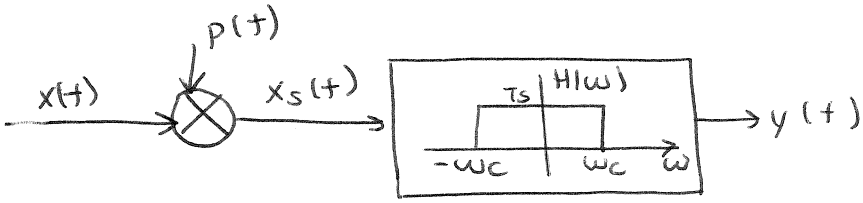
ECE 360 11/13

Practice Problems for Sampling.

(1)



$$X_s(t) = \sum_{k=-\infty}^{\infty} x(kT_s)\,\delta(t-kT_s)$$

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT_s)$$

This is an impulse based sampler.

a) Suppose that $\mathcal{F}\{x(t)\}$ has absolute bandwidth of 200π radians ($\omega_B = 200\pi$) Select values for T_s and w_c so that $y(t) = x(t)$

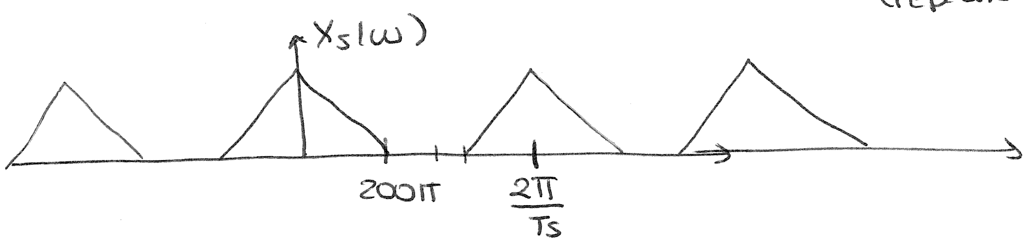
Sampling frequency in radians : $\left(\frac{\cancel{2}\pi}{T_s}\right) \geq \cancel{2}\omega_B$ (Sampling Theorem)

$$\frac{\pi}{\omega_B} \geq T_s$$

Therefore, $\boxed{T_s \leq \frac{1}{200}}$

The spectrum of $X_s(\omega) = \frac{2\pi}{T_s} \sum_{k=-\infty}^{\infty} X\left(\omega - \frac{2\pi k}{T_s}\right)$

(repeats itself every $\frac{2\pi}{T_s}$ radiar



Therefore, $\boxed{200\pi \leq w_c \leq \frac{2\pi}{T_s} - 200\pi}$

b) Suppose that $x(t) = \text{sinc}(10t)$. Select values for T_s and w_c so that $y(t) = x(t)$

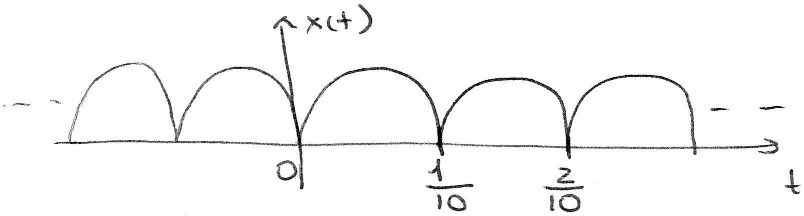
$$X(\omega) = \frac{1}{10}\,\text{rect}\left(\frac{\omega}{20\pi}\right)$$ $\Rightarrow$ The absolute bandwidth (the highest frequency component) is 10π

Therefore, $\omega_s \geq 2\omega_B = 20\pi$

$$T_s \leq \frac{\pi}{\omega_B} = \frac{\pi}{10\pi} \qquad \boxed{T_s \leq \frac{1}{10}}$$

$$\boxed{10\pi \leq \omega_c \leq \frac{2\pi}{T_s} - 10\pi}$$

c) Suppose $x(t) = |\sin(10\pi t)|$. Select values for T_s and ω_c so that $y(t) = x(t)$



$g(t) = \sin(10\pi t)\,\mathrm{rect}\left(10t - \frac{1}{2}\right)$ $\quad T_0 = \frac{1}{10} \rightarrow f_0 = 10$

