ECE 802-601 Homework 4 Solutions

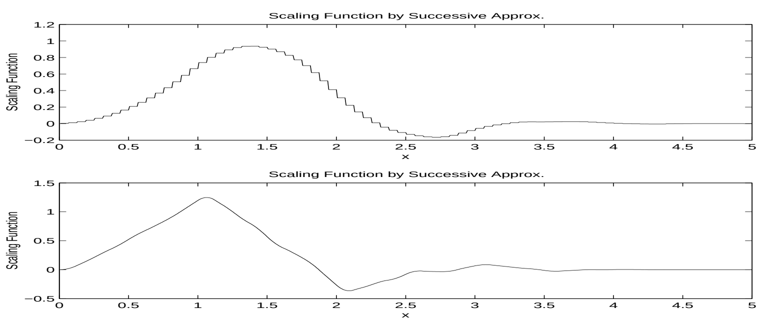
1. a)For length 6 Daubechies filter, $N = 6$ and the number of vanishing moments $K = 3$. Therefore, $H(\omega)$ can be written as

$$
\begin{aligned}
H(\omega) &= \sqrt{2}(\frac{1+e^{-j\omega}}{2})^3 R(e^{-j\omega}) \\
R(z)R(z^{-1}) = Q(z^{-1}) &= P(\frac{2-z-z^{-1}}{4}) \\
P(y) &= \sum_{k=0}^{2} \binom{2+k}{k} y^k \qquad (1)
\end{aligned}
$$

Inserting $\frac{2-z-z^{-1}}{4}$ for y gives the following polynomial for $Q(z^{-1})$.

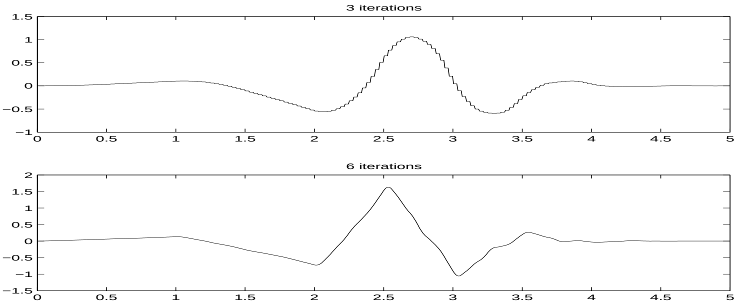
$$Q(z^{-1}) = 3z^2 - 18z + 38z - 18z^{-1} + 3z^{-2} \qquad (2)$$

The roots of this polynomial are found to be $2.7127+1.4439j, 2.7127-1.4439j, 0.2873+0.1529j, 0.2873-0.1529j$. Since we are interested in designing the minimum phase filter, we choose the roots that are inside the unit circle, $0.2873+0.1529j, 0.2873-0.1529j$. Now, we can write $R(e^{-jw}) = A(e^{-j\omega} - r_1)(e^{-j\omega} - r_1^*)$, where r_1 equals to $0.2873 + 0.1529j$. Since $R(e^{-j\omega}) = 1$ at $\omega = 0$, A comes out to be equal to 1.8821. Writing $H(z)$ back and reading off the coefficients gives, $h(0) = 0.3327, h(1) = 0.8069, h(2) = 0.4598, h(3) = -0.1351, h(4) = -0.0854, h(5) = 0.0352$. Therefore, $h_1(n)$ is given as $h_1(0) = 0.0352, h_1(1) = 0.0854, h_1(2) = -0.1351, h_1(3) = -0.4598, h_1(4) = 0.8069, h_1(5) = -0.3327$. The scaling and wavelet functions at the different levels are generated by iterating the synthesis filterbank that many times.



