

Introduction to ECE 802

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Introduction

- Classical signal analysis focuses on two domains
 - Time Domain: Amplitude, Energy, Correlation
 - Frequency Domain: Power, Spectrum, Bandwidth
- Frequency domain analysis is done through **FOURIER TRANSFORM**

Time Domain Analysis

- In the time domain, any signal can be expressed as:

$$x(t) = \int_{-\infty}^{\infty} x(\tau) \delta(t - \tau) d\tau$$

- Expanded in terms of an infinite number of delta functions over the real axis with expansion coefficients equal to amplitude.

Spectrum Analysis

- Fourier transform is defined to be:

$$S(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} s(t) e^{-j\omega t} dt$$

$$s(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} S(\omega) e^{j\omega t} d\omega$$

- Decompose a given time signal in terms of *eternal* sinusoids (complex exponentials)
- Expansion Coefficients=Spectral Power
- What is wrong with this??

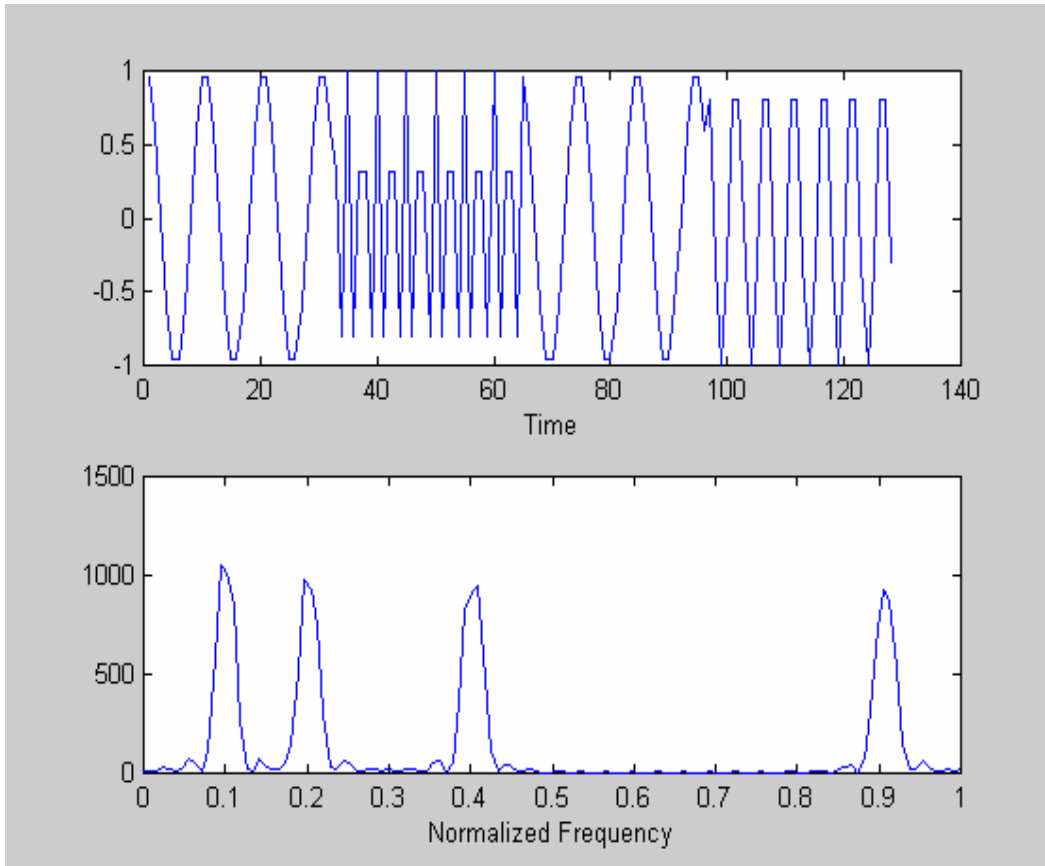
Disadvantages

- Not all signals are eternal → Most signals are time-limited.
- Gives us the *global* picture, no local information
- Most real life signals are not combinations of sinusoids.
- Fourier transform is sufficient for stationary signal analysis and LTI systems.
- How does the frequency change with time?

