Abstract—A simplified equivalent circuit of electric powertrain system of hybrid electrical vehicle (HEV) is proposed to research and investigate into the impact of pulse width modulation (PWM) strategies of voltage source inverter (VSI) on the DC-link capacitor in HEVs. Then, analytical solutions to the switching function are introduced based on space vector PWM (SVPWM) and discontinuous PWM (DPWM) schemes. Furthermore, the DC-link capacitor current is characterized with analytical solution and simulation method to quantify the impact of various PWM schemes on the current ripple. Finally, both the reliability and the lifetime of DC-link capacitor are discussed based the spectrum of DC-link capacitor current.

Keywords—DC-link capacitor, current ripple, pulse width modulation, equivalent Circuit, third harmonic injection method

I. INTRODUCTION

DC-link circuit in power electronic conversion systems is used to balance the instantaneous power difference between source and load, and to minimize current ripple from the switching action to ensure adequately stiff DC-link voltage. A large portion of current ripple from both backward circuit and forward circuit are absorbed by DC-link circuit so that there is less interference effect between input source and output load [1][2]. There are two typical configurations in power electronic conversion systems of HEV, rectifier→DC-link→inverter and DC/DC-converter→DC-link→inverter, as shown in Fig. 1. The DC-link circuit in HEV is generally designed as a capacitor bank composed of aluminum electrolytic capacitors (Al-Caps) or metalized polypropylene film capacitors (MPPF-Caps) [3][4].

In order to lower the power loss of the inverter in HEV, modulation schemes of inverter are usually based on space vectors, which mainly cover continuous pulse width modulation (CPWM) and discontinuous pulse width modulation (DPWM). CPWM includes sinusoidal pulse width modulation (SPWM), which is not included in this paper, and space vector pulse width modulation (SVPWM). DPWM involves DPWM0, DPWM2, SPWM1, DPWM3, DPWMMAX and DPWMMIN according to the choice and placement of zero space vectors.

The reference voltage of each phase leg under DPWM strategies, compared with that under traditional CPWM strategies, is clamped to positive or negative direct-current bus and maintains or intermittently maintains one third of fundamental period $T_0$ so that DPWM strategies can effectively reduce switching loss and enhance efficiency of the system to a certain degree. For SVPWM and DPWM, the realization principle mainly involves space vector modulation (SVM) method based on the coefficient distribution of space vectors [5][6] and the third harmonic injection (THI) method [7][8]. Despite different approaches of generating the switching states using these two methods, both the essence and the final effect are consistent with each other [9]. The THI method, which is easier to be realized than the SVM method, is commonly adopted in VSI of HEV.

Some research shows that DPWM strategies can effectively reduce VSI switching losses of HEV [10][11]. Other research demonstrates that HEV DC-link capacitor ripple current spectrum reflects the reliability and frequency characteristic of switching function to a certain degree [12]. The research in [13] and [14] aims at minimizing DC-link capacitor current ripple in HEV under SPWM. However, the research about DC-link capacitor current ripple under SVPWM and DPWM strategies cannot be found up to now. As we all know, for VSI of HEV, either SVPWM or DPWM has higher power efficiency than SPWM. What happens to DC-link capacitor current under SVPWM and different DPWM strategies is the study topics in the paper.

The switching states series of VSI vary with different modulation strategies. The input current of VSI depends on switching states series and load. DC-link capacitor current is the difference between the VSI input current and the output current of backward circuit, which is rectifier of DC/DC converter. Therefore, modulation strategies have impact on DC-link capacitor and its reliability. It is practically important to investigate the impact of modulation schemes on HEV DC-link capacitor.

978-1-5090-4281-4/17/$31.00 ©2017 IEEE
II. SIMPLIFIED EQUIVALENT CIRCUIT OF ELECTRIC POWERTRAIN SYSTEM OF HEV

A simplified equivalent circuit of electric powertrain system of HEV, as shown in Fig. 2, is developed to model the current ripple of DC-link capacitor. As the equivalent circuit, the fully controlled IGBTs are used as switching devices and a typical three-phase bridge topology is used in VSI.