2. First we show that it corresponds to a valid scaling function: $\sum h(n) = \frac{8}{4\sqrt{2}} = \sqrt{2}$. Then, we check the equation $\sum_n h(n)h(n-2k) = \delta(k)$. For k=0, we have $h^2[0]+h^2[1]+h^2[2]+h^2[3] = \frac{4+12+12+4}{32} = 1$. For $k = 1$, we get $h[2]h[0]+h[3]h[1] = 0$. Therefore, this is a QMF. $h_1[0] = \frac{1-\sqrt{3}}{4\sqrt{2}}, h_1[1] = \frac{-3+\sqrt{3}}{4\sqrt{2}}, h_1[2] = \frac{3+\sqrt{3}}{4\sqrt{2}}, h_1[3] = \frac{-1-\sqrt{3}}{4\sqrt{2}}$.

3. Frequently signals have a bias, which is a polynomial part added to a bounded rapidly oscillating part, such as $f(t) = a + bt + ct^2 + g(t)$, where g(t) is a sinusoidal signal.



Suppose that we have 1024 samples of f on the interval $[-1, 1]$. In order to analyze this signal with Daubechies family of wavelets, the smallest length filter we should choose is N = 6 which has 3 vanishing moments. This is due to the fact that the polynomial is of order 2, and using wavelets with more than 3 vanishing moments will ensure that the wavelet is orthogonal to the polynomial part of the signal. Based on the decomposition in wavemenu, the polynomial part of the signal lives in V_6. The sinusoid part of the signal lives in $W_6, W_7, W_8, W_9, W_{10}$.

4. a) For this question we need to show, $\sum_k |\Phi(\omega + 2\pi k)|^2 = 1$. Simplifying the expression, $\Phi(\omega) = \begin{cases} 0, & \omega < -4\frac{\pi}{3} \\ \sqrt{2 + \frac{3\omega}{2\pi}}, & -4\pi/3 \le \omega \le -2\pi/3 \\ 1, & -2\pi/3 < \omega < 2\pi/3 \\ \sqrt{2 - \frac{3\omega}{2\pi}}, & 2\pi/3 \le \omega \le 4\pi/3 \\ 0, & \omega > 4\pi/3 \end{cases}$. It is then easy to prove $\sum_k |\Phi(\omega + 2\pi k)|^2 = 1$ graphically.

b)$\Phi(\omega) = \frac{1}{\sqrt{2}} H(\frac{\omega}{2})\Phi(\frac{\omega}{2})$. From this relationship, $H(\omega) = \sqrt{2}\Phi(2\omega)$ and $H_1(\omega) = e^{-j\omega}\sqrt{2}\Phi(2\omega + 2\pi)$. Therefore, $\Psi(\omega) = \frac{1}{\sqrt{2}} H_1(\frac{\omega}{2})\Phi(\frac{\omega}{2}) = e^{-j\omega/2}\Phi(\omega + 2\pi)\Phi(\omega/2)$. Therefore, $\Psi(\omega) = \begin{cases} e^{-j\omega/2}\sqrt{2 + 3\omega/4\pi}, -8\pi/3 \le \omega \le -4\pi/3 & \\ e^{-j\omega/2}\sqrt{2 - \frac{3(\omega+2\pi)}{2\pi}}, & -4\pi/3 \le \omega \le -2\pi/3 \\ 0 & otherwise \end{cases}$

c)$V_j \subset V_{j+1}$ since the scaling function at scale $j + 1$ contains the scaling function at scale j, thus has a larger span. As $j \to \infty$, $V_j = L_2(R)$ and as $j \to -\infty$, V_j becomes the empty set.

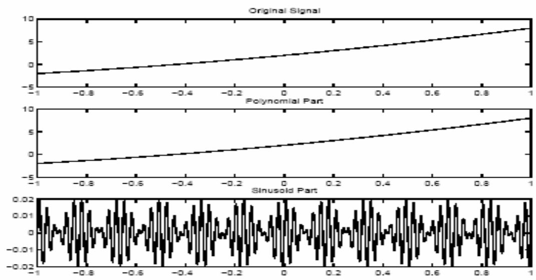


Figure 1: The separated sinusoid and polynomial