# ECE 202

## Exam II

**July 28, '06**

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**NOTE:**

1. **You must place your answers on the lines provided.**

2. **Some answers may have little or no partial credit. Please check your work.**

3. **Some answers may have partial credit, so please show your work.**

4. **This exam has 4 problems, check to see you have 4 problems.**

5. **All answers that have units must be indicated. All answers must be in decimal form and in engineering notation.**

6. **When you stand-up to turn-in your exam, you will not be allowed to change your answers.**
Form the mesh equations by inspection in terms of only $\vec{I}_1$ and $\vec{I}_2$ as unknowns. Do not solve for $\vec{I}_1$ and $\vec{I}_2$. Express final entries as a single complex number in rectangular form.

$12 \angle 60^\circ = 6 + j10.39$

$$\begin{bmatrix} 6 + j10.39 \\ -16 \vec{I}_x \end{bmatrix} = \begin{bmatrix} 80 + j63 + 33 - j18 \\ -33 + j18 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$$

$16 \vec{I}_x = 16 \vec{I}_1 - 16 \vec{I}_2$

$$\begin{bmatrix} 6 + j10.39 \\ 0 \end{bmatrix} = \begin{bmatrix} 113 + j45 \\ -33 + j18 + 16 \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$$

$$\begin{bmatrix} 6 + j10.39 \\ 0 + j0 \end{bmatrix} = \begin{bmatrix} \frac{113}{(-17)} + j \frac{45}{18} \\ \frac{-33}{92} + j \frac{18}{26} \end{bmatrix} \begin{bmatrix} \vec{I}_1 \\ \vec{I}_2 \end{bmatrix}$$

$-2 + 2j/Answer$
2) **Find the waveform $f(t)$ from $F(s)$ by doing a PFE**

A) \[ F(s) = \frac{11(s+3)}{s(s+4)} = \frac{k_1}{s} + \frac{k_2}{s+4} \]

\[ k_1 = 11, \quad \frac{11(s+3)}{s(s+4)} \bigg|_{s=0} = \frac{11(0+3)}{(0+4)} = \frac{33}{4} = 8.25 \]

\[ k_2 = \frac{(s+4)11(s+3)}{s(s+4)} \bigg|_{s=-4} = \frac{11(-4+3)}{-4} = \frac{11}{4} = 2.75 \]

\[ F(s) = \frac{8.25}{s} + \frac{2.75}{s+4} \]

\[ f(t) = (8.25 + 2.75e^{-4t})u(t) \]

\[ f(t) = \left(\frac{8.25 + 2.75e^{-t/250m}}{3} \right) u(t) \] \[(10)\]

B) \[ F(s) = \frac{(s+7)}{(s+6)(s^2+6s+25)} \]

\[ k_{10} = -2, \quad F(\infty) = -6 \pm \frac{\sqrt{36-4(25)}}{2} = -6 \pm j8 \]

\[ F(s) = \frac{k_1}{s+6} + \frac{k_2}{s+3+j4} + \frac{k_3}{s+3-j4} \]

\[ k_1 = \frac{(s+6)}{(s+6)(s^2+6s+25)} \bigg|_{s=-6} = \frac{-6+7}{36-36+25} = \frac{1}{25} = 0.04 \]

\[ k_2 = \frac{(s+3-j4)}{(s+6)(s^2+6s+25)} \bigg|_{s=-3+j4} = \frac{-3+j4+7}{(-3+j4+6)(j8)} = \frac{4+j4}{(3+j4)(j8)} = \frac{4+j4}{-32+j24} = \frac{5.66}{40} \frac{45^\circ}{43.1} \]

\[ 141.5m \angle -98.13^\circ \]

\[ f(t) = \left[40e^{-6t} + 2(141.5m)e^{-3t} \cos(4t - 98.13^\circ) \right] u(t) \]

\[ f(t) = \left[40e^{-t/166m} + 283m e^{-t/333m} \cos(4t - 98.13^\circ) \right] u(t) \] \[(15)\]
Use Laplace Transforms and a PFE to find $V_c(t)$.

