Name: ______________________________

Student ID: ___________________________

ECE 360 FINAL EXAM
December 9, 2002

- No textbooks, notes or HW solutions.
- One page of hand-written notes.
- Calculators are allowed.
- Exam is 2 hours.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [10] Determine whether the following statements are true or false.

a) The exponential Fourier series coefficients, $X[k]$ of a real and periodic signal, $x(t)$, have the following property: $X[-k] = X^*[k]$

T

b) The system defined by $y[n] = r^n x[n]$ is time-invariant.

F

c) The discrete-time signal $x[n] = e^{jn}$ is periodic.

F

d) Decimation by a factor of N implies a N – fold reduction in the sampling rate.

T

e) The input signal $x(t)$ determines the natural response of a continuous time LTI system.

F

2. [30] A causal LTI system has impulse response, $h[n]$, for which the z-transform is

$$H(z) = \frac{1+z^{-1}}{(1-0.5z^{-1})(1+0.25z^{-1})}$$

a) [5] What is the region of convergence of $H(z)$?
b) [5] Is the system stable? Explain.
c) [10] Find the impulse response $h[n]$.

d) [10] Find the output $y[n]$ of the system if the input is $x[n] = \left(\frac{1}{2}\right)^n u[n]$ in the time domain using convolution.
Note: You will not receive full credit if you solve this problem in the z-domain.

Hint: $\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}$

a) $|z| > 0.5$ (anything else gets at most 2 pts.)

b) stable (3)
Exp. (2)

c) $H(z) = \frac{K_1}{1-0.5z^{-1}} + \frac{K_2}{1+0.25z^{-1}}$ → 4 2

$K_1 = \frac{1+2}{1.5} = 2$ → 2

$K_2 = \frac{1-4}{3} = -1$ → 2

$h[n] = 2(0.5)^n u[n] - (-0.25)^n u[n]$ 4 3 4

d) $y[n] = x[n] * h[n]$ 1

$y[n] = (0.5)^n u[n] * \left[2(0.5)^n u[n] - (-0.25)^n u[n]\right]$ 4

$\sum_{k=0}^{n} (0.5)^k \cdot 2(0.5)^{n-k} - \sum_{k=0}^{n} (0.5)^k (-0.25)^{n-k}$

$= 2(0.5)^n (n+1) - (-0.25)^n \frac{1-(-2)^{n+1}}{3}$ 5

$(0.5)^n \quad \frac{1-(-0.5)^{n+1}}{0.5}$

$\sum_{k=0}^{n} (0.5)^{n-k} (-0.25)^k$

$(0.5)^n \quad \left(\frac{-0.25}{0.5}\right)^k$

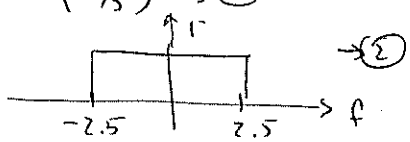
3. [30] For the signal $x(t) = 5sinc(5t)$,

a) [4] Sketch the Fourier transform of this signal, $X(f)$.
b) [4] What is the Nyquist sampling frequency in Hz ?
c) [4] If the signal is sampled at 10Hz, write the sampled signal $x[n]$.
d) [10] Sketch the Fourier transform of the sampled signal.

Hint: The sampled signal is $x_s(t) = x(t)\sum_{k=-\infty}^{\infty}\delta(t-kT)$, where T is the sampling period corresponding to the sampling frequency given in part c.

e) [8] Suggest a method for recovering the spectrum of the continuous time signal from the spectrum of the sampled signal.
Hint: This will require filtering of the spectrum in part d. Specify the type of the filter, the cutoff frequencies and the appropriate magnitude and phase responses.

a) $X(f) = rect(f/5) \rightarrow 2$



$\rightarrow 2$

b) $f_s = 5$ Hz.

