ECE 360 Frequency Response and Filtering Examples

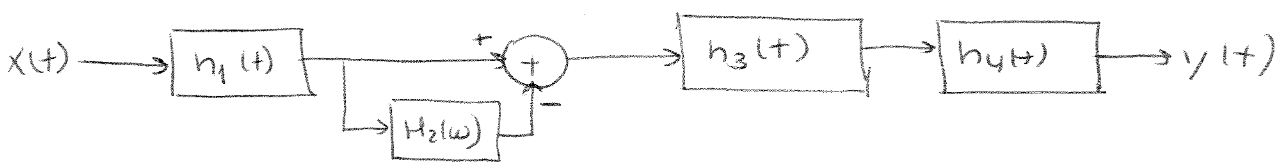
① Consider the interconnection of 4 LTI systems, where

$$h_1(t) = \frac{d}{dt}\left[\frac{\sin \omega_c t}{2\pi t}\right]$$

$$H_2(\omega) = e^{-j\frac{2\pi\omega}{\omega_c}}$$

$$h_3(t) = \frac{\sin 3\omega_c t}{\pi t}$$

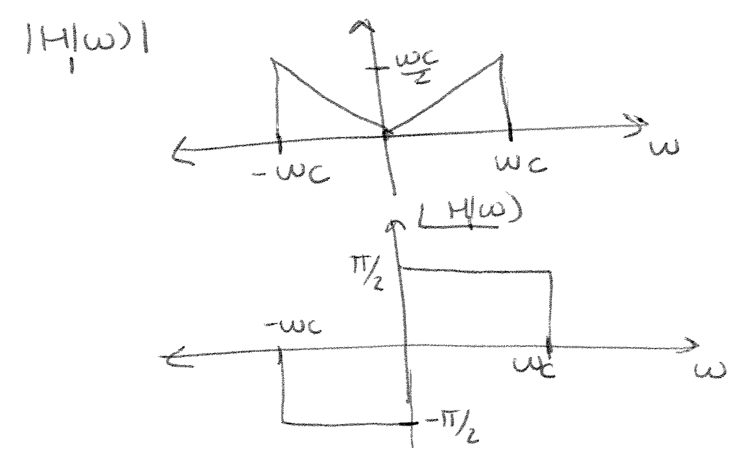
$$h_4(t) = u(t)$$



a) Determine and sketch $H_1(\omega)$

$$\mathcal{F}\left\{\frac{d}{dt}\left[\frac{\sin\omega_c t}{2\pi t}\right]\right\} \rightarrow j\omega\, \mathcal{F}\left\{\frac{\sin\omega_c t}{2\pi t}\right\}$$

$$= \frac{j\omega}{2}\, \text{rect}\left(\frac{\omega}{2\omega_c}\right)$$



b) What's the impulse response of the entire system?

In time-domain : $(h_1(t) - h_1(t) * h_2(t)) * h_3(t) * h_4(t)$

In frequency domain: $(H_1(\omega) - H_1(\omega) H_2(\omega))\, H_3(\omega)\, H_4(\omega)$

$$H_3(\omega) = \text{rect}\left(\frac{\omega}{6\omega_c}\right)$$

$$H_4(\omega) = \frac{1}{j\omega} + \pi\delta(\omega)$$

$$H_3(\omega)\, H_4(\omega) = \frac{1}{j\omega}\, \mathrm{rect}\left(\frac{\omega}{6\omega_c}\right) + \pi\,\delta(\omega)$$

$$H_1(\omega)\,(1 - H_2(\omega)) = \frac{j\omega}{2}\, \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right)\left(1 - e^{-j\frac{2\pi\omega}{\omega_c}}\right)$$

$$H(\omega) = \frac{1}{2}\, \mathrm{rect}\left(\frac{\omega}{6\omega_c}\right) \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right) \left(1 - e^{-j\frac{2\pi\omega}{\omega_c}}\right)$$

$$H(\omega) = \frac{1}{2}\, \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right) \left(1 - e^{-j\frac{2\pi}{\omega_c}\omega}\right)$$

