)
17: \( R_t \leftarrow \{P_t \cup Q_t\} \)
18: rank \( R_t \) into fronts, \( R_t \rightarrow \{F_1, F_2, \ldots, F_n\} \)
19: \( P_{t+1} \leftarrow \emptyset \)
20: \( i \leftarrow 1 \)
21: while \( |P_{t+1}| + |F_i| \leq N \) do
22: assign crowding distances on the front \( F_i \)
23: \( P_{t+1} \leftarrow \{P_t \cup F_i\} \)
24: \( i \leftarrow i + 1 \)
25: end while
26: sort \( F_i \) in descending order using \( <_n \)
27: \( P_{t+1} \leftarrow \) the first \( N - |P_{t+1}| \) solutions from \( F_i \)
28: \( Q_{t+1} \leftarrow \) select, crossover and mutate \( P_{t+1} \)
29: randomly insert all \( x_i \in O_t \) into \( Q_{t+1} \)
30: \( t \leftarrow t + 1 \)
31: end while

Figure 3: The illustration of lines 9–12 in Algorithm 3. The right axes are the variable space and the left axes are the corresponding objective space. The point \( \mathbf{x}_c \) is the child (black circle) and \( \mathbf{x}_p \) is the parent (white circle). The point \( \mathbf{v} \) are the pivot points (grey circles). The operation will choose one of the directions denoted by \( L_1, L_2 \) or \( L_3 \). If \( \mathbf{x}_c \) violates the variable bound then it is reverted back to the vicinity of the corresponding pivot point \( \mathbf{v} \).
Figure 4: This figure illustrates how our algorithm deterministically identifies which front needs to be explored first and gradually discovers the entire PF. The example here demonstrates a case of ZDT3 problem (Figure 4a). The outlier dots (orange) represents the deterministically generated solutions that did not survived because they are weakly dominated. The light green dots are those that are deterministically generated and survived. In the case of ZDT4 (Figure 4b), we can see how the bias and gaps have been corrected by our approach.

Figure 5: The convergence test of Algorithm 3 (onsga2r) vs. NSGA-II on problem ZDT1. The curves are actually consisted of box-plots, and for the Algorithm 3 (onsga2r), the medians are joined with a line. Here we can see that HV is 0 upto 1.8k SE and then abruptly reaches to 2 (the long vertical line at the beginning of Algorithm 3’s curve).

Table 1: The ideal HV, corresponding reference points. The last two columns represent the total number of SEs to attain 90% of the ideal HVs for both algorithms over the benchmark problems. For DTLZ7 problem, the ideal HV is approximated using a Monte-Carlo sampling.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Ideal HV (HV)</th>
<th>Reference Points: $(f_1, f_2, f_3)$</th>
<th>NSGA-II 90%-HV / SE</th>
<th>Algorithm3 90%-HV / SE</th>
</tr>
</thead>
<tbody>
<tr>
<td>ZDT1</td>
<td>3.67</td>
<td>(2.0, 2.0)</td>
<td>5100</td>
<td>1909</td>
</tr>
<tr>
<td>ZDT2</td>
<td>3.34</td>
<td>(2.0, 2.0)</td>
<td>7700</td>
<td>2164</td>
</tr>
<tr>
<td>ZDT3</td>
<td>4.82</td>
<td>(2.0, 2.0)</td>
<td>4900</td>
<td>2019</td>
</tr>
<tr>
<td>ZDT4</td>
<td>3.67</td>
<td>(2.0, 2.0)</td>
<td>14500</td>
<td>4950</td>
</tr>
<tr>
<td>ZDT6</td>
<td>15.35</td>
<td>(4.0, 4.0)</td>
<td>10500</td>
<td>1539</td>
</tr>
<tr>
<td>DTLZ1</td>
<td>999.98</td>
<td>(10.0, 10.0, 10.0)</td>
<td>22800</td>
<td>11602</td>
</tr>
<tr>
<td>DTLZ2</td>
<td>7.48</td>
<td>(2.0, 2.0, 2.0)</td>
<td>1200</td>
<td>10602</td>
</tr>
<tr>
<td>DTLZ3</td>
<td>3374.98</td>
<td>(15.0, 15.0, 15.0)</td>
<td>30600</td>
<td>11802</td>
</tr>
<tr>
<td>DTLZ4</td>
<td>7.48</td>
<td>(2.0, 2.0, 2.0)</td>
<td>1800</td>
<td>2022</td>
</tr>
<tr>
<td>DTLZ5</td>
<td>6.1</td>
<td>(2.0, 2.0, 2.0)</td>
<td>1200</td>
<td>1226</td>
</tr>
<tr>
<td>DTLZ6</td>
<td>55.6</td>
<td>(4.0, 4.0, 4.0)</td>
<td>29400</td>
<td>24602</td>
</tr>
<tr>
<td>DTLZ7</td>
<td>134.20</td>
<td>(10.0, 10.0, 10.0)</td>
<td>9200</td>
<td>3606</td>
</tr>
</tbody>
</table>

at a certain time instance, the overshoot-correction on the line 13 of Algorithm 3 have a good chance of creating a valid neighbouring solution. To mute such an un-warranted exploration, we have changed the problem definition in such a way that all the solutions on the true-PF will have $x_1 = 0.5$, so that the Pareto-set is consisted of a straight line located at $x_1 = 0.5$ instead of $x_1 = 0.0$. Such modifications are done on the problems where appropriate.