$\mathrm{rect}\left(10\left(t - \frac{1}{20}\right)\right)$

$\downarrow$

$$G(f) = j0.5[\delta(f+5) - \delta(f-5)] * \frac{1}{10}\,\mathrm{sinc}\left(\frac{f}{10}\right) e^{-j\frac{\pi f}{10}}$$

$$= \frac{j0.5}{10}\left[\mathrm{sinc}\left(\frac{f+5}{10}\right) e^{-j\frac{\pi(f+5)}{10}} - \mathrm{sinc}\left(\frac{f-5}{10}\right) e^{-j\frac{\pi(f-5)}{10}}\right]$$

$$X(f) = \sum_{k=-\infty}^{\infty} \left(\frac{0.5j}{\cancel{10}}\right)\cdot \cancel{10}\left[\mathrm{sinc}\left(\frac{10k+5}{10}\right) e^{-j\frac{\pi(10k+5)}{10}} - \mathrm{sinc}\left(\frac{10k-5}{10}\right) e^{-j\frac{\pi(10k-5)}{10}}\right]\cdot \delta(f-10k)$$

$$= \sum_{k=-\infty}^{\infty} (0.5j)\left[\mathrm{sinc}(k+1/2)\, e^{-j\pi k}\cdot(-j) - \mathrm{sinc}\left(\frac{10k-5}{10}\right) e^{-j\pi k}\cdot j\right]\delta(f-10k)$$

$$= \sum_{k=-\infty}^{\infty} 0.5\left[\mathrm{sinc}(k+1/2)\, e^{-j\pi k} + \mathrm{sinc}(k-1/2)\, e^{j\pi k}\right]\delta(f-10k)$$

Since this is not bandlimited, it is not possible to reconstruct the signal from its samples.

(2) Let the signal $x_1(t)$ be band-limited to 2kHz and $x_2(t)$ be bandlimited to 3kHz. Using properties of FT., find the Nyquist rate of sampling for the following signals.

a) $x_1(2t)$ $\rightarrow$ will correspond to $\frac{1}{2} X_1\left(\frac{f}{2}\right)$ bandlimited to 4kHz.

$f_s \geq 8$kHz.

b) $x_2(t-3) \rightarrow e^{-j6\pi f} X_2(f)$ $\quad$ $f_s \geq 6$kHz (same as for $x_2(t)$)

$\downarrow$

just the phase changes.

c) $x_1(t)\, x_2(t) \rightarrow X_1(f) * X_2(f)$ extends to 5kHz.

$f_s \geq 10$kHz.

d) $x_1(t)\cos(1000\pi t) \rightarrow$ by modulation shifts to ± 500Hz.

stretches to 2.5kHz. $\quad$ $f_s \geq 5$kHz.

(3) Sketch the spectra at the output of the following cascaded systems. Assume that $x(t) = 5\,\text{sinc}(5t)$, the sampler has $f_s = 10$Hz and performs ideal sampling (i.e. multiplication by a train of impulses) and the cutoff frequency of the ideal lowpass filter is 5Hz.

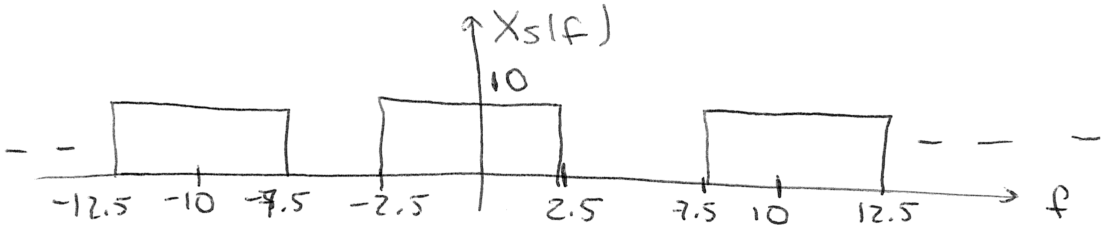
$\sum_{k=-\infty}^{\infty} \delta(t-kT_s)$

$X(f) = \text{rect}\left(\frac{f}{5}\right)$ $\quad$ $x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t-kT_s) \xrightarrow{\mathcal{F}} X(f) * \frac{1}{T_s} \sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T_s}\right)$

$T_s = \frac{1}{10}$

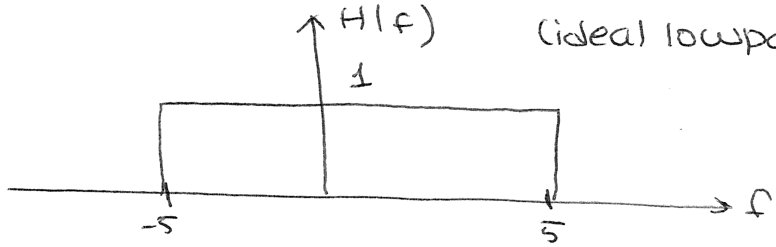
output of the sampler: $X_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T_s}\right)$

$X_s(f) = 10 \sum_{k=-\infty}^{\infty} X(f - 10k)$



LPF

(ideal lowpass filter with cutoff 5Hz)



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