ABSTRACT
This paper presents a finite element based numerical study on controlling the postbuckling behavior of thin-walled cylindrical shells under axial compression. With the increasing interest of various disciplines for harnessing elastic instabilities in materials and mechanical systems, the postbuckling behavior of thin-walled cylindrical shells may have a new role to design materials and structures at multiple scales with switchable functionalities, morphogenesis, etc. In the design optimization approach presented herein, the mode shapes and their amplitudes are linearly combined to generate initial geometrical designs with predefined imperfections. A nonlinear postbuckling finite element analysis evaluates the design objective function, i.e., the desired postbuckling force-displacement path. Single and multi-objective optimization problems are formulated with design variables consisting of shape parameters that scale base eigenvalue shapes. A gradient-based algorithm and numerical sensitivity evaluations are used. Results suggest that an optimized shape for a cylindrical shell can achieve a targeted response in the elastic postbuckling regime with multiple mode transitions and energy dissipation characteristics. The optimization process and the obtained geometry can be potentially used for energy harvesting and other sensing and actuation applications.

INTRODUCTION
The design optimization of thin-wall axially compressed cylindrical shells with buckling as a constraint [1, 2] typically focuses on maximizing the critical buckling load in the primary static branch because postbuckling behavior has long been regarded as an undesirable phenomenon due to the significant loss of load-carrying capacity and catastrophic failure that follows once it occurs [3]. Yet, a paradigm has shift evolving over the past decade and has turned its attention to utilize such instable behavior for new purposes [4-6]. It is well known that a load drop in the stable load-deformation path relates to the dissipation of strain energy as the shell transitions into a stable post-buckled shape. Several studies on axially compressed cylindrical shells have focused on the response near the first bifurcation point for purposes of residual capacity estimation [7-9], see path (a) in Fig. 1. Physically, mode transitions imply changes in the buckling waves on the cylinder’s surface. Under certain geometric and stiffness constraints multiple buckling events can still occur under increasing axial shortening. The curvature in cylindrical shells provides a natural geometric constraint that allows the attainment of multiple critical points, as shown in path (b) of Fig. 1. While path (b) has a relatively lower stiffness it features multiple bifurcation points due to changes in the deformed geometry after each critical point.
In spite of the noted interests on postbuckling behavior, full characterization of the postbuckling response with multiple mode jumping characteristics, which may provide uses beyond load-carry capacity, is a less explored topic. Koiter [10] once noted that load-carry capacity in the far post-buckling regime is of little practical importance for a unstiffened cylindrical shell because material or geometric imperfections are highly unpredictable and lead to configurations at a lower strength and state of strain energy. It is well known that the existence of imperfections have a significant effect on the critical load, and this has challenged the development of analytical and numerical models. However, emerging smart applications have recreated new opportunities for using response in the postbuckling regime for energy harvesting, energy dissipation sensors, actuators, etc. Rather than maximizing strength as an objective, research studies on structural optimization of cylindrical shells have also explored form finding for smart purposes. For example, Chen et al. [11] used optimization techniques to identify the suitable geometry of a cylinder that maximized piezoelectric energy conversion efficiency. Kim et al. [12] investigated the finding of effective partial placement of damping material on the cylindrical shell to maximize the damping effect via optimization tools.

Optimization studies towards tailoring the postbuckling response of cylindrical shell has not been equally explored compared to plates because it always fails to the clichés of imperfection sensitivities and manufacturing difficulties. However, several recent studies have explored the opportunity of designing the pattern and amplitude of imperfections. Kegl et al. [13] proposed a gradient-based optimization approach for finding asymmetric imperfection patterns and amplitudes such that the nonlinear response of the shell can follow a certain equilibrium path without having a bifurcation point. Lindgaard et al. [14] carried out shape optimization with buckling mode shapes as the design variables. The resulting mode shapes were used to define a “worst” imperfection pattern for a thin-walled cylindrical shell such that the axially compressed cylinder would minimize the buckling load. Ning and Pellegrino [15] explored the concept of imperfection-insensitive cylindrical shells by designing a wavy cross-section using structural optimization. In current study, buckling mode shapes from eigenvalue analyses were used as base geometrical forms to optimize a seeded imperfection.