All the results are collected from 31 independent runs started with non-identical random seeds. In all plots, onsga2r stands for Algorithm 3. The extra cost to find the extreme points are indicated with a “T” arrow on the x-axis. During computation of the HV measure, we have set the reference point to $(2.0, 2.0)$ for all problems except ZDT6, where it has been set to $(4.0, 4.0)^7$. The Table 1 summarizes the performance measure for our test was Hypervolume (HV) [14], and we are interested to see which algorithm can reach to a desired HV within less solution evaluations (SE). For all problems, we have seen our algorithm can demonstrate a very steep convergence to the true PF, even when the extra SE from Algorithm 1 are compensated.
performance measuring parameters, the second column shows the ideal hypervolume of each of the PF of the corresponding problems. The second and the third columns present the number of SE have been spent to reach 90% of the hypervolume by each of the algorithms. To ensure a completely fair comparison, for all instances, we have excluded the solutions in $E^*$ during the hypervolume computation.

The experiment with ZDT1 is illustrated in Figure 5, here we can see that the Algorithm 1 takes up to around 2K of solution evaluations. Given that, NSGA-II still lags behind with a multiple factors to reach the desired PF. We have seen similar effect on all the rest of the problems ZDT2, ZDT3, ZDT4 and ZDT6. Except for ZDT3, we have observed some fluctuations due the disconnected nature of the true PF. Due to the space constraints, we present a subset of the results in the Figure 6.

There is another interesting observation we have made, once this opposite point generation scheme is used, the search process becomes more focused and works in a more predictive manner. As an example, in the case of ZDT3 problem (where the true PF consists 5 disconnected curves), the algorithm first tries to fill up the first partition and gradually moves to the next. The algorithm automatically detects which portion of the PF needs to be addressed first and try to fill the gaps by deliberately injecting points to the vicinity of those gaps. As a result our model can infer which objective is hard to solve and deterministically decides which one needs to be explored more. This scenario is illustrated in Figure 4, where we can see how the point generation algorithm moves from one disconnected front to the next.

Moreover, our approach can also efficiently solve the issue of search trajectory bias, if we look at the Figure 4b, we can see that the new solutions are deterministically generated where the explorations are not done thoroughly yet.

In the next experiment, we have carried out the similar tests with the scalable problem sets – DTLZ1, DTLZ2, DTLZ3, DTLZ4, DTLZ5, DTLZ6 and DTLZ7 [4]. For all cases we have considered 3-objectives. The control was NSGA-II results and similarly we compensate the measure with the extra SE to find extremes. All the results are collated from 31 independent runs. The reference point for HV computation has been set to $\{2.0,2.0,2.0\}$ for DTLZ2, DTLZ4 and DTLZ5. For DTLZ1 and DTLZ7 it has been set to $\{10.0,10.0,10.0\}$, for DTLZ3 it was $\{15.0,15.0,15.0\}$ and for DTLZ6, it was $\{4.0,4.0,4.0\}$. Due to the space constraints, we present a subset of the results in the Figure 7.

In this paper, we only present the results for those problems that are comparatively harder to solve, i.e. problems with local-optima, disconnected fronts, non-uniform densities in the true PF etc.

6. CONCLUSIONS AND FUTURE WORKS

The main contribution of this paper is the incorporation of opposite point generation scheme in a different perspective. Our approach also shows that a simple and a deterministic scheme can aid the EMO optimization algorithm in a very interesting way. Our technique is also easy to implement and offers less overhead to the host algorithm. This approach can also correct the search bias introduced by the problem difficulty in an automated and predictable manner. The original idea of the Opposition Based Learning (OBL) is quite interesting; and we can make more of it if this idea is utilized in a more meaningful way – we think this is the main contribution of our study.

However, in the future we want to address some other interesting issues with our current study, especially in the "many-objective" problems. Moreover, we think there are also a possible scope in changing the idea opposition for reference point based many-objective algorithms like MOEA/D and NSGA-III [2]. In the future research, we hope to investigate these ideas.

7. REFERENCES

Figure 7: These plots illustrate the comparative analysis of the convergence rates for 2 3-objective problems (DTLZ3 and DTLZ7), the curves are actually consisted of box-plots. Here onsga2r denotes our algorithm and nsga2r is NSGA-II. For DTLZ7, our algorithm shows better convergence in terms of mean and median hypervolume.