Optional HW solutions

1) 11.2

11.2 (Solution)

(a) $x(t) = e^{-2t+4}u(t) = e^4 e^{-2t}u(t), \quad X(s) = \frac{e^4}{s+2}$

(b) $x(t) = te^{-2t+4}u(t) = e^4 te^{-2t}u(t), \quad X(s) = \frac{e^4}{(s+2)^2}$

(c) $x(t) = e^{-2t+4}u(t-1) = e^2 e^{-2(t-1)}u(t-1), \quad X(s) = \frac{e^2 e^{-s}}{s+2}$

(d) $x(t) = [1-e^{-t}]u(t) = u(t) - e^{-t}u(t), \quad X(s) = \frac{1}{s} - \frac{1}{s+1}$

(e) $x(t) = (t-2)u(t-1) = (t-1)u(t-1) - u(t-1), \quad X(s) = \frac{e^{-s}}{s^2} - \frac{e^{-s}}{s}$

(f) $x(t) = (t-2)^2u(t-1) = (t-1-1)^2u(t-1) = [(t-1)^2 - 2(t-1) + 1]u(t-1)$

So, $X(s) = 2\frac{e^{-s}}{s^3} - 2\frac{e^{-s}}{s^2} + \frac{e^{-s}}{s}$

2) 11.7 c,e,f

(c) $H(s) = \frac{4s}{(s+3)(s+1)^2} = \frac{A}{s+3} + \frac{K_0}{(s+1)^2} + \frac{K_1}{s+1} = \frac{-3}{s+3} - \frac{2}{(s+1)^2} + \frac{3}{s+1}$

Note that $K_0 = \frac{4s}{s+3}|_{s=-1}$ and $K_1 = \frac{d}{ds}[\frac{4s}{s+3}]|_{s=-1}$

So, $h(t) = (\cancel{+}{-3}e^{-3t} - 2te^{-t} + 3e^{-t})u(t)$

(e) $H(s) = \frac{2}{(s+2)(s^2+4s+5)} = \frac{A}{s+2} + \frac{K}{s+2+j} + \frac{K^*}{s+2-j} = \frac{2}{s+2} - \frac{1}{s+2+j} + \frac{1}{s+2-j}$

So, $h(t) = [2e^{-2t} - 2e^{-2t}\cos(t)]u(t)$

(f) $H(s) = \frac{4(s+1)}{(s+2)(s^2+2s+2)} = \frac{A}{s+2} + \frac{K}{s+1+j} + \frac{K^*}{s+1-j} = \frac{-2}{s+2} + \frac{1+j}{s+1+j} + \frac{1-j}{s+1-j}$

So, $h(t) = -2e^{-2t}u(t) + 2e^{-t}[\cos(t) + \sin(t)]u(t)$

Or, since $K = 1 + j = \sqrt{2}\angle 0.25\pi$, $h(t) = -2e^{-2t}u(t) + 2\sqrt{2}e^{-t}\cos(t - 0.25\pi)u(t)$

(g) $H(s) = \frac{2(s^2+2)}{(s+2)(s^2+4s+5)} = \frac{A}{s+2} + \frac{K}{s+2+j} + \frac{K^*}{s+2-j} = \frac{12}{s+2} + \frac{-5-j4}{s+2+j} + \frac{-5+j4}{s+2-j}$

So, $h(t) = 12e^{-2t}u(t) + 2e^{-2t}[-5\cos(t) - 4\sin(t)]u(t)$

Or, since $K = -5 - j4 \approx 6.4\angle -141.3°$, $h(t) = [12e^{-2t} + 2(6.4)e^{-2t}\cos(t + 141.3°)]u(t)$

3

11.19 (Properties) $X(s) = \frac{4}{(s+2)^2}$. So, $x(t-2) \Leftrightarrow \frac{4e^{-2s}}{(s+2)^2}$ and $x(2t) \Leftrightarrow \frac{(0.5)4}{(0.5s+2)^2} = \frac{8}{(s+4)^2}$

