Feedback Linearization Control of Three-Phase Buck-Boost Inverters

Heng Gao and Bingsen Wang
Department of Electrical Engineering
Arizona State University
PO Box 875706
Tempe, Arizona 85287-5706 USA
Heng.Gao@asu.edu; Bingsen@asu.edu

Abstract— This paper presents a nonlinear control algorithm for a single-stage three-phase buck-boost inverter. In contrast to the linear control law that is formulated according to small signal model that is linearized around an operating point, the exact linearization that is directly based on the large signal model allows for the formulation of the control law over a wide operating range. Moreover, the proposed control algorithm delivers superior dynamic performance due to the decoupled control inputs.

Index Terms-- buck-boost inverter, feedback linearization, nonlinear control, pulse width modulation

I. INTRODUCTION

The utilization of the renewable energy is becoming increasingly important as the result of the escalated demands for clean, efficient and non-emission energy supplies. Solar and wind are two of the viable energy sources suitable for distributed power generation. Unlike the traditional fossil-fuel based power generation, for the renewable sources, the maximum output power is uncontrollable and difficult to predict precisely. The power output characteristic of a Photovoltaic (PV) module is a function of the amplitude of the luminous intensity, the temperature, the humidity and so on as described in [1]; while the output of a wind turbine varies by the speed and the direction of the wind. Various maximum power point tracking (MPPT) techniques have been developed to enable the wind or solar generation systems to approach the optimal operating point and harvest maximum power, which often leads to time-varying DC voltages. Accordingly, it is necessary for the interface between the DC side and the grid to have the ability of handling a wide range of voltage transfer ratio. With pulse width modulation (PWM) of the switching bridge, the magnitude of the fundamental component of the output line-line voltage of a voltage source converter (VSC) will be unable to exceed the voltage across the dc link capacitor unless over-modulation schemes are resorts. In a dual manner, the fundamental component of the output line current of a current source converter (CSC) will be limited to the magnitude of the dc link inductor current.

The limited voltage transfer ratio and current transfer ratio that are inherently associated with VSCs and CSCs have become a technical barrier for applications that require a wide operating range. A power converter capable of operating over a wide range of the voltage/current transfer ratio is highly desired to maximize the captured energy and consequently reduce effective energy cost and to shorten the payback period for the investment of generation assets.

Research work has been undertaken to overcome the buck- or boost-only capability of power converters. Although the cascaded connection of multiple power conversion stages is a straightforward approach to achieve a wide operating range, an evaluation of power efficiency and reliability and cost would not favor such a multiple-stage solution. Hence, the single-stage dc/ac power conversion with buck and boost capability is focused in this study. Derived from the applications of Cuk and Sepic-Zeta buck-boost converters topology, the ideas of the similar topology in the three-level inverters are brought forward to be the solutions respectively in [2] and [3]. [2] proposed the Cuk bidirectional ac-dc converter topology. Given the topology and the operating principles, the state space PWM strategy is proposed in [3], and the small signal analysis is carried out in [2]. Similar to dc/dc topologies of Cuk and the Sepic-Zeta converters, an inductor and a capacitor are required on the dc side of the dc/ac power converter for proper power transfer.

As an alternative approach to achieving buck-boost capability, Z-source converters (ZSCs) have been proposed as a viable solution [4-5] without cascading multiple stages. The ZSC topology features a dc link that consists of a symmetrical lattice network, i.e. two inductors and two capacitors. This lattice network was primarily studied in the field of filter design to synthesize specified network transfer functions [6-8]. In a power conversion system, the fourth-order symmetric lattice network constituting the DC link of ZSC introduces undesired complexities, which not only results in high component-count, but also exerts extra burden in modeling the system and designing suitable controls.

To achieve the buck-boost ability while simultaneously minimizing the passive components on the dc side and reducing the order of the system, this paper proposes a new topology that is based on current-sourced inverters. The reduced complexity renders the proposed topology best suitable for applications that demand high efficiency, reliability and cost effectiveness. Interfacing renewable energy with power grids is one of the targeted application areas.

The control of the converter is based on the feedback linearization. In state space average models for power converters operated under PWM, there exist products of the states and inputs, which causes the model inherently coupled...
and nonlinear. The feedback linearization is an exact control
and it addresses the disadvantages associated with the linear
control based on small signal analysis around an operating
point. The employment of feedback linearization in control
of PWM VSCs has been reported in [9-11].

The rest of the paper is organized as follows. The
converter topology and its operating principles are presented
in Section II. The modulation strategy is proposed in Section
III followed by the nonlinear state space model in Section
IV. The feedback linearization control algorithm with
decoupled control inputs is derived in Section V. Numerical
simulations are conducted on a detailed Saber Model and the
results validating the effectiveness of the control are
presented in Section VI. A summary presented in Section VII
concludes the paper.