![Simplified equivalent circuit of electric powertrain system of HEV](image)

Fig. 2. Simplified equivalent circuit of electric powertrain system of HEV

A. Backward Equivalent Circuit of DC-link Capacitor

It is assumed that the current ripple only originates from switching actions of the PWM-operated VSI bridge and the harmonic contribution from the upstream source is neglected. It is worth noting that the methodology adopted in this study is equally applicable to the source harmonics. The sum of the current \( i_{\text{CAP}} \) that flows into DC-link capacitor and the current \( i_{\text{VSI}} \) that flows into VSI is a constant \( I_{\text{EQ}} \), which implies that the DC-link capacitor and VSI bridge share the same pulsating current although in opposite phase. The ideal steady-state current is zero when the DC-link capacitor voltage \( 2V_{\text{DC}} \) is constant. So, the backward equivalent circuit is a DC voltage source or DC-link capacitor bank in parallel with a DC current source.

B. Forward Equivalent Circuit of VSI

The forward circuit is generally 3-phase permanent magnet synchronous motor (PMSM), each phase of which can be equivalent to a resistance-inductance load in series with an induced electromotive force in proportion to rotor speed of PMSM. Note that the induced electromotive force can’t be ignored, when the vehicle reaches a much higher running speed, especially.

It is assumed that the root-mean-square (rms) magnitude of output line current of VSI is \( I_i \) and the power factor angle is \( \phi \). For typical inductance of a traction motor of HEV and high switching frequency of VSI, the line current \( i_{\text{a}}, i_{\text{b}} \) and \( i_{\text{c}} \) are reasonably assumed symmetrical 3-phase current free of ripple.

\[
\begin{align*}
    i_{\text{a}} &= \sqrt{2}I_{i}\sin(\omega_{\text{t}}t - \phi) \\
    i_{\text{b}} &= \sqrt{2}I_{i}\sin(\omega_{\text{t}}t - \frac{2}{3}\pi - \phi) \\
    i_{\text{c}} &= \sqrt{2}I_{i}\sin(\omega_{\text{t}}t + \frac{2}{3}\pi - \phi)
\end{align*}
\]

(1)

C. DC-link Capacitor Current Equation

The instantaneous current \( i_{\text{VSI}} \) flowing into VSI bridge can be calculated by (2). And DC-link capacitor current \( i_{\text{CAP}} \) can also be calculated by application of Kirchhoff’s current law to Fig. 2, as shown in (3).

\[
\begin{align*}
    i_{\text{VSI}} &= S_a i_a + S_b i_b + S_c i_c \\
    i_{\text{CAP}} &= I_{\text{EQ}} - i_{\text{VSI}}
\end{align*}
\]

(2)

(3)

Where \( S_a, S_b, \) and \( S_c \) are the switching functions of the upper switches of the VSI bridge. From (1), (2) and (3), frequency components of DC-link capacitor current \( i_{\text{CAP}} \) depend on switching function \( S_a, S_b \) and \( S_c \). The current \( i_{\text{VSI}} \) has the same AC components as that of the current \( i_{\text{CAP}} \) with opposite phase.

III. IMPACT OF SVPWM SCHEMES ON SWITCHING FUNCTION OF VSI WITH ANALYTICAL SOLUTION

A. Switching State Analysis of SVPWM and DPWM Strategies

There are eight switching modes, with complementary states for the two switching devices in any phase leg, corresponds to eight stationary space vectors \( V_i \) \((i=0,1,\ldots,7)\) and seven input current mode of VSI, as shown in Table I and Fig.3.. So, DC-link capacitor current is correlated with switching states.

| TABLE I. eight switches state, space vectors and the input current \( I_{\text{VSI}} \) of VSI |
|---|---|---|---|---|---|---|---|---|
| \( V_i \) | \( V_0 \) | \( V_1 \) | \( V_2 \) | \( V_3 \) | \( V_4 \) | \( V_5 \) | \( V_6 \) | \( V_7 \) |
| \( \theta \) | 0 | 1/8 | 5/8 | 3/8 | 7/8 | 6/8 | 2/8 | 4/8 |

Among them, six effective space vectors \( V_i \) \((i=1,2,\ldots,6)\) divide the d-q plane into six 60\(^\circ\)-segments. Reference space vector \( v^* \) rotates counterclockwise from d axis when \( t=0 \) and covers a circle every fundamental period \( T_0 \).

