ECE 360 Practice Final Exam
Solutions

(1) a) F

b) F $\left(\frac{4 \cancel{n} \cancel{\pi}}{3} = 2\cancel{\pi} F \cancel{n} \rightarrow F = \frac{4}{6} = \frac{2}{3} \rightarrow N = 3 \right)$

c) T

d) F

e) F

(2) a) $H(z) = \dfrac{K\, z(z-2)}{(z-0.5j)(z+0.5j)}$

$H(1) = \dfrac{K(\cancel{-1})}{(1-0.5j)(1+0.5j)} = \cancel{-}\,2$

$\dfrac{K}{1+0.25} = 2 \quad \rightarrow K = 2.5$

$H(z) = \dfrac{2.5z(z-2)}{(z-0.5j)(z+0.5j)}$

$\dfrac{H(z)}{z} = \dfrac{2.5(z-2)}{(z-0.5j)(z+0.5j)}$

$= \dfrac{A_1}{(z-0.5j)} + \dfrac{A_1^*}{(z+0.5j)}$

$A_1 = \dfrac{2.5(0.5j-2)}{j} = \dfrac{1.25j-5}{j}$

$= 1.25 + 5j$

$\dfrac{(1.25+5j)z}{(z-0.5j)} + \dfrac{(1.25-5j)z}{(z+0.5j)}$

$\boxed{h[n] = 2(5.15)(0.5)^n \cos\left(\frac{\pi}{2} n + 76^\circ\right) u[n]}$

b) System is stable, since poles are inside the unit circle (causal system)

c) $H(z) = \dfrac{Y(z)}{X(z)} = \dfrac{2.5z^2 - 5z}{z^2 + 0.25}$

$z^2 Y(z) + 0.25 Y(z) = 2.5 z^2 X(z) - 5z X(z)$

$\downarrow$

$y[n+2] + 0.25 y[n] = 2.5 x[n+2] - 5x[n+1]$ //

d) $x[n] = (0.5)^n u[n]$

$X(z) = \dfrac{z}{z-0.5}$

$Y(z) = H(z) \cdot X(z) = \dfrac{2.5z^2(z-2)}{(z-0.5)(z-0.5j)(z+0.5j)}$

$\dfrac{Y(z)}{z} = \dfrac{2.5z(z-2)}{(z-0.5)(z-0.5j)(z+0.5j)} = \dfrac{K_1}{(z-0.5)} + \dfrac{A_1}{(z-0.5j)} + \dfrac{A_1^*}{(z+0.5j)}$

$K_1 = \dfrac{(1.25)(-1.5)}{0.5} = -3.75$ //

$A_1 = \dfrac{2.5(0.5\cancel{j})(0.5j-2)}{(0.5j-0.5)(\cancel{j})} = \dfrac{-5+1.25j}{-0.5+0.5j}$

$$A_1 = \frac{-5+1.25j}{-0.5+0.5j} \cdot \frac{(-0.5-0.5j)}{(-0.5-0.5j)} = \frac{2.5+0.625-j0.625+2.5j}{0.25}$$

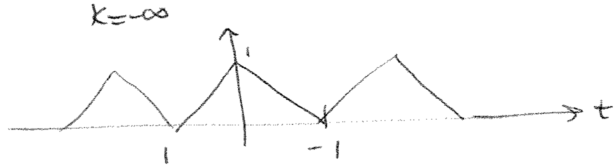
$$= 12.5 + j7.5 \;//$$

$$A_1^* = 12.5 - j7.5$$

$$\frac{Y(z)}{z} = \frac{-3.75}{(z-0.5)} + \frac{12.5+j7.5}{(z-0.5j)} + \frac{12.5-j7.5}{(z+0.5j)}$$

$$y[n] = -3.75\,(0.5)^n u[n] + 2(14.6)\,(0.5)^n \cos\left(\frac{\pi}{2}n + 31^\circ\right) u[n] \;//$$