II. TOPOLOGY AND OPERATING PRINCIPLE

As shown in Fig. 1, the semiconductor-realized topology
is utilized as an inverter that converts a dc voltage to three-
phase ac voltages and is then connected to the utility grid that
is represented by the three-phase voltage sources through an
LC filter. Fig. 2 shows the semiconductor realization of same
topology for a rectifier application. The polarity of the dc
link inductor and the polarity of the load voltage are
indicated in the figure. The operation of the converter will be
explained with regard to the inverter application.

![Fig. 1 Semiconductor realization of the proposed topology as a dc/ac inverter.](image1)

![Fig. 2 Semiconductor realization of the proposed topology as an ac/dc rectifier.](image2)

The operating principle of the inverter will be explained
with reference to the two different intervals corresponding to
the on-off states of the dc link switch \( S_{dc} \) within a switching
period as shown in Fig. 3. Fig. 3a and Fig. 3b illustrate the
conduction path of load current when \( S_{dc} \) is on and when \( S_{dc} \)
is off, respectively.

![Fig. 3 Conduction path of load current when (a) Sdc is on; (b) Sdc is off.](image3)

A. When the switch \( S_{dc} \) is on

As illustrated in Fig. 3a, when \( S_{dc} \) is on for an interval of
\( d_{dc} T_s \) with \( d_{dc} \) being the duty ratio and \( T_s \) being the switching
period, the dc link voltage source \( V_{dc} \) charges the dc link
inductor. In order to avoid the formation of a loop consisting
of only of voltage source and capacitors, either one of the
switch sets of \( \{S_{ap}, S_{bp}, S_{cp}\} \) and \( \{S_{an}, S_{bn}, S_{cn}\} \) has to stay
off.

B. When the switch \( S_{dc} \) is off

As illustrated in Fig. 3, when \( S_{dc} \) is off for an interval of
\( (1-d_{dc}) T_s \), the dc link voltage source is isolated from the rest
of the circuitry. The dc link inductor and the switching
bridge form a topology that is identical to a conventional
CSC. Based on the operating principle of CSC, the constraint
of allowing a conduction path for the dc link current without
shorting the voltages on the ac side has to be satisfied.
Consequently, one and only one switch from each of the two
sets of \( \{S_{ap}, S_{bp}, S_{cp}\} \) and \( \{S_{an}, S_{bn}, S_{cn}\} \) has to stay on during
the interval when \( S_{dc} \) is off.

The modulation functions for phase-legs \( a, b, \) and \( c \) are
further assumed to be:

\[
m_a = M \cos(\alpha + \theta)
\]

\[
m_b = M \cos(\alpha + \theta - \frac{2}{3} \pi)
\]

\[
m_c = M \cos(\alpha + \theta + \frac{2}{3} \pi)
\]

With consideration of the power balance in the steady
state and the dc/ac current relation derived from the topology,

\[
d_{dc} V_{dc} I_{dc} = \frac{3}{2} V_m I_m \cos \phi
\]

\[
M I_c = I_m
\]
where \( V_m \) is the magnitude of the voltages \( v_a, v_b, \) and \( v_c \). \( I_m \) is the amplitude of the fundamental component of the currents \( i_a, i_b, \) and \( i_c \).

Therefore, the general buck-boost characteristic of the topology is described as:

\[
V_m = \frac{2d_{dc}}{3M_I \cos \varphi} V_{dc}
\]

Due to the fact that the CSC operating mode of the bridge is only limited to the switching interval of \( (1-d_{dc})T \) when the switch \( S_{dc} \) is off, the modulation index is subject to the following constraint in order to ensure the realization of the modulation schemes.

\[
0 \leq MI < (1 - d_{dc})
\]

### III. MODULATION STRATEGY

A carrier-based modulation scheme is adopted in this paper. The modulation functions for the switches \( S_{ap}, S_{bp} \) and \( S_{cp} \) are \( m_{ap}, m_{bp}, \) and \( m_{cp} \), respectively. The modulation functions for the switches \( S_{an}, S_{bn} \) and \( S_{cn} \) are \( m_{an}, m_{bn}, \) and \( m_{cn} \), respectively. These modulation functions satisfy the following constraints:

\[
\begin{align*}
  m_{ap} + m_{bp} + m_{cp} &= 1 - d_{dc} \\
  m_{an} + m_{bn} + m_{cn} &= 1 - d_{dc} \\
  m_{ap} - m_{an} &= m_a \\
  m_{bp} - m_{bn} &= m_b \\
  m_{cp} - m_{cn} &= m_c 
\end{align*}
\]