![Location of eight segments in d-q plane](image)

Fig. 3. Location of eight segments in d-q plane

The vector \( v^* \) in a switching period \( T_i \) (\( T_i \) is far less than \( T_0 \)) is considered as constant \( V \) and is approximated by composite vector of two adjacent effective space vectors \( V_0, V_1 \) and two zero space vectors \( V_6, V_7 \). Actuation time \( T_i \) for \( V_1, V_5 \) and \( T_i \) for \( V_0 \) and \( V_7 \) in any segment can be calculated according to the volt-second balance \( v^* T_i=V_1 T_i+V_7 T_i+0 T_0 \).

For the conventional SVPWM, both \( V_0 \) and \( V_7 \) are chosen with total actuation time of \( T_0 \) and symmetrical space vectors series are introduced to synthesize a reference output vector in
a segment. The commonly adopted series mode is \( V_0 \rightarrow V_{ODD} \rightarrow V_{EVEN} \rightarrow V_{ODD} \rightarrow V_0 \).

Zhou et. al. show that SVPWM is essentially equivalent to carrier-based PWM with carrier frequency \( \omega_c \) (equal to switching frequency \( \omega_s \)) [15]. The equivalency is obtained by injecting the sinusoidal reference signal with third harmonic \( u_{c3} \) as shown in (4) when \( k=0.5 \) to form the modulation signal \( u_z = u_k z_k u_k z_k u_k z_k \) in carrier-based PWM, shown in Fig.4.

\[
\begin{align*}
\mu^*_z &= -\mu_{\text{min}}^* - (1 - k)\mu_{\text{min}}^* + (2k - 1) \\
& \quad (4)
\end{align*}
\]

Where, \( \mu_{\text{max}}^* \) is the maximal value of \( u_c^* \), \( \mu_{\text{min}}^* \) is the minimal value of \( u_c^* \), \( k \) is the zero vector selection factor. SVPWM and DPWM strategies are formed with different values of \( k \) in different intervals.

![Fig. 4. Reference wave \( u_c^* \), sine wave; third harmonic \( u_c^* \), wave by dotted line modulation wave \( u_c^* \), non-sine wave by solid line for SVPWM](image)

For DPWM, only one zero space vector is chosen every 60° phase interval based on series mode for SVPWM with symmetrical space vectors. Taking away \( V_3 \) and keeping the total actuation time \( t_c \) for \( V_0 \) will result in commonly adopted series mode \( V_0 \rightarrow V_{ODD} \rightarrow V_{EVEN} \rightarrow V_{ODD} \rightarrow V_0 \), which has the same modulation effect as the modulation wave of PWM being sinusoidal reference injected with third harmonic \( u_{c3}^* \) in (4) when \( k=0 \). Taking away \( V_0 \) and keeping the total actuation time \( t_c \) for \( V_3 \) lead to commonly adopted series mode \( V_{ODD} \rightarrow V_{EVEN} \rightarrow V_3 \rightarrow V_{EVEN} \rightarrow V_{ODD} \), which has the same modulation effect as the modulation wave of PWM being sinusoidal reference injected with third harmonic \( u_{c3}^* \) in (4) when \( k=1 \). The six DPWM strategies are defined in terms of the choice and the placement of zero space vectors, i.e. the value of the zero vector selection factor \( k \), in six segments, as shown in TableII.

**TABLE II. ZERO SECTOR SELECTION FACTOR FOR 6 DPWM STRATEGIES CORRESPONDING EVERY 30° IN A REFERENCE INTERVAL**

<table>
<thead>
<tr>
<th>Range of ( \omega )</th>
<th>MIN</th>
<th>MAX</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–π/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>π/6–2π/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2π/6–3π/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3π/6–4π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>4π/6–5π/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5π/6–6π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>6π/6–7π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>7π/6–8π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>8π/6–9π/6</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>9π/6–10π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>10π/6–11π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>11π/6–12π/6</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

The choice and placement of zero vectors, corresponding to SVPWM or DPWM, determine switching function \( S \), and have impact on current ripple of DC-link capacitor. So does the third harmonic. Reference signal \( u_{c3}^* \), third harmonic \( u_c^* \) and modulation signal \( u_{\text{max}}^* \) of SVPWM and DPWM are shown in Fig.4 and Fig.5.

