Solutions to Practice Exam

Part I

1. D
2. A
3. a) False
   b) False
   c) True
   d) False

4. B
5. B

Part II

1. a)

$$Y(s) = \frac{1}{s} - \frac{1}{s+1}$$

$$X(s) = \frac{1}{s} \qquad (unit \quad step)$$

$$H(s) = \frac{Y(s)}{X(s)} = 1 - \frac{s}{s+1} = \frac{1}{s+1}$$

$$h(t) = e^{-t}u(t)$$

b) $h(t) = \frac{ds(t)}{dt} = e^{-t}u(t)$

c)

$$y(t) = x(t) * h(t)$$

$$= tu(t) * e^{-t}u(t)$$

$$y(t) = 0, \qquad t < 0$$

$$y(t) = \int_0^t (t-\tau)e^{-\tau}\,d\tau, \qquad t \geq 0$$

$$= t + e^{-t} - 1$$

$$y(t) = (t + e^{-t} - 1)u(t)$$

2. a)

$$x(t) = 2\frac{dy(t)}{dt} + y(t)$$

$$2s + 1 = 0, s = \frac{-1}{2}$$

$$y_N(t) = Ke^{-0.5t}$$

$$y_F(t) = C_1\cos(4t) + C_2\sin(4t)$$

$$2\cos(4t) = -8C_1\sin(4t) + 8C_2\cos(4t) + C_1\cos(4t) + C_2\sin(4t)$$

$$C_1 = \frac{2}{65}, C_2 = \frac{16}{65}$$

$$y(t) = Ke^{-0.5t} + \frac{2}{65}\cos(4t) + \frac{16}{65}\sin(4t)$$

$$y(0) = 0 \Rightarrow K = \frac{-2}{65}$$

$$y(t) = \frac{-2}{65}e^{-0.5t} + \frac{2}{65}\cos(4t) + \frac{16}{65}\sin(4t)$$

b) Transient response: $\frac{-2}{65}e^{-0.5t}$

Steady-state response: $\frac{2}{65}\cos(4t) + \frac{16}{65}\sin(4t)$

c) $y_{ZIR}(t) = 2e^{-0.5t}$