Concept of Transients

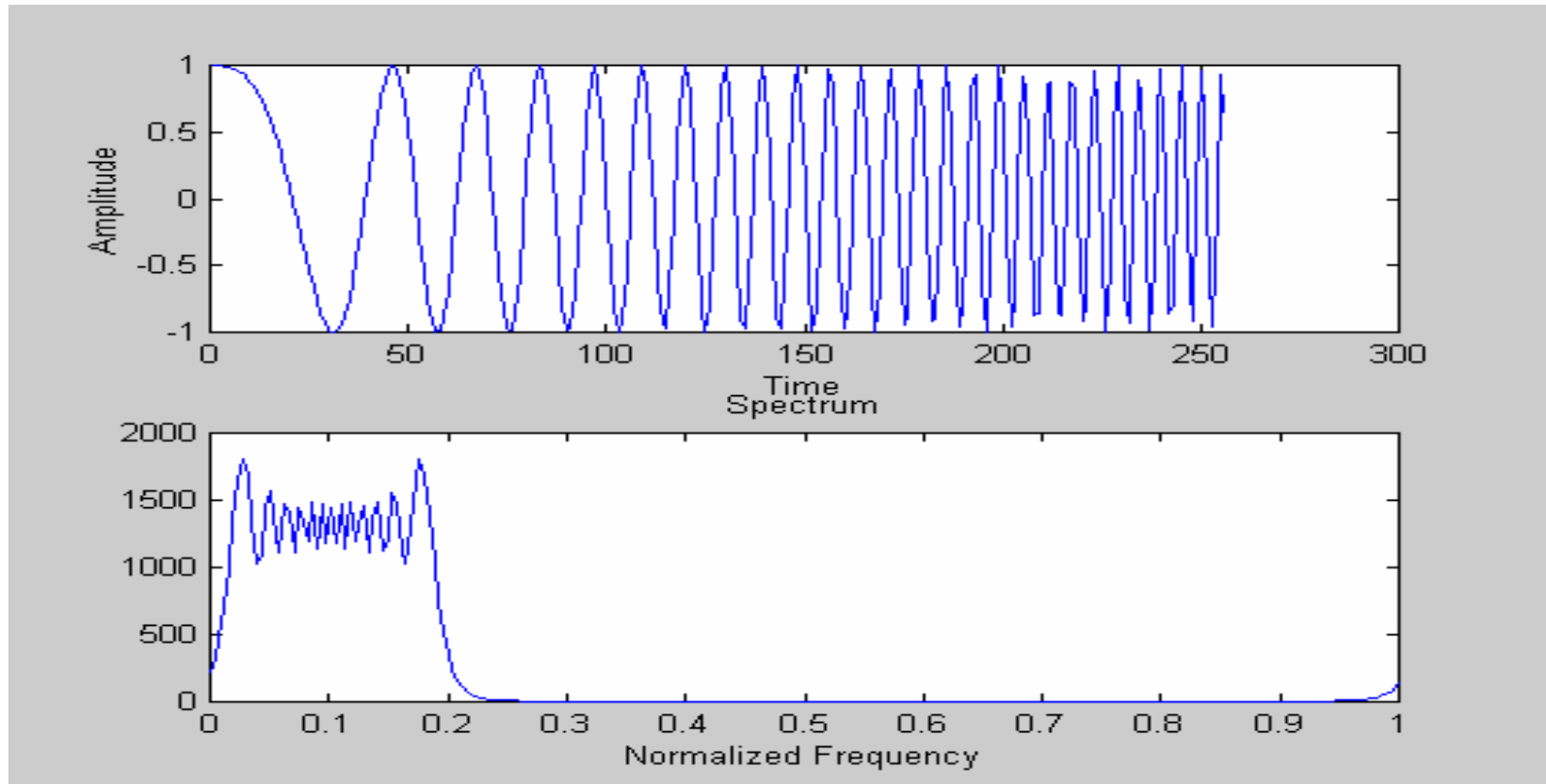
- Transient: Any phenomena that is not stationary, has finite duration
- The human sensory system concentrates on the transients rather than the stationary for selecting important information.
- Examples:
 - A word pronounced at a particular time
 - Epilepsy pattern in brain waves
 - An object located in the corner of an image
- Most of signal processing is devoted to analyzing stationary signals and LTI systems.

Example



➔ No temporal localization

Example



Wavelet Analysis

- Therefore, expand signals in terms of localized functions (wavelets)

$$x(t) = \sum_k a_k \psi_k(t)$$

- $a_k \rightarrow$ expansion coefficients
- $\psi_k(t) \rightarrow$ set of decomposing functions
- Compare this to Fourier series

Properties

- We will study how to choose these functions
 - Are there some particular functions to choose?
 - Is it signal adaptive?
 - General properties
 - How to design wavelets?

Tools

- In order to determine, the ‘best’ set of functions to expand our signals, we need to define a space for our signals → **Vector space**
- After the ‘signal space’ is defined, we need to define a measure for quantifying how good any approximation is → **Inner product**
- Different sets of expansion functions → **Basis**

Applications

- Biomedical Signal Processing
- Speech Processing
- Radar Analysis, Target Detection
- Blind Source Separation
- Image Compression (JPEG2000)
- Object Recognition