3) $$x(t) = \sum_{k=-\infty}^{\infty} \text{tri}(t-2k)$$



a) $g(t) = \text{tri}(t)$ $\qquad T = 2 \rightarrow f_0 = 1/2$

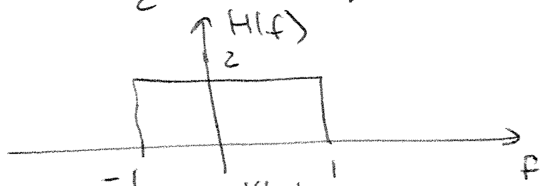
$$G(f) = \text{sinc}^2(f)$$

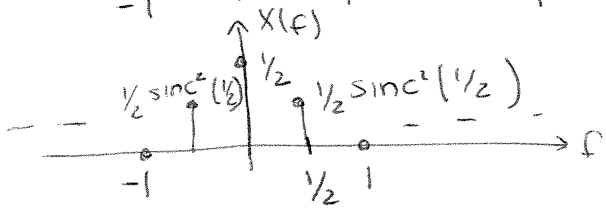
$$X(f) = \sum_{k=-\infty}^{\infty} f_0\, G(kf_0)\, \delta(f-kf_0)$$

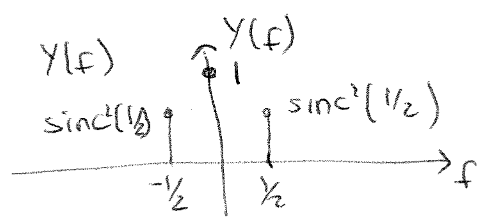
$$X(f) = \sum_{k=-\infty}^{\infty} \frac{1}{2}\, \text{sinc}^2\left(\frac{k}{2}\right) \delta(f-kf_0)$$

b) $X[k] = \frac{1}{2}\,\text{sinc}^2\left(\frac{k}{2}\right)$

c)





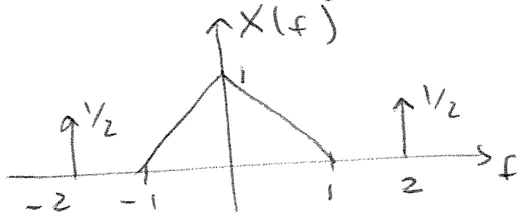


d) $$Y(f) = \delta(f) + \text{sinc}^2\left(\frac{1}{2}\right)\left[\delta\left(f-\frac{1}{2}\right) + \delta\left(f+\frac{1}{2}\right)\right]$$

$$y(t) = 1 + 2\,\text{sinc}^2\left(\frac{1}{2}\right)\cos(\pi t) \;//$$

4) $x(t) = \mathrm{sinc}^2(t) + \cos(4\pi t)$

a) $X(f) = \mathrm{tri}(f) + \frac{1}{2}[\delta(f-2) + \delta(f+2)]$



Highest freq: 2Hz.

Nyquist rate: 4Hz.

b) $f_s = 8\text{Hz} \rightarrow T = 1/8 \text{ sec}$

$$x[n] = x(nT) = \mathrm{sinc}^2\left(\frac{n}{8}\right) + \cos\left(\frac{\pi n}{2}\right)$$

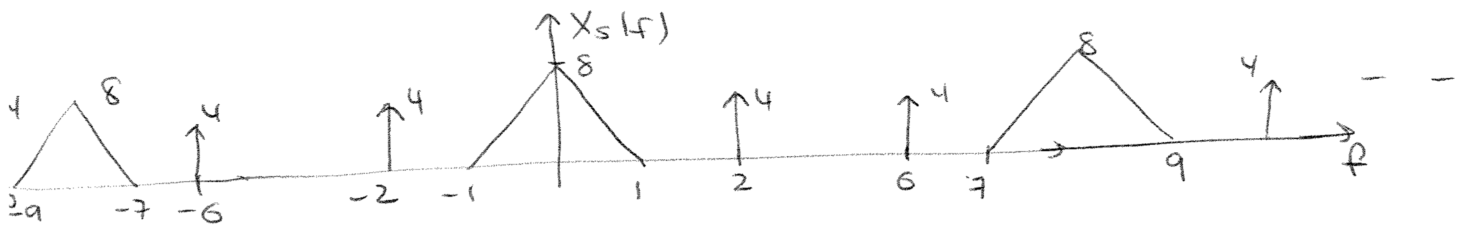
c) $x_s(t) = x(t) \cdot \sum_{k=-\infty}^{\infty} \delta(t - kT)$

sampled signal

$$X_s(f) = X(f) * \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f - \frac{k}{T}\right)$$

$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(f - \frac{k}{T}\right)$$

$$= 8 \sum_{k=-\infty}^{\infty} X(f - k8)$$



d) Lowpass filter $H(f)$ with cutoff freq. between 2 and 6Hz and magnitude 1/8

for example,