The constraints in (5) indicate that non-unique solutions exist for the modulation functions. One set of the modulation functions is constructed as follows:

\[
\begin{align*}
m_{ap} &= \Phi(m_a) m_a + m_0 \\
m_{an} &= \Phi(m_a) m_a + m_0 \\
m_{bp} &= \Phi(m_b) m_b + m_0 \\
m_{bn} &= \Phi(m_b) m_b + m_0 \\
m_{cp} &= \Phi(m_c) m_c + m_0 \\
m_{cn} &= \Phi(m_c) m_c + m_0
\end{align*}
\]

where the zero sequence \( m_0 \) is given by:

\[
m_0 = \frac{1 - d_{dc}}{3}
\]

The function \( \Phi(\cdot) \) in (6) is defined as

\[
\Phi(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}
\]

### IV. NONLINEAR STATE SPACE MODEL

In order to control the output power, the average state space model of the inverter is derived as follows. First, during each switching period, the currents through the inductor and the voltages across the capacitor are treated to be constants. The switching period can be divided into two sub periods: Period One is when the switch \( S_{dc} \) conducts, all the six switches of the switching bridge are forced open; the other is Period Two when the switch \( S_{dc} \) is open and the switching bridge is switched according to the PWM gating signals. In Period One, the dc voltage \( V_{dc} \) charges the dc link inductor for \( d_{dc}T \) (\( T \) is the switching period); In Period Two, the average voltage across the dc link inductor is

\[
-(m_{V_a} + m_{V_b} + m_{V_c}).
\]

Thus, during one switching period, the average voltage across the dc-link inductor is given by:

\[
V_{dc} = L_{dc} \frac{di_{dc}}{dt}
\]

\[
= -MI \left[ \cos(\alpha + \theta - \frac{2\pi}{3}) \right] \left[ v_a - v_{dc} \right] + d_{dc} V_{dc}
\]

For the ac side, the differential equations can be derived from the Kirchhoff’s Law given by:

\[
\frac{di_{ap}}{dt} = -R \left[ i_{ap} + i_{an} \right] + \frac{m_a}{L_{ap}}
\]

\[
\frac{di_{bp}}{dt} = -R \left[ i_{bp} + i_{bn} \right] + \frac{m_b}{L_{bp}}
\]

\[
\frac{di_{cp}}{dt} = -R \left[ i_{cp} + i_{cn} \right] + \frac{m_c}{L_{cp}}
\]

where \( R \) is the resistance of the filter inductor.

The averaged phase currents flowing out of the inverter bridge are given by:

\[
i_a = m_a \cdot i_{dc} = MI \cdot \cos(\alpha + \theta) \cdot i_{dc}
\]

\[
i_b = m_b \cdot i_{dc} = MI \cdot \cos(\alpha + \theta - \frac{2\pi}{3}) \cdot i_{dc}
\]

\[
i_c = m_c \cdot i_{dc} = MI \cdot \cos(\alpha + \theta + \frac{2\pi}{3}) \cdot i_{dc}
\]

The state space model is subsequently transformed to the \( dq \) reference frame, and the system can be described by the following set of state equations:

\[
\begin{align*}
  i_d &= \frac{R}{L_f} i_d + \omega q + \frac{1}{L_f} v_{dc} = \frac{1}{L_f} v_{dc} \\
  i_q &= -\omega q + \frac{R}{L_f} i_q + \frac{1}{L_f} v_q \quad (13) \\
  v_d &= \frac{1}{C_f} i_d + \omega \cdot q + \frac{MI \cdot \cos(\theta)}{C_f} \cdot i_{dc} \\
  v_q &= \frac{1}{C_f} i_q + \omega \cdot v_d + \frac{MI \cdot \sin(\theta)}{C_f} \cdot i_{dc} \\
  i_{dc} &= \frac{3}{2} \frac{MI \cos(\theta)}{L_{dc}} v_d - \frac{3}{2} \frac{MI \sin(\theta)}{L_{dc}} v_q + \frac{d_{dc} V_{dc}}{L_{dc}}
\end{align*}
\]

where \( v_{dc} \) is the amplitude of the grid phase voltage.