- **Step 1) Find Initial Conditions**

For $t < 0$

$V_c(0^-) = 9V$

- **Step 2) Transform Circuit to the s-Domain**

For $t > 0$

$$\frac{1}{s(0.33\mu F)} = \frac{3.03(10^6)}{s}$$

- **Step 3) Perform s-Domain Analysis**

Using Superposition,

$$V_c(s) = V_c'(s) + V_c''(s)$$
3) **ADDITIONAL WORK SPACE**

a) \[ V_c'(s) \text{ (SET } \frac{9}{s} = 0) \]

\[
V_c'(s) = \frac{12}{s} \cdot \frac{\frac{3.03 \times 10^6}{24 + \frac{3.03 \times 10^6}{s}}}{s} = \frac{12}{s} \cdot \frac{3.03 \times 10^6}{24s + 3.03 \times 10^6} = \frac{12}{s} \cdot \frac{126.25K}{5 + 126.25K}
\]

b) \[ V_c''(s) \text{ (SET } \frac{12}{s} = 0) \]

\[
V_c''(s) = \frac{9}{s} \cdot \frac{24}{24 + \frac{3.03 \times 10^6}{s}} = \frac{9 \cdot 24}{24s + 3.03 \times 10^6} = \frac{9}{s} \cdot \frac{126.25K}{5 + 126.25K}
\]

c) \[ V_c(s) = V_c' + V_c'' = \frac{(12)(126.25K) + 9s}{s(5 + 126.25K)} = \frac{9s + 1.515 \times 10^6}{s(5 + 126.25K)} \]

- **STEP 4)** PERFORM A PFE

\[
V_c = \frac{\frac{k_1}{s}}{5 + 126.25K}
\]

\[
k_1 = \left. \frac{9s + 1.515 \times 10^6}{s(5 + 126.25K)} \right|_{s=0} = \frac{1.515 \times 10^6}{126.25K} = 12
\]

\[
k_2 = \left. \frac{9s + 1.515 \times 10^6}{s(s + 126.25K)} \right|_{s=-126.25K} = \frac{378.75K}{126.25K} = -3
\]

- **STEP 5)** FIND THE INVERSE LAPLACE TRANSFORM

\[
V_c = \frac{12}{s} - \frac{3}{s + 126.25K}
\]

\[
V_c(t) = \left[ 12 - 3 \ e^{-126.25Kt} \right] u(t)
\]

\[
V_c(t) = \left[ 12 - 3 \ e^{-t/7.92\mu} \right] u(t) \ V \quad (26)
\]
4) Answer the following:

A) In your own words and in a complete sentence, what condition is needed for a circuit to be at its resonant frequency?

5) **The input impedance must be purely real for a circuit to be at resonance.**

B) If a resistor is dissipating the following amount of power: 80 \([4 + 4 \cos (2\pi \cdot 200 \cdot t)]\) watts, what is the average power dissipated?

\[
P_{\text{ave}} = \frac{1}{T} \int_0^T p(t) \, dt = (80)(4)
\]

\[
P_{\text{ave}} = 320 \text{ W}
\]

C) In your own words and in a complete sentence, what condition is needed for a rational function to be improper?

5) **A rational function is improper when the order of the numerator polynomial equals or exceeds the order of denominator polynomial.**

D) What is \(f(t)\) if \(F(s) = \frac{25}{s^2 + 600}\)?

\[
F(s) = \frac{25}{s^2 + 600} = \frac{25}{\sqrt{600}} \frac{1}{s^2 + 600} = 1.021 \frac{\sqrt{600}}{s^2 + 600}
\]

\[
\sqrt{600} = 24.5 \quad \sin(\theta) = \frac{B}{B^2 + P^2}
\]

\[
f(t) = 1.021 \sin(24.5t) u(t)
\]

E) Sketch \(v(t) = 6u(t) - 3u(t - 2) - 3u(t - 4)\) for \(0 < t < 8 \text{ sec.}\)