Six Step Modulation of Matrix Converter with Increased Voltage Transfer Ratio

Bingsen Wang                Giri Venkataramanan
Department of Electrical and Computer Engineering
University of Wisconsin-Madison
1415 Engineering Drive
Madison, WI 53706 USA
bingsen@cae.wisc.edu; giri@engr.wisc.edu

Abstract— Conventional modulation strategies for matrix converters based on trigonometric transformations and space vector techniques have their voltage transfer capability limited at about 0.866 p.u. This leads to a derating of motor capability by 25% in the realization of variable frequency drives using off-the-shelf line-fed motors, or requires custom designed motors for operation at reduced voltage. In this paper, a six-step variable frequency modulation strategy for the matrix converter that is capable of providing more than 1 p.u. fundamental component of output voltage is presented. Six step operation suitable for the conventional 9-switch topology and the reduced switch dual bridge topologies are presented, with capability of unity displacement factor operation at the input and output simultaneously. It is demonstrated that by varying the switching angles, control of fundamental component of output voltage and input current is possible, with concomitant generation of reactive power at the input and output terminals of the converter, with bidirectional power flow. The paper presents analytical results, computer simulation results along with laboratory scale experimental results.

I. INTRODUCTION

Potential attractive features of the matrix converter, such as the absence of dc link energy storage, high power density, high quality of input output waveforms are leading to continued research efforts aimed at translating their promise into reality [1-9]. Although the matrix converters were first introduced as a class of frequency changers realized using controlled switches by Gyugyi [10], significant progress began with the introduction of high frequency synthesis proposed by Venturini in 1980 [11-13]. Although the initial approach was limited to a voltage transfer ratio of one half, improved modulation strategies with voltage transfer ratio of $\sqrt{3}/2$ was published in 1989 [14]. Further along, indirect modulation methods based on fictitious dc link concept have been proposed [15-19]. Except the programmed switching pattern in [19], all these modulation strategies for the matrix converter, based on high frequency synthesis have not been able to break the upper bound given by $\sqrt{3}/2$ p.u. Since motor output power is proportional to the square of applied voltage, this limitation leads to reduced output power, limited to $\frac{1}{4}$ of the motor rating, when fed from a matrix converter operating from the line nominally rated at motor voltage. Otherwise, a custom designed motor designed for the appropriate output voltage level would be required. This remains one among the many roadblocks limiting the viability of the matrix converter. In this paper, a low switching frequency modulation technique for frequency conversion using matrix converters is presented. The proposed strategy is capable of providing more than unity voltage transfer ratio. The modulation strategy is easier developed on the basis of operation of the dual bridge topology with a fictitious dc link. However, the approach is applicable for the conventional 9-switch topology as well, as will be demonstrated through appropriate topological mapping. The paper presents the topological mapping, followed by the modulation strategy, performance characterization including harmonic and reactive power flow analysis. Simulation results are presented in the paper, and will be supplemented with laboratory experimental results in the paper.

Fig. 1(a) illustrates the conventional matrix converter using conventional three single-pole-triple-throw (SPTT) switches $S_u$, $S_v$, $S_w$. Fig. 1(b) illustrates the reduced switch two SPTT switches $S_p$ and $S_n$ and three single-pole-double-throw (SPDT) switches. The two realizations are labeled as conventional matrix converter (CMC) and indirect matrix converter (IMC) respectively. If the switching function of the throws $T_{xy}$, as illustrated in Fig. 1, is defined to be $h_{xy}$, the input-output transposition properties of the CMC can be expressed as

$$V_o = H_{CMC} V_i$$

$$I_o = H_{CMC} I_i$$

where $V_i$, $V_o$, $I_i$, $I_o$ are defined as $[v_a \ v_b \ v_c]^T$, $[v_u \ v_v \ v_w]^T$, $[i_a \ i_b \ i_c]^T$, $[i_u \ i_v \ i_w]^T$. $H_{CMC}$ is the switching matrix with elements being switching functions of various throws shown in Fig. 1a.