Rather than single scalar objective, this paper aims to obtain a load-deformation curve with a desired elastic postbuckling response. Previous studies have investigated the opportunities of obtaining targeted response curve in large deformation problems. Sekimoto and Noguchi [16] proposed two algorithms to obtain an optimal shape that could follow a predefined load-displacement curve, namely a postbuckling response, as the objective trajectory. The optimization process was formulated through the sequential linear programming method and implemented in the form finding of a toggle structure. Bruns et al. [17] modified an arc-length algorithm and embedded it into an optimization problem to solve a load-displacement constraint problem. The method was implemented in the search of optimal material distribution for a fix-fix beam that would follow a pre-defined nonlinear response exhibiting snap-through behavior.

Pilot experimental and numerical studies by the authors [18, 19] have shown that the force-deformation response in the elastic postbuckling region can be modified to feature multiple mode jumps, but identifying the postbuckled deformed shape in a unique and repeatable manner was difficult due to the high degree of imperfection sensitivity. Though limited from such challenge, imperfection design and its optimization can be a promising opportunity to tailor and control the postbuckling response of cylindrical shells. Consequently, if the elastic postbuckling response can be modified and made less sensitive through strategic geometric patterns then shells can be designed with optimized seeded deformations to obtain the desired postbuckling response.

The objective of this paper is thus to explore the design of geometrical imperfections on cylindrical shell for a targeted postbuckling response by using standard numerical optimization and analysis tools. Single and multi-objective optimization cases were investigated to provide insight on imperfection designs and identify critical areas that require special attention for design and manufacturing.

METHOD AND PROCEDURE

The postbuckling of thin cylindrical shells is a notoriously unstable response and thus difficulty to predict through simulations. Nonetheless, advances in computational mechanics and optimization methods have made the noted challenge quite feasible. In this study, numerical methods for structural shape optimization proceeded in three major steps (see Fig. 2): (1) A finite element eigenvalue analysis and a nonlinear analysis using the finite element program Abaqus [20]; (2) response evaluation through a self-developed code in Matlab [21]; and (3) sensitivity analysis and design updates through the design optimization software HEEDS [22]. Detailed descriptions for each step are given in the following sections.

Geometric Design and Structural Analysis

Numerical studies [23, 24] have shown that equilibrium states in the postbuckling regime can be tracked and predicted by using general-purpose finite element programs, such as ABAQUS [20]. The chaotic behavior of a cylindrical shell is typically captured by conducting a second-order nonlinear analysis on a shell with imperfection considerations. Geometric imperfections on the shell surface can be introduced in the numerical model in three ways: from actual measurements, by a mathematically determined “worst” pattern, or through eigenvalue-based simulated shapes [25]. Typically, the mode shapes from the eigenvalue buckling analyses are commonly used as artificial imperfections for perturbation purposes.
The specific approach followed in this work is that rather than opening the solution domain to arbitrary shapes, which would need some kind of geometrical interpolation, the seeded geometrical design follow a combination of the linear eingenvalue buckling shapes. This approach, i.e., the linear combination of base shapes, has been shown to be effective in the work by Lindgaard et al. [26], who used the method to define the “worst” imperfection pattern in shells for purposes of nonlinear buckling optimization.

The process starts with the eigenvalue analysis of a perfect cylinder for a given material system. Buckling eigenvalues and mode shapes were predicted by the Lanczos method. The mode shapes and their amplitudes are to be linearly combined to generate an initial seeded geometrical design. Abaqus has a built-in tool for seeding imperfections into a perfect geometry based on mode shapes. For the traditional design goal of maximizing buckling capacity the first eigenmode is commonly chosen to simulate an imperfection with a symmetric shape because the lowest mode is assumed to have the largest effect on the first buckling load. The seeded imperfection amplitude can range from 5% to 50% of the shell thickness and is selected based on knowledge of the experimental response [23]. However, it has been shown that improved agreement with experimental data is obtained by seeding imperfections from the superposition of multiple mode shapes [27]. In this study, higher-order modes were considered to simulate the random nature of geometrical imperfections.