$\downarrow$

$$h(t) = \frac{1}{2}\left[\frac{\sin \omega_c t}{\pi t} - \frac{\sin\left(\omega_c\left(t - \frac{2\pi}{\omega_c}\right)\right)}{\pi\left(t - \frac{2\pi}{\omega_c}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{\sin \omega_c t}{\pi t} - \frac{\sin(\omega_c t - 2\pi)}{\pi\left(t - \frac{2\pi}{\omega_c}\right)}\right]$$

$$= \frac{1}{2}\left[\frac{\sin \omega_c t}{\pi t} - \frac{\sin \omega_c t}{\pi\left(t - \frac{2\pi}{\omega_c}\right)}\right] //$$

c) What's the output $y(t)$ when the input is

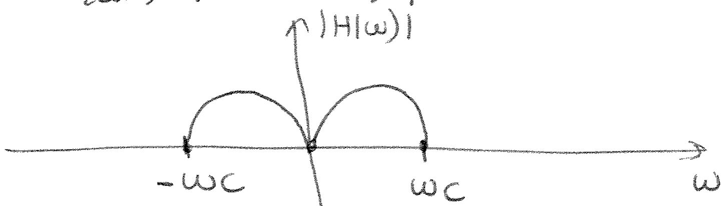
$$x(t) = \sin 2\omega_c t + \cos\left(\frac{\omega_c t}{2}\right)$$

$$H(\omega) = \frac{1}{2}\, \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right) e^{-j\frac{\pi}{\omega_c}\omega} \underbrace{\left(e^{j\frac{\pi}{\omega_c}\omega} - e^{-j\frac{\pi}{\omega_c}\omega}\right)}_{2j\sin\left(\frac{\omega\pi}{\omega_c}\right)}$$

$$H(\omega) = \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right) \sin\left(\frac{\omega\pi}{\omega_c}\right) e^{-j\left(\frac{\pi}{\omega_c}\omega + \frac{\pi}{2}\right)}$$

$$|H(\omega)| = \mathrm{rect}\left(\frac{\omega}{2\omega_c}\right) \left|\sin\left(\frac{\omega\pi}{\omega_c}\right)\right|$$

$$\angle H(\omega) = \begin{cases} -\left(\frac{\pi}{\omega_c}\omega + \frac{\pi}{2}\right) & 0 < \omega < \omega_c \\ -\frac{\pi}{\omega_c}\omega + \frac{\pi}{2} & -\omega_c < \omega < 0 \end{cases}$$



$\sin 2\omega_c t$ will not pass through the system since $2\omega_c$ is not in the passband.

$$\cos\left(\frac{\omega_c t}{2}\right) \rightarrow \left|H\left(\frac{\omega_c}{2}\right)\right| \cos\left(\frac{\omega_c t}{2} + \angle H\left(\frac{\omega_c}{2}\right)\right)$$

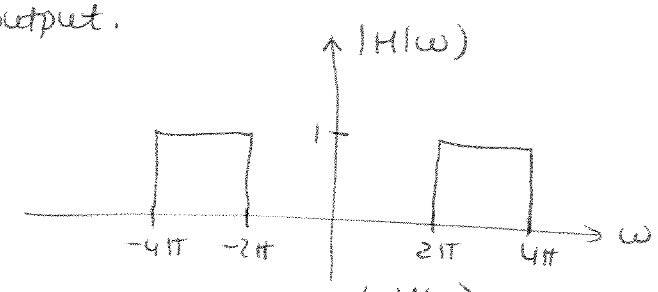
$\left|H\left(\frac{\omega_c}{2}\right)\right| = 1$ $\quad \angle H(\omega_c/2) = 0$ $\quad$ output: $\cos\left(\frac{\omega_c t}{2}\right)$

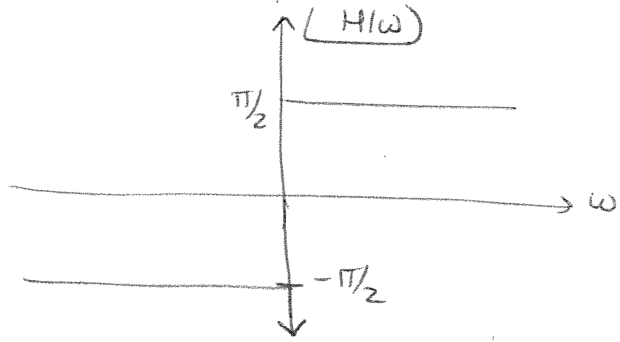
② A periodic signal $x(t)$ with period $T=2$ has F.S. coefficients

$$X[k] = \begin{cases} 0 & k=0 \\ 0 & k \text{ is even} \\ 1 & \text{if } k \text{ is odd} \end{cases}$$

The signal $x(t)$ is passed through the following filter. Determine the system output.