$$H_{CMC} = \begin{bmatrix}
    h_{uu} & h_{uv} & h_{uw} \\
    h_{vu} & h_{vv} & h_{vw} \\
    h_{wu} & h_{wv} & h_{ww}
\end{bmatrix}$$
The input-output transposition properties of the IMC can be expressed as

\[ V_o = H_{CSB} H_{VSB} V_i \]

\[ I_i = H_{CSB}^T H_{VSB}^T I_o \]  

(3)

where \( H_{CSB} \) is the switching matrix of the current source bridge (CSB) connected with the stiff voltages. \( H_{VSB} \) is the switching matrix of the voltage source bridge (VSB) connected with the stiff currents. They are given by

\[ H_{CSB} = \begin{bmatrix} h_{ap} & h_{bp} & h_{cp} \\ h_{an} & h_{bn} & h_{cn} \end{bmatrix} \]

\[ H_{VSB} = \begin{bmatrix} h_{ap} & h_{an} \\ h_{bp} & h_{bn} \end{bmatrix} \]

(4)

Through terminal equivalency for CMC and IMC it can be shown that \( H_{CMC} = H_{CSB} H_{VSB} \). Thus any modulation strategy that provides a solution for IMC can be applied for CMC through appropriate switching function mapping. In the subsequent section, the modulation process is derived for the IMC.

![Fig. 1 Representation of (a) IMC and (b) CMC using SPDT and SPTT switches](image)

II. SWITCHING FUNCTIONS AND SYNTHESIZED VOLTAGES/CURRENTS

Let the three phase input voltages and three phase output currents be assumed to be balanced stiff ac quantities and expressed as

\[ v_i = V_i \cos(\alpha_i(t)) \]

\[ i_o = I_o \cos(\beta_o(t)) \]  

(5)

\[ v_i = V_i \cos(\alpha_i(t) - 2\pi/3) \]

\[ i_o = I_o \cos(\beta_o(t) - 2\pi/3) \]

\[ v_i = V_i \cos(\alpha_i(t) + 2\pi/3) \]

\[ i_o = I_o \cos(\beta_o(t) + 2\pi/3) \]

where \( V_i \) is source voltage amplitude; \( I_o \) is the current source amplitude; \( \alpha_i(t) \) is the phase angle of the voltage source, given by \( \alpha_i(t) = \omega t + \alpha_{i0} \); \( \beta_o(t) \) is the phase angle of the current source, given by \( \beta_o(t) = \omega t + \beta_{o0} \).

Let the modulation functions for the CSB and VSB be defined as

\[ m_i = \cos(\beta_i(t)) \]

\[ m_o = \cos(\alpha_i(t)) \]

\[ m_o = \cos(\alpha_i(t) - 2\pi/3) \]

\[ m_o = \cos(\alpha_i(t) + 2\pi/3) \]

(6)

where \( \beta_i(t) = \omega t + \beta_{i0}; \alpha_i(t) = \omega t + \alpha_{i0} \).

The displacement angle \( \phi_i \) on the CSB side and the displacement angle \( \phi_o \) on the VSB side are defined as

\[ \phi_i = \alpha_{i0} - \beta_{i0} \]

\[ \phi_o = \alpha_{o0} - \beta_{o0} \]  

(7)

The switching functions for the various throws of the CSB and VSB are determined by (8) and (9), respectively.

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > m_o) \text{ and } (m_o > m_i) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > m_o) \text{ and } (m_o > m_i) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > m_o) \text{ and } (m_o > m_i) \\ 0 & \text{otherwise} \end{cases} \]

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\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > m_o) \text{ and } (m_o > m_i) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_o > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_o > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > 0) \\ 0 & \text{otherwise} \end{cases} \]

(8)

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_o > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_o > 0) \\ 0 & \text{otherwise} \end{cases} \]

\[ h_{ap} = \begin{cases} 1 & \text{if } (m_i > 0) \\ 0 & \text{otherwise} \end{cases} \]

(9)

The switching functions for the CSB and VSB are depicted in Fig. 2. With this choice of switching functions, the
resulting output voltages and input currents for unity displacement factor operation at input and output, and the corresponding Fourier spectra are shown in Fig. 3. It can be observed that waveforms of output voltages and the input currents are very similar to those of the voltage source inverter and current source inverter except for the presence of the ripple on the 60-degree segments of each waveform.