Simulating the postbuckling response requires conducting a second order nonlinear analysis. Options include a static solver, arc-length method (e.g., Riks), or a dynamic solver. All of these methods are suitable and offer advantages and disadvantages. The numerical simulations presented in this paper were conducted using an explicit dynamic solver. The cylinders were modeled with 4-node quadrilateral finite-membrane-strain elements with reduced integration (S4R), for which the bending strain formulations are approximations to those in Koiter-Sanders shell theory. The cylindrical shells were subjected to uniform axial compression under displacement control to a targeted end-shortening and then unloaded.

**Objective Response and Characteristic Features**

Nonlinear finite element analyses were conducted to determine the load-deformation response path far into the elastic postbuckling regime. The desired load-deformation response and the specific postbuckling response features of interest are schematically shown in Fig. 3. The equilibrium path is expected to have multiple postbuckling mode transitions rather than a single bifurcation point. From a physical perspective, the number of multiple mode transitions indicates localized elastic interactions of the cylindrical shell while a single larger jump may indicate damage. It should be noted that...
the magnitude of the first bifurcation event is not of primary interest. Rather, maximizing the number of load drops is of more importance. Lastly, it is of interest to maximize the enclosed area in the force-displacement response as it is associated with the dissipated energy from the equilibrium path transitions. Thus, key response features are thus the change in the axial stiffness of the cylinder from its initial value ($K_i$) to the unloading point ($K_u$), the magnitude of single load drop events ($\Delta P_i$), the spacing between load drop events ($\delta$), and the hysteretic area ($A$). Single and multi-objective optimization problems were formulated. The optimization variables are the parameters (e.g., $\alpha$, $\beta$, $\delta$) scaling the base eigenvalue shapes. Linear constraints control the amplitude of the overall imperfection. A routine developed in Matlab was used to evaluate the objective function by comparing the simulated response curve with the targeted response.

![Diagram](image)

**Fig. 3.** Schematic of multiple mode transitions in the postbuckling response of a compressed cylindrical shell.

**Sensitivity Analysis**

The design optimization program HEEDS [22] was used to conduct the sensitivity analysis, update the design variables and modify the nonlinear analysis input for the ABAQUS model. HEEDS (Hierarchical Evolutionary Engineering Design System) is an optimization package that utilizes an iterative design process and an adaptive search strategy (SHERPA) to efficiently find optimized solutions [22]. During a single parametric optimization study, SHERPA uses elements of multiple search methods simultaneously. Attributes from combinations of global and local search methods are used, and each participating approach contains internal tuning parameters that are modified automatically during the search according to knowledge gained about the nature of the design space. SHERPA is a direct optimization algorithm in which all function evaluations are performed using the actual model, as opposed to using an approximate response surface model [28]. SHERPA does not require solution gradients to exist and the only parameter that must be specified is the number of allowable evaluations.

**Evaluation**

Before conducting the optimization studies for targeted postbuckling response, the full numerical procedure was evaluated against a published experimental test. Research studies [29, 30] have shown that the geometry of a cylindrical shell dictates their buckling response and that obtaining multiple local buckling patterns requires cylinders with a small length to radius ratio ($L/R \leq 2$) and a large radius to thickness ($R/t \geq 200$) ratio. The test of a polyester film cylinder was chosen [31]. The cylinder had a length of 113.9 mm, a radius of 100 mm and a thickness of 0.247 mm. Thus, the cylinders had a $L/R$ ratio of 1.14 and a $R/t$ ratio of 405. The Young’s modulus $E$ was 5.56 GPa and the Poisson’s ratio $\nu$ is 0.3. The classic buckling critical load is given by [24]:

$$P_{cl} = \frac{2 \pi E t^2}{\sqrt{3(1 - \nu^2)}} = 1290 \, N$$

where $E$ is in MPa and $t$ is the shell thickness (in mm).

The numerical model had a fine mesh resolution, with 400 elements in the circumferential direction and 80 elements in the axial direction. The cylinder was clamped along both edges. The buckling eigenvalue and mode shapes were predicted by the Lanczos method and the postbuckling response was simulated using an explicit dynamic solver. The cylinder was subjected to axial compression under displacement control to a shortening of 1.0 mm (same as in the experiment). It should be noted that only elastic response was considered.