For notational clarity, the input, the states and the outputs are rewritten as the vector of \( u, x \) and \( y \) where

\[
\begin{align*}
x_1, x_2 &= \text{d- and q-component of the current that flow into the grid, respectively} \\
x_3, x_4 &= \text{d- and q-component of the voltage across the filter capacitor on the ac side} \\
x_5 &= \text{dc link inductor current}
\end{align*}
\]
With the newly defined state and input/output variables, the state space model in (13) can be rewritten in the matrix form given by (14).

\[
\begin{bmatrix}
\mathbf{x}_1 \\
\mathbf{x}_2 \\
\mathbf{x}_3 \\
\mathbf{x}_4 \\
\mathbf{x}_5
\end{bmatrix} = \begin{bmatrix}
\frac{R}{L_f} & \omega & \frac{1}{L_f} & 0 & 0 \\
-\omega & -\frac{R}{L_f} & 0 & \frac{1}{L_f} & 0 \\
-\frac{1}{C_f} & 0 & 0 & \omega & 0 \\
0 & -\frac{1}{C_f} & \omega & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\mathbf{x}_1 \\
\mathbf{x}_2 \\
\mathbf{x}_3 \\
\mathbf{x}_4 \\
\mathbf{x}_5
\end{bmatrix} + \begin{bmatrix}
\mathbf{u}_1 \\
\mathbf{u}_2 \\
\mathbf{u}_3 \\
\mathbf{u}_4 \\
\mathbf{u}_5
\end{bmatrix} + \begin{bmatrix}
\mathbf{v}_{\text{ref}} \\
0 \\
0 \\
0 \\
0
\end{bmatrix}
\]  

(14)

It is readily observed that the model is nonlinear due to the product of the state and the input on the right-hand side of the first equation of (14).

V. FEEDBACK LINEARIZATION AND CURRENT CONTROL

Nonlinear phenomena take place in modeling the PWM converters since there exists multiplicative nonlinearity between system states and inputs in the differential equation.

Then general nonlinear state space model is given by:

\[
x = F(x) + G(x)u
\]

\[
y = H(x)
\]

(15)

The fundamental idea of the feedback linearization is to transform the original system models into an equivalent model of a simpler form. The transformation is generally described by the Lie Derivative in [9]. However for a 5th order system as obtained from the model, there are general steps to achieve the feedback linearization. The steps are conducted by first differentiating the output \( v \) until the input \( u \) appears at the \( r \)-th order derivative of the output \( v \) given by:

\[
y^{(r)}(x) = \alpha(x) + \beta(x)u
\]

(16)

where \( \alpha(x) \) and \( \beta(x) \) are mono-variable arithmetic functions of input \( x \). Reconstruct the expression by introducing a new variable \( v = \alpha(x) + \beta(x)u \) which is of nonlinear relation with the states and the inputs. The transfer function of the new system is a pure \( r \)-th order integral given by:

\[
s^r Y(s) = F(s)
\]

(17)

Thereby the control can be designed by means of the methods of the linear models. According to the procedure, the resulting form of the transform is given by:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3
\end{bmatrix} = \begin{bmatrix}
F(x) + G(x)u_2 \\
G(x)^{-1} - F(x) + G(x)u_3 \\
0
\end{bmatrix}
\]

(18)

where \( G(x)^{-1} \) and \( F(x) \) are given by

\[
G(x)^{-1} = \begin{bmatrix}
\frac{L_ic_f}{x_4} & 0 & 0 \\
0 & \frac{L_i}{L_f} & 0 \\
1.5 & \frac{x_5c_f}{L_d} & \frac{x_5c_f}{x_5L_f}
\end{bmatrix}
\]

\[
F(x) = \begin{bmatrix}
\frac{R^2}{L_f} - \omega^2 & -1 & \omega \frac{R}{L_f} & \frac{1}{L_f} & 0 \\
-\omega^2 & R^2 & -\frac{R}{L_f} & \frac{1}{L_f} & 0 \\
0 & 0 & \frac{2\omega R}{L_f} & \frac{1}{L_f} & 0 \\
0 & 0 & \frac{2\omega R}{L_f} & \frac{1}{L_f} & 0 \\
0 & 0 & \frac{2\omega R}{L_f} & \frac{1}{L_f} & 0
\end{bmatrix}
\]

Upon the equations above, the control law is designed so that the outputs can track their references in a decoupled fashion. In the application, it means the output current can be controlled in magnitude so that the whole system’s output power can be adjusted, while the phase is also controlled to achieve the unity power factor. Further, the dc link current is expected to stay constant.