![Fig. 5. Reference wave \( u_c^* \), sine wave; third harmonic \( u_c^* \), wave by dotted line modulation wave \( u_c^* \), non-sine wave by solid line for DPWM](image)

For SVPWM, DPWMMAX or DPWMMIN, \( k \) is constant 0.5, 1 or 0, respectively. For DPWM0, DPWM1, DPWM2 and DPWM3, zero vector selection factor \( k \) is integer 0 or integer 1 and alternates at the border line given by adjacent segments, which is the center or the edge of every segment. According to TABLE II and (4), six conceivable analytical solutions to modulation \( u_{z_k} \) are determined under all DPWM strategies as follows.

Solution I: \( u_{z_k}^* - u_{z_k}^* \)

Solution II: \( u_{z_k}^* + u_{z_k}^* + 1 \)

Solution III: \( u_{z_k}^* - u_{z_k}^* - 1 \)

Solution IV: \( u_{z_k}^* - u_{z_k}^* - 1 \)

Solution V: 1

Solution VI: -1

**TABLE III. SOLUTION TO MODULATION WAVE FOR SIX DPWM STRATEGIES EVERY 30° IN A REFERENCE INTERVAL**

<table>
<thead>
<tr>
<th>Range of ( \omega )</th>
<th>MIN</th>
<th>MAX</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–π/6</td>
<td>I</td>
<td>IV</td>
<td>I</td>
<td>I</td>
<td>IV</td>
<td>I</td>
</tr>
<tr>
<td>π/6–2π/6</td>
<td>I</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>2π/6–3π/6</td>
<td>I</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>3π/6–4π/6</td>
<td>III</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>4π/6–5π/6</td>
<td>III</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>5π/6–6π/6</td>
<td>III</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>6π/6–7π/6</td>
<td>III</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>7π/6–8π/6</td>
<td>VI</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>8π/6–9π/6</td>
<td>VI</td>
<td>V</td>
<td>V</td>
<td>I</td>
<td>V</td>
<td>I</td>
</tr>
<tr>
<td>9π/6–10π/6</td>
<td>VI</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>10π/6–11π/6</td>
<td>VI</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
<td>IV</td>
</tr>
<tr>
<td>11π/6–12π/6</td>
<td>I</td>
<td>IV</td>
<td>I</td>
<td>I</td>
<td>IV</td>
<td>I</td>
</tr>
</tbody>
</table>
There are three solutions to the modulation wave under SVPWM strategy.

\[
\begin{align*}
    u_c &= \begin{cases} 
        1.5u_{x_c}^* & [0, \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, 2\pi] \\
        0.5(u_{x_c}^* - u_{y_c}^*) & [\frac{\pi}{2}, \frac{3\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{3\pi}{6}, \frac{\pi}{2}] \\
        0.5(u_{x_c}^* - u_{y_c}^*) & [\frac{\pi}{2}, \frac{3\pi}{6}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{3\pi}{6}, \frac{\pi}{2}] 
    \end{cases}
\end{align*}
\]  

(5)

B. Analytical Solution of Switching Function

Switch state series in a reference period \(T_0\) can be obtained according to the common series mode of DPWM and SVPWM. So, the frequency \(\omega\) of \(S_i\) (i =a,b,c) by the Fourier decomposition can be described as \(n\omega_0 + m\omega_c\) (n and m are integers). Switching functions \(S_i\) are obtained by comparing the modulation wave \(u_c\) (\(u_c = u_{x_c}^* + u_{y_c}^*\), \(u_{x_c}^* = Msin\theta\)) with a triangular carrier with frequency \(\omega_c\) and peak 1. The state of 3 switches in a carrier period \(T_c\) repeats every reference period \(T_0\). Two variables, \(x = \omega_c t + \theta_c\), and \(y = \omega_c t + \theta_c\) are introduced. The value of \(S_i\) is 1 only when \(u_c\) is greater than the carrier, which covers from \(x_l\) to \(x_u\) in a carrier interval \(T_c\), shown in Fig.6.