![Diagram](image)

**Fig. 4.** Comparison of postbuckling response from numerical simulations and experiment.

The imperfection selection for the initial design followed the classic approach, i.e. the first buckling mode shape was superposed into a perfect shell as an imperfection with a scale of 10% of the shell thickness. Fig. 4 shows the comparison between the initial design and test data from Yamaki et al. [31]. The first buckling load in the test was 908.2 N while the initial design had a predicted buckling capacity of 1194.9 N. The predicted buckling load may seem to be close to the
Experimental value but it can be seen from Fig. 4 that the magnitude of the load drop is quite different. Minimizing this discrepancy is of importance for this study in which the focus is to tailor the detailed response features in the postbuckling response region.

Arbitrary choice of eigenmodes as imperfections may lead to a reasonable first buckling load, but postbuckling paths would be drastically affected by the different choices of imperfections. It is thus of interest to solve the problem in an inverse manner to find an optimal combination of mode shapes that yield the best curve fitting with the test data. Pilot study by the authors [18] showed that mode transitions in the postbuckling regime were more likely to occur when the seeded imperfection mode shapes had a relatively small number of waves in the longitudinal direction but a large number of waves in the circumferential direction. Thus, 18 modes were chosen as design variables after eliminating repeated shapes (i.e., with the same eigenvalue). Some of selected mode shapes are shown in Fig. 5, where $m$ and $n$ are the number of full sine waves along the axial and circumferential direction, respectively.

![Fig. 5. Sample seeded buckling mode shapes](image)

The curve fitting option within HEEDS can update design variables in the parameter optimization problem. In this study, the optimization problem can be written as:

Objective: $\min \left| P_{\text{test}} - P_{\text{sim}} \right|$ 
Subject to: $0 \leq a_i \leq t$, $i = 1, \ldots, 18$

where $P_{\text{test}}$ is the area between the response curve of test and simulation (corresponds to the difference of works from the origin to the prescribed displacement) as shown in Fig. 2; $a_i$ is the scaling amplitude of each mode shape; and $t$ is the shell thickness. Previous initial model previously described with only a mode 1 imperfection was chosen as the initial design. The seeded geometric design was linearly superposed from 18 mode shapes with varying imperfection amplitudes with respect to the total shell thickness. Twenty five evaluations were requested in HEEDS and the best design from the optimization had a $\Pi$ value of 33.2. The optimized superposed geometry included the contributions from 5% of mode 2, 5% of mode 15 and 10% of mode 63. The postbuckling response is plotted in Fig. 4 along with test data and baseline initial design. The comparison of postbuckling response parameters is given in Table 1, from which it can be seen that the optimized results better capture the key response features shown in Fig. 3, including the initial stiffness ($K_i$), magnitude of first mode jump ($\Delta P_1$) and the total number of mode transitions ($n$). Although there are still differences between the simulated curve with optimal imperfections and the test curve, it can be seen that the area ($A_{\text{total}}$) below the force-deformation curves (which corresponds to the total work) are very close to each other. In addition, the postbuckling shape obtained by the numerical model matched well the diamond-shaped buckling wave pattern observed in the test (as shown in Fig. 4). This implementation confirmed the validity of using the nonlinear analysis and optimization approach to match a targeted postbuckling response and indicates potential for using the approach toward the efforts of tailoring and controlling the postbuckling equilibrium path and mode transitions in a cylindrical shell under axial compression.

![Fig. 6. Postbuckling response of cylinders with identical mode imperfection seeding but different amplitudes](image)

### Table 1. Postbuckling response of cylinders.

<table>
<thead>
<tr>
<th>Item</th>
<th>$K_i$ (N/mm)</th>
<th>$P_1$ (N)</th>
<th>$\Delta P_1$ (N)</th>
<th>$\delta_1$ (mm)</th>
<th>$A_{\text{total}}$ (J)</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test</td>
<td>7297</td>
<td>908</td>
<td>535</td>
<td>0.13</td>
<td>211</td>
<td>4</td>
</tr>
<tr>
<td>Baseline</td>
<td>7448</td>
<td>1195</td>
<td>791</td>
<td>0.16</td>
<td>269</td>
<td>6</td>
</tr>
<tr>
<td>Optimization</td>
<td>7177</td>
<td>869</td>
<td>516</td>
<td>0.12</td>
<td>215</td>
<td>4</td>
</tr>
</tbody>
</table>