Also, $x(2t-2) \Leftrightarrow \frac{8e^{-s}}{(s+4)^2}$ and $x'(t) \Leftrightarrow \frac{4s}{(s+2)^2}$

(a) $x'(2t-2) \Leftrightarrow \frac{8se^{-s}}{(s+4)^2}$

(b) $e^{-2t}x(t) \Leftrightarrow \frac{4}{[(s+2)+2]^2} = \frac{4}{(s+4)^2}$

(c) $e^{-2t}x(2t) \Leftrightarrow \frac{8}{[(s+2)+4]^2} = \frac{8}{(s+6)^2}$

form

(d) $e^{-2t-2}x(2t-2) = e^{-2}e^{-2t}x(2t-2) \Leftrightarrow \frac{8e^{-2}e^{-(s+2)}}{[(s+2)+4]^2} = \frac{8e^{-(s+4)}}{(s+6)^2}$

(e) $tx'(t) \Leftrightarrow -\frac{d}{ds}\left[\frac{4s}{(s+2)^2}\right] = \frac{8s}{(s+2)^3} - \frac{4}{(s+2)^2}$

(f) $2te^{-2t}x'(2t-2) \Leftrightarrow -2\frac{d}{ds}\left[\frac{8e^{-(s+2)}}{(s+6)^2}\right] = 16e^{-(s+2)}\frac{(s+8)}{(s+6)^3}$

4

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 2x(t)$$

a) Transfer function: Take Laplace transform of both sides assuming zero initial conditions

$$s^2Y(s) + 3sY(s) + 2Y(s) = 2X(s)$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{2}{s^2+3s+2}$$

b) $h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{(s+1)(s+2)}\right\}$

$$= \frac{K_1}{s+1} + \frac{K_2}{s+2}$$

$$= \frac{2}{s+1} + \frac{-2}{s+2}$$

$h(t) = 2e^{-t}u(t) - 2e^{-2t}u(t)$ //

c) Unit step response.

Do in Laplace domain:

$x(t) = u(t)$

$X(s) = \frac{1}{s}$

$$Y(s) = H(s)X(s)$$

$$= \frac{2}{s(s+1)(s+2)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+1} + \frac{K_3}{s+2}$$

$$= \frac{1}{s} + \frac{-2}{s+1} + \frac{1}{s+2}$$

$$s(t) = y(t) = u(t) - 2e^{-t}u(t) + e^{-2t}u(t)$$

d) $$\frac{ds(t)}{dt} = (2e^{-t} - 2e^{-2t})u(t) = h(t)$$

5) 11.30 b

11.30 (Solution) $x(t) \longrightarrow \boxed{h(t) = 2e^{-t}u(t) - \delta(t)} \longrightarrow y(t)$

Now, $H(s) = \dfrac{2}{s+1} - 1$

(a) $x(t) = e^{-t}u(t)$, $\quad X(s) = \dfrac{1}{s+1}$, $\quad Y(s) = X(s)\left[\dfrac{2}{s+1} - 1\right] = \dfrac{2}{(s+1)^2} - \dfrac{1}{s+1}$

So, $y(t) = 2te^{-t}u(t) - e^{-t}u(t)$.

By convolution, $[2e^{-t}u(t) - \delta(t) \star e^{-t}u(t) = 2te^{-t}u(t) - e^{-t}u(t)$

(b) $x(t) = u(t)$, $\quad X(s) = \dfrac{1}{s}$, $\quad Y(s) = X(s)\left[\dfrac{2}{s+1} - 1\right] = \dfrac{2}{s(s+1)} - \dfrac{1}{s} = \dfrac{1}{s} - \dfrac{2}{s+1}$

So, $y(t) = (1 - 2e^{-t})u(t)$.

By convolution, $[2e^{-t}u(t) - \delta(t) \star u(t) = 2(1 - e^{-t})u(t) - u(t) = (1 - 2e^{-t})u(t)$

(c) $x(t) = \cos(t)$, $\quad \omega = 1$ rad/s. So, $H(s \Rightarrow j) = \dfrac{2}{1+j} - 1 = -j$. So, $y(t) = \sin(t)$