![Fig. 6. Detailed illustration of switching constant determination for PWM strategies based on space vector realized by the third harmonic injection method.](image)

In Fig. 6, \(x_u\) is the upper limit of integration and \(x_l\) is the lower limit of integration in every carrier period \(T_c\) of the \(x\)-axis direction, which repeats \(T_0/T_c\) times in a fundamental period of \(y\)-axis direction. From Fig.6., \(x_u\) and \(x_l\) can be deduced, shown in (6) and (7), under the assumption that \(u_c\) is a constant in a carrier interval. In (6) and (7), \(p\) is integer \(u_c\) and is fragment function.

\[
x_u = 2\pi n + 0.5\pi (u_c - 1), p = 1,2,\ldots
\]

(6)

\[
x_l = 2\pi n - 0.5\pi (u_c + 1), p = 1,2,\ldots
\]

(7)

From (6) and (7), we can get two contour lines described by \(x_u\) and \(x_l\) and the region between them covers the all the states when \(S_i\) is 1. So, a unit cell is introduced in \(x-y\) plane with \(x\) in a carrier interval \(T_c\) and \(y\) in a fundamental period \(T_0\). Unit cells of \(S_i\) for different PWM schemes based on space vectors are shown in Fig.7, whose horizontal coordinate \(x\) is \(\omega_c t\) and vertical coordinate \(y\) is \(od\).

![Fig. 7. Unit cells of switching function \(S_i\) for double-edge triangular modulation under different PWM schemes.](image)

Taking double FFT on switching function \(S_i\) in a fundamental period \(T_0\) results in the spectrum of the switching function. \(S_i\) can be expressed as a summation of double-Fourier series.

\[
S_i(t) = \frac{A_{00}}{2} + \sum_{n=1}^{\infty} \left[ A_{nn} \cos(ny) + B_{nn} \sin(ny) \right] + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left[ A_{nm} \cos(mx + ny) + B_{nm} \sin(mx + ny) \right]
\]

(8)

The Fourier coefficients are determined by

\[
A_{00} = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(0+0)} dy 
\]

(9)

where, in \(x\) the axis direction, \(x_u\) is the upper limit of integration and \(x_l\) is the lower limit of the inner integration; in \(y\) axis direction, the range of the outer integration, divided into sub-ranges according to table III, covers a period \(T_0\), also can be any other range covering \(2\pi\).

The key to determining the summation of harmonic components of switching function \(S_i\) is to calculate the Fourier coefficient \(C_{nm}\) in (9). For the inner integral, it is critical to determine the upper limit function \(x_u\) and the lower limit function \(x_l\), which are the linear function about modulation waves \(u_{x_c}\). For the outer integral, it is critical to determine the integration sub-ranges based on table III.

The calculation of the Fourier coefficients is illustrated by the switching function \(S_i\) with DPWMMAX, shown in (10)-(16).

When \(m=0\), coefficients are

\[
A_{00} = A_{00}^* = A_{00}^0 = 2 - \frac{3\sqrt{3}M}{2\pi}
\]

(10)

When \(m=0, n\neq 0\), coefficients are simplified to
It can be seen from (17) that baseband harmonics with third-harmonic frequency carrier harmonics and sideband harmonics with third-harmonic frequency are left in components of DC-link capacitor current, which can be verified by simulation, shown in Fig.8 and Fig.9. And RMS of those can be calculated by (11) to (17).

![Fig.8 Spectrums of i_{c,s} for different DPWMs based on analytical solution, f_c=50Hz, f_0=1200Hz, I=78A, φ=0 rad, M=0.9](image)

For different DPWM strategies, there is no difference in the RMS of DC-link capacitor current because actuation time \( T_e \) and \( T_c \) are not impacted by the selection and placement of zero space vectors. However, the position of either of effective space vectors has an offset with 0.5\( T_e \). So, there is some difference in the THD of DC-link capacitor current for different DPWM strategies. Simulation verification is done, shown in Table III.

| TABLE III RMS AND THD OF DC-LINK CAPACITOR UNDER DPWMS |
|---------------------------------|--------|--------|---------|---------|---------|---------|
| DPWM | MAX   | MIN   | 0      | 1      | 2      | 3       |
| RMS/A | 57.82 | 57.64 | 57.72  | 57.66  | 57.63  | 57.59   |
| THD%  | 57.82 | 57.64 | 57.72  | 57.66  | 57.63  | 57.59   |

It can be seen from Table III that the RMS of DC-link capacitor current under different DPWM are almost equal to each other, but there is obvious difference among THD of them.