Note: $P_1$, $\Delta P_1$ and $\delta_1$ are associated with first bucking event. $A_{\text{total}}$ is the area below each curve (correspond to the total work).
Three additional simulations were conducted using the same modeling approach on cylinders with all of the pre-selected 18 mode shapes for imperfection seeding but with different scaling factors (0.01%, 1% and 100% of the shell thickness for each mode). Fig. 6 shows the resulting simulated postbuckling responses. For the upper bound case (nearly perfect cylinder), the postbuckling response is essentially linear elastic with a critical buckling load of 1279 N. For the lower bound case (cylinder with large imperfections), the response has no critical points. For the cylinder with small imperfections (1% case), the postbuckling response has the desirable features noted in Fig. 3. It is clear that the simulation results are very sensitive to the prebuckling mode shape selection and imperfection amplitudes, but such uncertainty also offers a variety of opportunities with the resulting design domain, which is identified as a hatched area in Fig. 6.

OPTIMIZATION CASE STUDIES

This section presents three optimization cases aimed at finding a seeded imperfection shape for a targeted elastic postbuckling response under uniform compressive axial loading and unloading. The analysis and optimization approach followed was the same as presented in the previous sections. One hundred evaluations for each case were requested during the optimization process with HEEDS. For consistency, the cylinders’ geometry and material properties were the same as those used in the evaluation study. The objectives are defined according to the schematic load-deformation response presented in Fig. 3. Three most important features associated with potential smart applications are (1) the maximum value of single load drop ($\Delta P_{\text{max}}$); (2) the number of mode jumps ($n$); and (3) the hysteretic area ($A$). Single- and multiple-objective cases were considered.

The objective of the first optimization case was to maximize a single load drop in the load-deformation response. A large snap-through buckling event is of interest because of the associated significant change of strain energy in the system and its transformation (primarily) into kinetic energy. The load-displacement response along with the time history of kinetic energy release of the best design is plotted in Fig. 7. It can be observed that $\Delta P_{\text{max}}$ occurs in the first buckling event, which is typically the case, followed by a large loss of stiffness. The postbuckling response does feature other critical points, each leading to successive decreases in stiffness and axial capacity. Also plotted in Fig. 7 is the kinetic energy history (as a function of axial shortening), from which the significant energy release can be observed after each load drop in the load-deformation response. The key postbuckling response values of two best designs ($\Delta P$-1 and $\Delta P$-2) are given in Table 2. It can be seen that the obtained imperfections of both designs were seeded from a single mode with relatively low amplitude.

The objective of the second optimization case was to maximize the number of mode jumps or critical points in the postbuckling region. The load-deformation response along with the time history of kinetic energy release of the best design is plotted in Fig. 8. The key postbuckling response values of the best two designs (n-1 and n-2) are given in Table 2. It can be observed that the obtained imperfections were seeded from multiple mode shapes with relatively high amplitudes. The number of mode jumps ($n$) obtained in this case was much higher than was obtained in the initial/baseline design conducted in the evaluation section (see in Table 1). It can be noted that compared to cases $\Delta P$-1 and $\Delta P$-2, the n-optimized designs had relatively lower values for other key features ($\Delta P$, A, K). These results are in agreement with previous findings [18] that $\Delta P$, A and n have a strong correlation with the structural initial stiffness $K$. Seeking a higher n in the postbuckling regime requires the seeded imperfection to be larger. As a result, the initial stiffness decreases, the $\Delta P$ values decrease and A is much smaller. Given such opposite effects between n and other key features, an optimization case for maximizing A was not be evaluated because the results are expected to be similar to those for the case of maximizing $\Delta P$.  

![Fig. 7. Obtained postbuckling response of cylinders with the objective of maximizing a single load drop ($\Delta P$).](image1)