![Waveforms of various switching functions](image1)

![Normalized output phase-to-load-neutral voltage and spectrum](image2)

![Input current and spectrum for the case of ϕ_i=0 and ϕ_o=0](image3)
III. VOLTAGE AND CURRENT TRANSFER CHARACTERISTICS

In addition to operation at unity displacement factor at both input and output ports, the fundamental component of the output voltage can be varied as the input displacement angle varies and it is independent of the output displacement angle. Similarly, if the output displacement angle varies, the fundamental component of the input current varies and it is independent of the input displacement angle. For the extreme cases, the fundamental component of the input current is zero if $\phi_i=-\pi/2$ and the fundamental component of the output voltage is zero if $\phi_o=\pi/2$. Between the two extreme cases, as illustrated in Fig. 4a, the output voltage fundamental component decreases as the input displacement factor decreases while the rms value of the harmonic component remains almost constant. Accordingly, the output voltage distortion factor, which is defined as the ratio of the fundamental component of the output voltage over the total rms value, decreases lower input displacement factor. In a dual manner, as illustrated in Fig. 4b, the input current fundamental component decreases as the output displacement factor decreases while the harmonic component rms value remains almost constant.

The results shown in Fig. 4 are obtained through numerical evaluations. These results can be well explained if we conduct Fourier analysis on the input currents and output voltages. The switching functions in (8) can be expressed in a Fourier series summation as

$$h_{ui} = \frac{1}{3} + \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \left( \frac{n\pi}{2} \right) e^{jn\phi_i},$$

$$h_{uo} = \frac{1}{3} + \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \left( \frac{n\pi}{2} \right) e^{jn(\pi/2)}.$$  \hspace{1cm} (10)

Similarly, the switching functions in (9) can be expressed in a Fourier series summation as

$$h_{oi} = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \left( \frac{n\pi}{2} \right) e^{jn\phi_o},$$

$$h_{oe} = \frac{1}{2} + \sum_{n=0}^{\infty} \frac{1}{2n+1} \sin \left( \frac{n\pi}{2} \right) e^{jn(\pi/2)}.$$  \hspace{1cm} (11)

Substituting (10) and (11) into (3), together with (5), results in the following relationships.

$$i_o = \sum_{n=2}^{\infty} \frac{1}{\pi} \frac{\sin(n\phi_o)}{2n+1} + \sum_{n=2}^{\infty} \frac{1}{\pi} \frac{\sin(n\phi_o)}{2n+1} e^{j(\pi/3)} \left( e^{j(3\phi_o)} - 1 \right)$$  \hspace{1cm} (12)

The fundamental component of the input current and output voltage can be determined to be:

$$I_{oi} = \frac{6\sqrt{3}I_0}{\pi} \cos(\phi_o); \quad V_{oe} = \frac{6\sqrt{3}V_o}{\pi} \cos(\phi_o)$$  \hspace{1cm} (13)

From (13), it can be observed that the maximum voltage transfer ratio occurs when the input displacement power factor is unity, which is $6\sqrt{3}/\pi=1.05$. The maximum current transfer ratio occurs when the output displacement factor is unity. With the analytical expression in (13), we can study the real and reactive power flow across the power converter under different operating conditions, which is discussed in the following section.
IV. POWER FLOW

To calculate the real power, only the fundamental component of the input currents and output voltages need to be considered since the active power transfer takes place with voltage and current of the same frequency. The input power and output power can be found with the expressions of the input and output quantities derived in (13) together with the input voltage sources and output current sources in (5).