### IV. Solution of Current Ripple of DC-Link Capacitor

#### A. Analytical Solution of PWMS’s Impact on the DC-Link Capacitor Current

Taking (1) and (8) into (2) and (3), DC-link capacitor current expression is

\[
\begin{align*}
\overline{\mathcal{I}_c} &= \frac{M}{\pi} \int_0^{\pi/3} e^{i\phi} d\phi + \frac{\sqrt{3} M}{2\pi} \int_0^{\pi/6} e^{i\phi} \cos(y - \frac{\pi}{3}) d\phi \\
& \quad + \frac{\sqrt{3} M}{2\pi} \int_0^{\pi/6} e^{i\phi} \cos(y + \frac{\pi}{3}) d\phi \\
& \quad + \frac{\sqrt{3} M}{2\pi} \int_0^{\pi/6} e^{i\phi} \cos (y + \frac{\pi}{3}) d\phi - \overline{\mathcal{I}_c} \\
& \quad + \frac{\sqrt{3} M}{2\pi} \int_0^{\pi/6} e^{i\phi} \cos (y - \frac{\pi}{3}) d\phi \\
& \quad + \frac{\sqrt{3} M}{2\pi} \int_0^{\pi/6} e^{i\phi} \cos (y + \frac{\pi}{3}) d\phi - \overline{\mathcal{I}_c} \\
\end{align*}
\]

When \( m \neq 0 \), coefficients are simplified to

\[
\begin{align*}
\overline{\mathcal{I}_c} &= \frac{1}{m_{\pi}} \left[ \int_0^{\pi/6} e^{i\phi} \sin [m \pi + 0.5 \sqrt{3} m \pi \cos (y - \frac{\pi}{3})] d\phi \right] \\
& \quad + \frac{1}{m_{\pi}} \left[ \int_0^{\pi/6} e^{i\phi} \sin [m \pi + 0.5 \sqrt{3} m \pi \cos (y + \frac{\pi}{3})] d\phi \right] \\
\end{align*}
\]

The choke point in calculating coefficients from (14) to (16) is that the solution to calculating inner integrals, which is about trigonometric transcendental equations. Ladder diagram is used to calculate the integral forms from (14) to (16).

#### B. Simulation Solution of PWMS’s Impact on the Ripple of DC-Link Capacitor Current

The system parameters are as follows: the fundamental frequency is 50Hz; the carrier frequency is 15 kHz; DC-link capacitor is 3000uF; the modulation index is 0.9; DC current source is deduced to ensure the voltage of DC-link capacitor maintains around 600V. Fig. 9 shows the simulation results of DC-link capacitor current spectra for different PWMS.
Some observations can be made from Fig. 9:

1. The carrier harmonics amplitudes of DC-link capacitor ripple current at integer carrier frequency are much higher than those of baseband harmonics and sideband harmonics. The maximum value of carrier harmonics is the highest at 30 kHz and then delay in non-linear with integer carrier frequency.

2. For SVPWM strategy, the maximum of baseband harmonics amplitudes, carrier harmonics amplitudes and sideband harmonics amplitude of DC-link capacitor ripple current are all lower than those of DPWM strategies, respectively.

3. For DPWM0, DPWM1, DPWM2 and DPWM3 strategies, the maximum of baseband harmonics amplitudes, carrier harmonics amplitudes and sideband harmonics amplitude of DC-link capacitor ripple current are all lower than those of DPWMMAX and DPWMMIN strategies.

4. There are obvious differences among the magnitude series for DPWM strategies.

V. THE IMPACT OF HARMONICS ON THE RELIABILITY OF DC-LINK CAPACITOR

A. The Hotspot Temperature and Reliability of DC-Link Capacitor

The internal temperature variations due to high current ripple, a critical stress to the failure of Al-Caps or MPPF-Caps in HEV, affect the reliability and lifetime of DC-Link capacitor [16].

\[ T = T_{\text{amb}} + R_{\text{ha}} \sum_{i} \text{ESR}(f_i) \times I_{\text{rms}}^2(f_i) \]  \hspace{1cm} (9)

Where, \( T \) is the hotspot temperature of DC-link capacitor, \( T_{\text{amb}} \) is the ambient temperature, \( R_{\text{ha}} \) is the equivalent thermal resistance from hotspot to ambient, \( \text{ESR}(f_i) \) and \( I_{\text{rms}}(f_i) \) are the equivalent series resistance of DC-link capacitor and the root-mean-square value of the ripple current at frequency \( f_i \), separately. Therefore, the spectrum of the ripple current of DC-link capacitor solution is essential for estimating temperature \( T \).