![Fig. 8. Obtained postbuckling response of cylinders with the objective of maximizing the number of mode jumps (n).](image2)
Due to the noted correlation between $\Delta P$ and $A$, only one of these two objectives was selected to study a multi-objective optimization case along with $n$. The last optimization is aimed at maximizing $n$ and $A$ and further validate the nature of their competing features. The numbers of mode jumps $n$ along with corresponding dissipated energy $A$ of all 100 evaluations are plotted in Fig. 9. The results for $A$ are sorted in ascending order to identify a the correlation with the results for $n$. The mean value of the dissipated energy is 36 kJ and the mean value of mode jumps is 9. It can be seen that the majority of postbuckling responses with larger $A$ have lower number of $n$ and vice versa. Table 2 summarizes characteristic features of the postbuckling response for the best two designs (nA-1 and nA-2). Compared with the best designs under a single optimization objective, it can be seen that the seeded imperfections include contributions from more mode shapes and their scaling amplitudes are between the levels found for the previous cases. Further, all of the key postbuckling response features seem to be in between the responses of case $\Delta P$ and case $n$. As expected, lower imperfection amplitude did significantly increase the dissipated energy and magnitude of $\Delta P_{max}$. The end stiffness for all cases was similar, yet the initial stiffness was quite different.

**Table 2. Postbuckling response of imperfect cylinders and their imperfection selections and amplitudes.**

<table>
<thead>
<tr>
<th>$\Delta P$</th>
<th>$A$</th>
<th>$k$</th>
<th>$\alpha$</th>
<th>$a_i$</th>
<th>$a_j$</th>
<th>$a_k$</th>
<th>$a_l$</th>
<th>$a_m$</th>
<th>$a_n$</th>
<th>$a_o$</th>
<th>$a_p$</th>
<th>$a_q$</th>
<th>$a_r$</th>
<th>$a_s$</th>
<th>$a_t$</th>
<th>$a_u$</th>
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<tr>
<td>$\Delta P_{max}$ (N)</td>
<td>514</td>
<td>472</td>
<td>64.2</td>
<td>29.1</td>
<td>126.4</td>
<td>116.7</td>
<td>11.7</td>
<td>7</td>
<td>13</td>
<td>12</td>
<td>9</td>
<td>9</td>
<td>85.3</td>
<td>101</td>
<td>32.5</td>
<td>24.1</td>
</tr>
<tr>
<td>n</td>
<td>11</td>
<td>7</td>
<td>13</td>
<td>12</td>
<td>9</td>
<td>9</td>
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<td>10%</td>
<td>300%</td>
<td>720%</td>
<td>180%</td>
<td>200%</td>
<td>a_i</td>
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<td>295</td>
<td>140.6</td>
<td>140.6</td>
<td>116.9</td>
<td>206</td>
<td>5346</td>
<td>5326</td>
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<td></td>
<td></td>
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<td></td>
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<td>$\alpha_{real}$</td>
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<td>10%</td>
<td>300%</td>
<td>720%</td>
<td>180%</td>
<td>200%</td>
<td>a_j</td>
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<td>0</td>
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<tr>
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<td>0.173</td>
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<tr>
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*Note: the percentage is with respect to the shell thickness of a perfect cylinder (0.247 mm)*

**CONCLUSIONS**

The postbuckling response of axially compressed cylindrical shells, a feature traditionally of little interest and/or to be avoided, has promising potential to generate/absorb energy for use in smart materials and structures. The occurrences of multiple mode transitions, their magnitude and spacing in the postbuckling regime depend on initial imperfections and vary from case to case.

The optimization studies presented in this paper have shown that the elastic postbuckling response of a thin cylindrical shell can be obtained through conventional numerical approaches with close correlation to testing results from published literature. A series of optimization cases were carried out to evaluate the possibility of tailoring the postbuckling response by designing imperfections seeded from buckling mode shapes. The results showed that ‘imperfections by design’ offer a variety of opportunities to achieve a specific single objective ($\Delta P$ and $n$) or multiple objectives ($n$ and $A$) in the elastic postbuckling response of cylindrical shells. Low-amplitude imperfections can lead to larger strain energy drops at critical points and consequently higher energy dissipation. A larger number of mode transitions can be achieved by designed imperfections based on the superposition of multiple modes. Additional constraints on each objective are required for further tailoring the postbuckling behavior.

The concepts presented in this paper provide insight into understanding the noted postbuckling behavior through optimization cases, which are thought to be an important step towards harnessing such instable behavior. Investigations focused in the postbuckling response of thin-walled shells are limited and thus further analytical and experimental studies are required to validate the concepts posed here. Nonetheless, the presented numerical studies indicate that the showcased postbuckling behavior can be achieved and designed.
ACKNOWLEDGMENTS

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REFERENCES