ECE 802-601 Homework 5
Spring 2008
Due April 10, 2008

1. [25] Suppose that $h, \tilde{h}$ define a pair of perfect reconstruction filters in a biorthogonal system.
a) Show that $H^*(\omega)\tilde{H}(\omega) + H^*(\omega+\pi)\tilde{H}(\omega+\pi) = 2$
b) Prove that $h_{new}[n] = \frac{1}{2}\left(h[n] + h[n-1]\right)$ and $\tilde{h}_{new}[n] = \frac{1}{2}\left(\tilde{h}[n] + \tilde{h}[n-1]\right)$ defines a new pair of perfect reconstruction filters.
c) The Deslauriers-Dubuc filters are $H(\omega) = 1$ and
$\tilde{H}(\omega) = \frac{1}{16}(-e^{-j3\omega} + 9e^{-j\omega} + 16 + 9e^{j\omega} - e^{j3\omega})$ .Compute $h_{new}, \tilde{h}_{new}$ as well as the corresponding biorthogonal wavelets.

2. [25] Wavelets can be used to remove noise from signal. Let $f(t) = \sin(8\pi t)\cos(3\pi t) + n(t)$, with $n(t)$ being the noise. Numerically, we can model $n(t)$ by using a random number generator, such as MATLAB's randn. Take 1500 samples on [-2, 3] of f and do an analysis with $N = 2$, 3 and 6 Daubechies wavelets. Experiment with different levels of decomposition and different thresholding methods (soft and hard thresholding).
a) For the same threshold and same thresholding method, which wavelet does the best job? Quantify your answer in terms of MSE.
b) Using the wavelet you determined in part (a), test different threshold values and compare the performance of soft vs. hard thresholding. Show your results using a MSE vs. threshold graph.

3. [25] Consider the following signal which is the sum of two sinusoids, $x(t) = \sin(8\pi t) + \sin(80\pi t)$, where t is discretized in the interval [0,1] using 1024 sample points. Perform a wavelet packet decomposition of this signal using Daubechies 4 wavelet and Shannon entropy as the cost function. Try to find the best decomposition level and the best tree such that the two sinusoids are separated from each other.

4. [25] 7.29 from the book. A smoothed image is given on the webpage.