According to (18), by utilizing the new variables \( t_1, t_2, t_3 \) as the inputs, the system is decoupled. Thus the controllers can be designed separately. Let \( y_1^*, y_2^*, y_3^* \) be the tracking references for the desired outputs and define the tracking error signals as

\[
e_1 = y_1 - y_1^* \\
e_2 = y_2 - y_2^* \\
e_3 = y_3 - y_3^*
\]

(19)

Then the control law for the new control inputs is given by:

\[
u_1 + k_{11} x_1 + k_{12} (x_1 - x_1^*) + k_{13} \int (x_1 - x_1^*) dt = 0
\]

\[
u_2 + k_{21} x_2 + k_{22} (x_2 - x_2^*) + k_{23} \int (x_2 - x_2^*) dt = 0
\]

\[
u_3 + k_{31} (x_3 - x_3^*) = 0
\]

(20)

The noise or the parameter inaccuracy can lead to steady-state error. Therefore the integral terms in (20) are added in to ensure that the steady-state error is nullified. Equations (20) can be rewritten as...
\[ v_1 = k_{13} x_1^2 + k_{13} \int (x_1' - x_1) dt + L_1(x) \]
\[ v_2 = k_{22} x_2^2 + k_{23} \int (x_2' - x_2) dt + L_2(x) \]
\[ v_3 = k_{33} x_3^2 + L_3(x) \]

where
\[ L_1(x) = \left( k_{11} - k_{12} \right) x_1 - k_{12} a x_2 - k_{11} \frac{1}{L_f} x_3 + k_{11} \frac{v_{sr}}{L_f} \]
\[ L_2(x) = k_{21} a x_1 + \left( k_{21} - k_{22} \right) x_2 - k_{21} \frac{1}{L_f} x_3 \]
\[ L_3(x) = -k_{32} x_3 \]

The coefficients \( k_{11}, k_{12}, k_{13}, k_{21}, k_{22}, k_{23}, \) and \( k_{32} \) are determined by pole-placement method in order to achieve a stable system with desired dynamic performance.

The block diagram of the feedback linearization control algorithm is shown in Fig. 4.

![Block diagram of feedback linearization](image)

**VI. SIMULATION RESULTS**

In order to verify the modulation strategy and the proposed nonlinear control algorithm, a detailed numerical simulation of a 4kW converter has been carried out utilizing the simulation software package Saber™.

The salient parameters of the power circuit and the operating condition are listed in Table I. The LC filter is designed such that the resonant frequency is at 1 kHz, which results in 40 dB attenuation of the switching harmonics that are close to the 10 kHz. It has been well understood the filter size can be further reduced with higher switching frequency.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC link inductor</td>
<td>( L_{dc} ) 10 mH</td>
</tr>
<tr>
<td>Filter inductor</td>
<td>( L_f ) 0.5 mH</td>
</tr>
<tr>
<td>Filter capacitor</td>
<td>( C_f ) 50 μF</td>
</tr>
<tr>
<td>Source voltage line-line</td>
<td>( V_{source} ) 208 Vrms</td>
</tr>
<tr>
<td>Source frequency</td>
<td>( f_{source} ) 60 Hz</td>
</tr>
<tr>
<td>DC link voltage</td>
<td>( V_{dc} ) 200 V</td>
</tr>
<tr>
<td>Carrier frequency</td>
<td>( f_{carrier} ) 10 kHz</td>
</tr>
</tbody>
</table>

During the simulation, the output power of the converter is varied by applying step changes to the tracking reference of the d-component of the current injected into the grid \( i_d \). In the meanwhile, the tracking references of \( i_q \) and \( i_{dc} \) are kept constant so that the unity power factor is maintained with a constant dc link current.

With reference to the simulation results in Fig. 5, the response of the d-component injected current \( i_d \) to its reference \( i_d^* \) is shown in the first panel of Fig. 5a. The \( q \) component injected current \( i_q \) and the dc link inductor current \( i_{dc} \) are shown in the second and third panel of Fig. 5a. The three-phase grid currents and the filter capacitor voltages are plotted in Fig. 5b and Fig. 5c, respectively. Fig. 5d illustrates the phase-a grid voltage and current, which clearly demonstrate that the operating condition of unity-power-factor has been well maintained during the transient.

![Simulation results](image)
Fig. 6 The Block diagram of the laboratory prototype that is under construction.

VII. CONCLUSIONS

This paper has presented a new topology of the current-sourced buck-boost dc-ac converter. A nonlinear control algorithm based on feedback linearization has been proposed to achieve superior dynamic performance over a wide operating range. In comparison to the existing research work on the three-phase buck-boost topology, the herein proposed topology features reduced count of passive components in the dc link and accordingly a reduced order average model. Moreover, the avoidance of electrolytic capacitors in this topology will eliminate various capacitor-related failure mechanisms. Hence, the reliability and lifetime are expected to improve. The numerical simulation results have confirmed that independent control of the outputs can be achieved. Both the steady state performance and transient response have been proved acceptable. The detailed experimental results will be further reported in future publications.

REFERENCES