$f_0 = 1/2$
$\omega_0 = 2\pi f_0 = \pi$ //

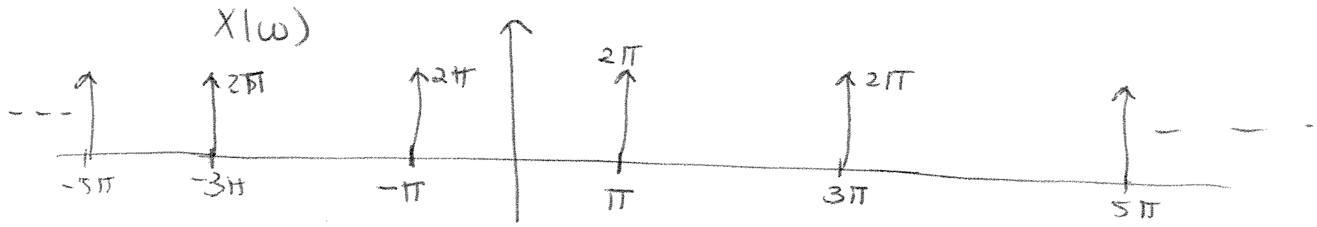




$X(\omega) \Rightarrow$

$$x(t) = \sum_{\substack{k=-\infty \\ k,\text{odd}}}^{\infty} e^{-j\omega_0 k t} = \sum_{\substack{k=-\infty \\ k,\text{odd}}}^{\infty} e^{-j\pi k t}$$

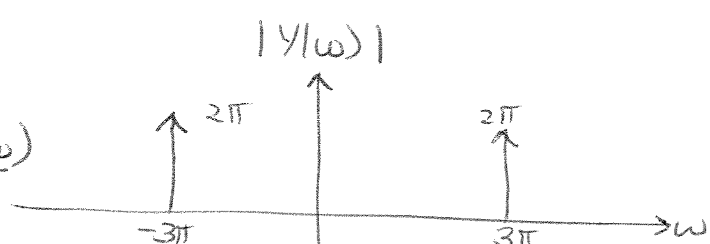
$$X(\omega) = 2\pi \sum_{\substack{k=-\infty \\ k,\text{odd}}}^{\infty} \delta(\omega + k\pi)$$

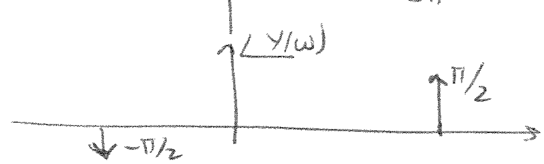


$Y(\omega) = X(\omega)\,H(\omega)$

$|Y(\omega)| = |X(\omega)|\,|H(\omega)|$

$\angle Y(\omega) = \angle X(\omega) + \angle H(\omega)$





$$Y(\omega) = 2\pi j\delta(\omega - 3\pi) - 2\pi j\,\delta(\omega + 3\pi)$$

$$Y(\omega) = 2\pi j\,[\delta(\omega - 3\pi) - \delta(\omega + 3\pi)]$$

$$\downarrow$$

$$y(t) = j\left[e^{j3\pi t} - e^{-j3\pi t}\right]$$

$$= j\,(2j\sin(3\pi t))$$

$$y(t) = -2\sin(3\pi t)$$

(3) The signal $x(t) = 4 + \cos(4\pi t) - \sin(8\pi t)$ is the input to a filter whose impulse response $h(t)$ is: Find $y(t)$

a) $h(t) = \mathrm{sinc}(5t)$ input freq: 0, 2, 4 Hz.

$$\downarrow$$

$$H(f) = \frac{1}{5}\,\mathrm{rect}\left(\frac{f}{5}\right)$$ (lowpass filter)

$$H(0) = \frac{1}{5}$$

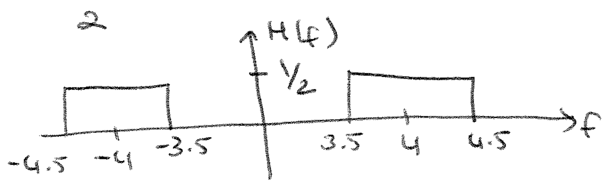
$$H(2) = 1/5$$

$$H(4) = 0$$

$$y(t) = \frac{4}{5} + \frac{1}{5}\cos(4\pi t)$$

b) $h(t) = \mathrm{sinc}(t)\cos(8\pi t)$ using the modulation property

$$H(f) = \frac{1}{2}\,[\mathrm{rect}(f+4) + \mathrm{rect}(f-4)]$$



$\Rightarrow$ bandpass filter

Input frequencies: 0, 2, 4 Hz. $\rightarrow$ only 4 Hz gets out of the filter

$$-\sin(8\pi t) = -\cos(8\pi t - 90°) \rightarrow \boxed{H(f)} \rightarrow \frac{-1}{2}\cos(8\pi t - 90°)$$

$$= -\frac{1}{2}\sin(8\pi t)$$

$$y(t) = -\frac{1}{2}\sin(8\pi t)$$