ECE 802-603 Homework 2
Spring 2010
Due February 11, 2010

- Please turn in all of your MATLAB code and your outputs with your solutions.
- You should turn in a hard copy on the due date of the assignment.

1. One way to address the issue of time-frequency resolution tradeoff of the spectrogram is to use multiple window functions in the computation, i.e.
$P_S(t,\omega)=\sum_{i=1}^{K}\lambda_i P_{S_i}(t,\omega)$, where individual spectrograms computed using different window functions are combined with some weighting coefficients, λ_i. For the following quadratic chirp test signal,

```
t=-2:0.001:2;
y=chirp(t,100,1,200,'q');
```

a) Use Gaussian windows at four different lengths (32, 64, 128, 256) and obtain a multiple window spectrogram using equal weights, i.e. $\lambda_i=\frac{1}{4}$ and show the result using imagesc. You need to be careful about the sizes of the different spectrograms.

b) Compute the individual spectrograms corresponding to each of the four Gaussian windows. Compare the results to the spectrogram in part a in terms of time and frequency resolution. Which spectrogram performs the best and why?

2. Prove that the spectrogram can be written as a two-dimensional convolution of the Wigner distribution of the signal and the Wigner distribution of the window function.

3. Consider the following signal, $s(t)=0.4e^{j10t}+e^{j20t}$.

   a) Find an expression for the instantaneous frequency of this signal. Plot your result for $0<t<5$. Does this result match your intuition? Why or why not?
   b) Compute the Wigner distribution of this signal by hand.
   c) Write a MATLAB function that will compute the instantaneous frequency from the Wigner distribution. Include your code and results. Compare with the results found in part a.
   d) Use the ridge extraction code on the web page and compare your results to the ones found in part c.

4. Load the crossing chirp signals from the web page.

a) Compute the Wigner distribution using the ‘wigner’ command in the time-frequency toolbox.

b) Compute the Choi-Williams distribution using the 'choiwil' command in the time-frequency toolbox. Use three different values of σ and comment on the effect of σ parameter on the distribution.

c) Choose the best σ value from part b and compute:

   i. The time and frequency marginals directly from the distribution. Verify that the marginals are satisfied.

   ii. Compute the initial and end frequencies from the distribution for the two chirp signals. Express your answers in terms of normalized frequency.

d) Find the ambiguity function for this signal using the '`ambigu`' command. Plot the absolute value of the ambiguity function.

e) Design a kernel function that will keep the actual signal and get rid off the cross-terms, give an equation for this kernel function. Hint: There is not a unique answer.

f) Apply your kernel in the ambiguity domain to the ambiguity function of the signal. Use the '`dochange`' command to transform the ambiguity domain to the time-frequency domain. Compare your result with the Wigner distribution and the Choi-Williams distribution obtained above.