\[
P_{in} = \frac{3}{2} \left( \frac{6 \sqrt{3} I_o}{\pi^2} \cos \phi_i \right) \cos(\phi) = \frac{9 \sqrt{3}}{\pi^2} V_i I_o \cos \phi_i \cos \phi_o
\]

\[
P_{out} = \frac{3}{2} \left( \frac{6 \sqrt{3} V_i}{\pi^2} \cos \phi_i \right) \cos(\phi_i) = \frac{9 \sqrt{3}}{\pi^2} V_i I_o \cos \phi_i \cos \phi_o
\]

(14)

The active power throughput, normalized to \(\frac{9 \sqrt{3}}{\pi^2} V_i I_o\), is plotted against \(\phi_i\) and \(\phi_o\) in Fig. 5. Obviously, the power throughput reaches its maximum when \(\phi_i = \phi_o = 0\), i.e. unity displacement power factor at both input and output ports.

In contrast to the active power, the reactive power across the converter is not conserved. Thus, both the input and output ports can supply reactive power or absorb reactive power simultaneously. If the reactive power associated with fundamental components alone is considered, the reactive power on the input side and output side can be found as follows.

\[
Q_{in} = \frac{3}{2} \left( \frac{6 \sqrt{3} I_o}{\pi^2} \cos \phi_i \right) \sin(\phi) = \frac{9 \sqrt{3}}{\pi^2} V_i I_o \sin \phi_i \cos \phi_o
\]

\[
Q_{out} = \frac{3}{2} \left( \frac{6 \sqrt{3} V_i}{\pi^2} \cos \phi_i \right) \sin(\phi_i) = \frac{9 \sqrt{3}}{\pi^2} V_i I_o \sin \phi_i \cos \phi_o
\]

(15)

Since the phase angle \(\phi_i\) and \(\phi_o\) are defined as the lagging phase angle of the input an output current, the input side absorbs reactive power or the converter looks inductive to the voltage source if \(\phi_i\) is positive. The output side supplies reactive power (with inductive load) if \(\phi_o\) is positive. Together with other quadrants, the characteristics of the input and output reactive power are illustrated in Fig. 6.

The variation of the input reactive power and output reactive power with the input and output displacement angle is plotted against \(\phi_i\) and \(\phi_o\) in Fig. 8. Again, both the input and output reactive power are normalized to \(\frac{9 \sqrt{3}}{\pi^2} V_i I_o\). In both plots, the absolute value of the reactive power is shown. It is easy identify the direction of the reactive power flow with reference to Fig. 6.

Fig. 6 Characteristic of input reactive power and output reactive power in different quadrants on the operation \(\phi_i, \phi_o\) plane.
V. SIMULATION AND EXPERIMENTAL RESULTS

Although the modulation process has been discussed with the sinusoidal input voltage and sinusoidal currents, the modulation process still applies if the output sinusoidal currents are replaced by the RL load. Waveforms from computer simulation of output voltages and output currents of a matrix converter fed from a 60 Hz, 70V, three phase source feeding a 70 V, 120 Hz to a three phase R-L load (R=8 $\Omega$, L=10 mH) using Simulink® are shown in Fig. 9. The similar operating condition is used in the experiment except the voltage amplitude has lower value. The experimental results are shown in Fig. 10.

VI. CONCLUSIONS

In this paper, a new modulation/control strategy for the matrix converter featuring up to unity voltage transfer ratio is proposed. The voltage/current transfer ratio is studied using Fourier analysis. Furthermore, the real and reactive power flow across the converter under the phase control mode has been presented based on the Fourier analysis results. The simulation and experimental results for an RL load case reveals the practical feasibility of this approach.

Such a low frequency modulation approach, combined with proper filter design and/or multi-pulse techniques, practical designs may be realized for application of the matrix converter at medium voltage high power applications using devices such as IGCTs. The combination of the proposed phase modulation together with conventional pulse width modulation provides independent specification of power factor at both the input and output ports of the matrix converter. Furthermore, the proposed approach makes it possible to develop, extend and apply appropriate modulation strategies beyond the limits of conventional scalar and vector PWM approaches in the transition region between linear operation and six-step operation.

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