ECE 802-603 Homework 4
Spring 2010
Due March 25, 2010

1. This problem is about designing Daubechies filters, scaling functions and wavelets.
a) Design the minimum-phase Daubechies filter of length 6. Find the corresponding wavelet filter.
b) Using the scaling equation, one can recursively compute the scaling function, i.e. have an initial estimate for $\phi(t)$ and then filter it to get the next estimate, use this estimate to get the next estimate and so on. The more you iterate this algorithm the closer you get to the actual scaling function. Download the psa.m and the downsampling & upsamling functions from the course website. Compute the corresponding scaling function for 3 and 6 iterations. Plot and compare for these iteration levels.
c) Compute the wavelet function.

2. Consider the length for filter with coefficients,
$h[0]=\frac{1+\sqrt{3}}{4\sqrt{2}}, h[1]=\frac{3+\sqrt{3}}{4\sqrt{2}}, h[2]=\frac{3-\sqrt{3}}{4\sqrt{2}}, h[3]=\frac{1-\sqrt{3}}{4\sqrt{2}}$. Show that this filter corresponds to a valid orthogonal scaling function and find the corresponding wavelet filter.

3. Frequently signals have a bias, which is a polynomial part added to a bounded rapidly oscillating part, such as $f(t)=a+bt+ct^2+g(t)$, where $g(t)$ is a sinusoidal signal. Suppose that we have 1024 samples of f on the interval [-1,1]. Come up with a strategy for separating the two signals via a Daubechies wavelet analysis. Download the signal from the course webpage. Use MATLAB to carry it out. Try to answer the following questions when you describe your strategy:
a) What is the smallest value of N, length of scaling filter, needed? Why? Hint: Think about vanishing moments.
b) In terms of the scaling and wavelet subspaces, describe in which subspaces the polynomial and the sinusoidal functions will live in.
c) Include your final result which shows the two reconstructed signals, one a polynomial, the other the oscillating signal.

4. Meyer wavelet is a smoothed version of the sinc wavelet and is designed in the frequency domain. Given

$$\Phi(\omega) = \begin{cases} \sqrt{\theta\left(2 + \frac{3\omega}{2\pi}\right)}, \omega \le 0 \\ \sqrt{\theta\left(2 - \frac{3\omega}{2\pi}\right)}, \omega > 0 \end{cases}, \text{ where } \theta(x) = \begin{cases} 0 & x \le 0 \\ x & 0 \le x \le 1 \\ 1 & 1 \le x \end{cases}$$

a. Derive the scaling function in the frequency domain and show that $\{\phi(t-k)\}$ is an orthogonal set. Hint: This may be easier to show in the frequency domain.
b. Derive the corresponding wavelet function in the frequency domain.
c. Show that the resulting subspaces V_j s form a multiresolution analysis by showing that the properties 2-5 on slide 7 of lecture 4 are satisfied.