c) $T = 1/10 \rightarrow 1$ $\quad x[n] = x(nT)$ 1

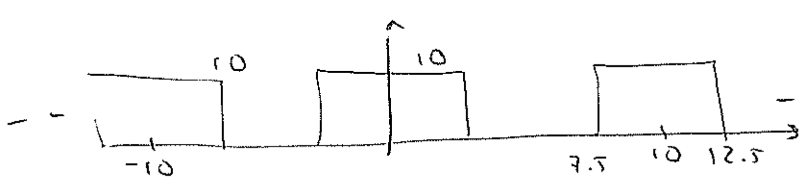
$x[n] = 5 \, sinc\left(\frac{5n}{10}\right) = 5 \, sinc\left(\frac{n}{2}\right)$ 2

d) $x_s(t) = x(t) \sum_{k=-\infty}^{\infty} \delta(t-kT)$

$X_s(f) = X(f) * \frac{1}{T}\sum_{k=-\infty}^{\infty} \delta\left(f-\frac{k}{T}\right) \rightarrow 2$

$= \frac{1}{T}\sum_{k=-\infty}^{\infty} X\left(f-\frac{k}{T}\right) \rightarrow 1$

$= 10 \sum_{k=-\infty}^{\infty} X(f-10k) \rightarrow 1$



$\rightarrow 5$

2 → amp.
2 → freq.
1 → periodic.

e) Lowpass filter 2

cutoff freq. 3

Magnitude 3 $= \frac{1}{10}$

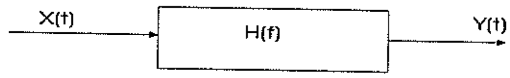
4. [30] Suppose that $x(t) = |\sin(\pi t)|$

a) [10] Find the Fourier transform, $X(f)$, of $x(t)$.

b) [5] Find the exponential Fourier series coefficients, $X[k]$, of $x(t)$.

Hint: Use the result you found in part a.

c) [15] Assume that this signal is passed through a filter with frequency response



$$H(f) = \begin{cases} 1, & -1.5 \le f \le 1.5 \\ 0, & otherwise \end{cases}$$

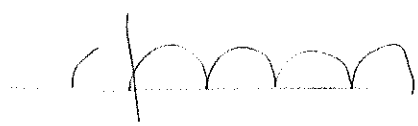
Sketch the spectrum of the output, $Y(f)$, and find the output signal $y(t)$.

Hints: $sinc(k) = 0$, when k is an integer and $k \neq 0$.

$\sin c(0.5) = \sin c(-0.5) = \frac{2}{\pi}$

$\sin c(1.5) = \sin(-1.5) = \frac{-2}{3\pi}$

a)



$f_0 = 1 \rightarrow$ 1

$g(t) = \sin(\pi t)\,rect(t - 1/2) \rightarrow$ 2

$G(f) = \cancel{\sin}\ 0.5j\,[\delta(f+0.5) - \delta(f-0.5)] * sinc(f)e^{-j\pi f} \rightarrow$ 2

$= 0.5j\,[sinc(f+0.5)e^{-j\pi(f+0.5)} - sinc(f-0.5)e^{-j\pi(f-0.5)}] \rightarrow$ 2

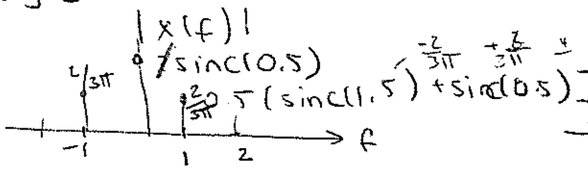
$= 0.5\,[e^{-j\pi f} sinc(f+0.5) + sinc(f-0.5)\, e^{-j\pi f}]$

$X(f) = \sum_{k=-\infty}^{\infty} f_0\, G(kf_0)\,\delta(f - kf_0)$

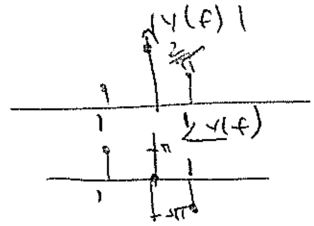
$= \sum_{k=-\infty}^{\infty} 0.5e^{-j\pi k}\,[sinc(k+0.5) + sinc(k-0.5)]\,\delta(f-k) \rightarrow$ 3

b) $X[k] = 0.5e^{-j\pi k}[sinc(k+0.5) + sinc(k-0.5)]$

c)



$\rightarrow$ 3



$\rightarrow$ 3

3

Extra Sheet for Question 4:

$$Y(f) = \frac{4}{\pi}\,\delta(f) - \frac{2}{3\pi}\left[\delta(f-1) + \delta(f+1)\right] \rightarrow 3$$

$$y(t) = \frac{4}{\pi} - \frac{4}{3\pi}\cos(2\pi t) \rightarrow 3$$