Statistical Partitioning of Wavelet Subbands for Texture Classification

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Wavelets and wavelet packets have been used extensively in texture classification problems. Different features such as maximum absolute amplitude, energy, and entropy have been extracted from the subbands. Therefore, we need to choose a subset of wavelet packet decomposition provides an overcomplete representation.

Problem Statement

How do we choose the 'best' set of subbands for classification?
The 2D discrete wavelet packet transform for an image \( A \) up to level \( P+1 \) is recursively defined as:

\[
\begin{align*}
(1) & \quad (f \in \mathbb{Z} : u, v, w) \mathbb{R} \int du \int dv \int dw |u, w| A_{p}(u,v,w) \hspace{0.5cm} = (f \in \mathbb{R})_{I+1} \mathbb{R} \\
(2) & \quad (f \in \mathbb{Z} : u, v, w) \mathbb{R} \int du \int dv \int dw |u, w| A_{p}(u,v,w) \hspace{0.5cm} = (f \in \mathbb{R})_{I+1} \mathbb{R} \\
(3) & \quad (f \in \mathbb{Z} : u, v, w) \mathbb{R} \int du \int dv \int dw |u, w| A_{p}(u,v,w) \hspace{0.5cm} = (f \in \mathbb{R})_{I+1} \mathbb{R} \\
(4) & \quad (f \in \mathbb{Z} : u, v, w) \mathbb{R} \int du \int dv \int dw |u, w| A_{p}(u,v,w) \hspace{0.5cm} = (f \in \mathbb{R})_{I+1} \mathbb{R}
\end{align*}
\]

The energy distribution over different subbands has been used for texture image classification, (e.g., [1, 2]).
Current Subband Selection Methods

- Bandividually based on a cost function or compare the discrimination power of each subband individually.
- Subband selection methods based on various criteria, such as entropy, energy magnitude, have been proposed (e.g., [3], [4]).
- Current methods evaluate the discrimination power of parent and children nodes in the decomposition tree.
- There is an intrinsic assumption of independence between the features at each node.
- Current methods evaluate the discrimination power of children subtrees and the parent node.

Goal: Incorporate the dependence between subbands into the subband selection algorithm.

Problem: There is an intrinsic assumption of independence between the features at each node.
Overview of the proposed method

1. Partition the subbands based on their dependence, i.e., the subbands in the same group are dependent whereas the subbands from different groups are independent.

2. Choose the subband with the highest energy from each group for classification.

3. Choose the subband with the highest energy from each group for classification.

4. This procedure combines choosing the most significant subbands in terms of energy with independence between selected subbands to increase discrimination power.

Therefore, the proposed algorithm can choose a compact feature set with high discrimination power.
Discovering the structure of statistical dependence.

* Approximation of actual dependencies.
  * Partition the subbands based on pairwise dependence.
  * K-Means:
    - Partition the subbands.
    - $O(NN)$, where $N$ is the number of subbands.

* May have a computational complexity of $O(N^2)$.
  * Compare different partitions using log-likelihood.

* Hypothesis testing:
  - Partitioning the subbands based on their dependence can be approached in different ways such as:

  • Partitioning the subbands based on their dependence can be approached in different ways such as:
For each image $A_i$ in the training set, extract the energy values of all subbands.

For any two subbands $S_i$, $S_j$, compute the mutual information:

$$I(S_i; S_j) = \sum_{s_i \in S_i} \sum_{s_j \in S_j} p(s_i; s_j) \log \frac{p(s_i; s_j)}{p(s_i)p(s_j)}$$

For each subband $S_i$, estimate the marginal p.d.f. of its energy.
For each subband, extract the energy values.

Subband Partitioning Algorithm.

$$\int \frac{\rho(s_i)}{\rho(s_i - s)} \exp \left( \frac{u^2}{2(s_i - s)^2} \right) \sum_{s \in S} \frac{u}{1} = (s)^i S d$$

$$\frac{\int_s d(s_i) d(s_j)}{\int_s d(s_i) d(s_j)} \log \int_s d(s_i) d(s_j) \sum_{s \in S} \sum_{s \in S} = (S_i S_j) I$$
Apply K-Means grouping algorithm to the subband set $\mathcal{U}$, generating subband partition $\mathcal{f} \mathcal{U}$ such that $\mathcal{M} \mathcal{S}$ $\mathcal{i} = 1$ $\mathcal{U}$ $\mathcal{i}$ $\mathcal{=} \mathcal{U}$ and $\mathcal{U}$ $\mathcal{i}$ $\mathcal{\cap}$ $\mathcal{U}$ $\mathcal{j}$ $\mathcal{=} \mathcal{\emptyset}$; $\mathcal{i}$ $\neq$ $\mathcal{j}$.