For different DPWM strategies, RMS at different frequency \( I_{\text{rms}}(f_i) \) have different characteristics, which leads to the summation result of the second part in (9) have of difference. Finally, the hotspot temperature is impacted by DPWM strategies. The hotspot temperature \( T \) is proportional to the result of summation formula. An hotspot temperature index \( \zeta \) is introduced to qualitatively describe modulation strategies’s influence on the hotspot. The greater the index \( \zeta \), higher the hotspot temperature \( T \). The estimated index \( \zeta \) for six DPWM strategies are tabulated in table IV.

\[ \zeta = \frac{\sum_{i} \text{ESR}(f_i) \times I_{\text{rms}}^2(f_i)_{\text{DPWM}}}{\sum_{i} \text{ESR}(f_i) \times I_{\text{rms}}^2(f_i)_{\text{SVPWM}}} \]  \hspace{1cm} (10)

| TABLE IV THE HOTSPOT TEMPERATURE INDEX FOR SIX DPWM STRATEGIES UNDER STEADY ROTATION SPEED BASE ON SIMULATION \( f_0=50\,\text{Hz}, f_c=15\,\text{kHz}, I=78\,\text{A}, M=0.9 \) |
|-----------------|-----|---|---|---|---|---|
| DPWM            | MAX | MIN | 0 | 1 | 2 | 3 |
| DPWM MAX        | 28.46 | 28.79 | 2.57 | 2.06 | 2.45 | 1.50 |

It can be seen from table IV, the hotspot temperature is obviously higher for DPWMMAX and DPWMMIN strategies than those for DPWM0, DPWM1, DPWM2 and DPWM3 strategies, which verifies that the later four strategies are more suitable to be employed of VSI in HEV rather than the former two strategies.

In the practical driving cycles of HEV, 3-phase line currents change with road condition and vehicle speed because the induced electromotive force varies with driving cycle. Therefore, the hotspot temperature is time-varying because the spectrum of the ripple current of DC-link capacitor changes with driving cycle. The operation reliability of DC-link capacitor decreases when the hotspot temperature \( T \) increases. If \( T \) is up to or more than the definite limit of hotspot
temperature, DC-link capacitor may be prematurely of malfunction.

**B. The Lifetime of DC-Link Capacitor**

The lifetime $L$ of DC-link capacitor depends on the maximum voltage $V$ across capacitor and the hotspot temperature $T$ when the vehicle runs in typical driving cycle [16].

\[
L = \frac{C}{\rho V^{-n} \times e^{\frac{E_a}{RT}}}
\]  
(11)

Where $\rho$ is a coefficient related to capacitor type and driving cycle; $n$ is the voltage stress exponent, $E_a$ is activation energy; $K_B$ is Boltzmann’s constant $(8.62 \times 10^{-5} \text{eV/K})$. Both $V$ and $T$ are influenced by equivalent thermal resistance $R_{th}$ and equivalent series resistance $ESR(f)$.

VI. SUMMARY

In this paper, the impact of modulation schemes on DC-Link capacitor of VSI is quantified with analytical solution and simulation method. Simplified equivalent circuit of the powertrain system of HEV is introduced to focus on the impact from switching action of VSI. Based on SVPWM schemes and their equivalent third harmonic injection patterns, unit cells for seven SVPWM schemes are developed to determine x- and y-axes integration region that is used for computation of Fourier coefficients, which constitute analytical solution to the spectrum of switching functions. DC-Link capacitor current spectrum is analytically calculated in terms of Fourier series of switching functions and the symmetry of 3-phase line currents. The analytical solution has been verified by simulation results. The impact of current ripple on the reliability of DC-link capacitor is investigated based on the hotspot temperature model and lifetime model.

ACKNOWLEDGMENT

We would like to greatly appreciate that Hebei University of Technology and Michigan State University provide us research platform. We are also grateful that the research is supported by the National Natural Science Foundation of China and is funded by China Scholarship Council.

REFERENCES