1. Select $\mathcal{M}$ subbands from $\mathcal{N}$ subbands and set them as initial means $\mathcal{f} \mathcal{¹}$ $\mathcal{1}$ $\mathcal{,}$ $\mathcal{¹}$ $\mathcal{2}$ $\mathcal{,}$ $\mathcal{\ldots}$ $\mathcal{,}$ $\mathcal{¹}$ $\mathcal{M}$ $\mathcal{g}$.

2. Classify the $\mathcal{N}$ subbands. Each subband is classified to the mean $\mathcal{¹}$ $\mathcal{i}$ with which it has the highest mutual information.

3. Recompute the means $\mathcal{f} \mathcal{¹}$ $\mathcal{1}$ $\mathcal{,}$ $\mathcal{¹}$ $\mathcal{2}$ $\mathcal{,}$ $\mathcal{\ldots}$ $\mathcal{,}$ $\mathcal{¹}$ $\mathcal{M}$ $\mathcal{g}$ based on the partition $\mathcal{f} \mathcal{\ldots}$ $\mathcal{,}$ $\mathcal{\Pi}$ $\mathcal{1}$ $\mathcal{,}$ $\mathcal{\Pi}$ $\mathcal{2}$ $\mathcal{,}$ $\mathcal{\ldots}$ $\mathcal{,}$ $\mathcal{\Pi}$ $\mathcal{M}$ $\mathcal{g}$.

4. Repeat steps (2) and (3) until the means do not change.

Select the subband with the highest energy value from each sub-set. From the sub-set $\mathcal{U}$ $\mathcal{i}$ of subbands, a subband $\mathcal{S}$ $\mathcal{S}$ $\mathcal{i}$ is selected, such that $\mathcal{i}$ $=$ $\mathop{\text{arg max}}_{\mathcal{j}}$ $\mathcal{E}$ ($\mathcal{S}$ $\mathcal{j}$) $\mathcal{j}$ $\mathcal{\in} \mathcal{U}$ $\mathcal{i}$ where $\mathcal{E}$ is the energy. The set of sub-bands with the highest energy value from each sub-set is formed.

Apply K-Means grouping algorithm to the subband set $\mathcal{N}$.
Classification experiments are conducted on Brodatz texture image database with 1200 images [5].

- The image size in the database is $128 \times 128$. Half of the database is used for training and the other half is used for testing.
- The wavelet packet decomposition is conducted using different wavelet bases ('haar', 'db6', 'sym4'), and the wavelet packet decomposition level is set to 3.
- For this wavelet packet decomposition, there are $N = 1 + 4 + 16 + 64 = 85$ subbands.

Selection based on multiple wavelet families is also considered.

- Selection based on energy-based selection.
- The results are compared to energy-based selection.

Performance Evaluation
Single Wavelet Basis Results

Figure 1: Classification Error Rate with SGS and Traditional Energy

Base Subband Selection
Multiple Wavelet Bases Results

Figure 2: Classification Error Rate with SGS and Traditional Energy Based Subband Selection using Multiple Wavelet Bases

Multie Wavelet Bases Results
Conclusions

A new subband grouping and selection (SGS) algorithm is proposed for subband selection in wavelet packet decomposition. The proposed method can be extended to overcomplete dictionaries. The proposed method can be extended to overcomplete dictionaries. The SGS algorithm provides improvement over standard subband selection algorithms. The algorithm takes the dependence between subbands into account as well as the information provided by each subband. The proposed method can be extended to overcomplete dictionaries. The proposed method can be extended to overcomplete dictionaries. The proposed SGS algorithm is defined as follows:

1. Calculate the subband energies for each subband.
2. Select the subband with the highest energy as the most discriminative subband.
3. Remove the selected subband and repeat the process until all subbands have been selected.
4. Discard subbands that have a high correlation with previously selected subbands.

The proposed SGS algorithm provides significant improvements over standard subband selection algorithms. The algorithm is capable of selecting subbands that are highly discriminative, while also achieving sparsity simultaneously.
References


