ECE 802-601 Homework 4
Spring 2008
Due March 18, 2008

1. [25] This problem is about designing Daubechies filters, scaling functions and wavelets.
a) Design the minimum-phase Daubechies filter of length 6. Find the corresponding wavelet filter.
b) Using the scaling equation, one can recursively compute the scaling function, i.e. have an initial estimate for *Á*(t) and then filter it to get the next estimate, use this estimate to get the next estimate and so on. The more you iterate this algorithm the closer you get to the actual scaling function. There is a MATLAB function that actually implements this approach at http://wwwdsp.rice.edu. Download the wavelet toolbox from that site and use the 'psa' function to generate the scaling function from the scaling filter. Compute the corresponding scaling function for 3 and 6 iterations. Plot and compare for these iteration levels.
c) Compute the wavelet function using the same MATLAB function, this time the filter is $h1$.

2. Frequently signals have a bias, which is a polynomial part added to a bounded rapidly oscillating part, such as $f(t) = a + bt + ct^2 + g(t)$, where $g(t)$ is a sinusoidal signal. Suppose that we have 1024 samples of f on the interval [-1,1]. Come up with a strategy for separating the two signals via a Daubechies wavelet analysis. Download the signal from the course webpage. Use MATLAB to carry it out.
Try to answer the following questions when you describe your strategy:
a) What is the smallest value of N, length of scaling filter, needed? Why? Hint: Think about vanishing moments.
b) In terms of the scaling and wavelet subspaces, describe in which subspaces the polynomial and the sinusoidal functions will live in.
c) Include your final result which shows the two reconstructed signals, one a polynomial, the other the oscillating signal.

3. Consider a length 4 orthogonal QMF filter, with coefficients $h[n] = [a, b, c, d]$.
a) Set up the constraint equations for a, b, c, d.
b) Assume $a = \frac{1}{2}, d = \frac{-1}{4}$, find the other two coefficients and find h[n].
c) What is the regularity of the filter and the support of the corresponding scaling function?

4. Meyer wavelet is a smoothed version of the sinc wavelet and is designed in the frequency domain. Given

$$\Phi(\omega) = \begin{cases} \sqrt{\theta\left(2 + \frac{3\omega}{2\pi}\right)}, \omega \le 0 \\ \sqrt{\theta\left(2 - \frac{3\omega}{2\pi}\right)}, \omega > 0 \end{cases}$$

a. Show that $\{\phi(t-k)\}$ is an orthogonal set. Hint: This may be easier to show in the frequency domain.
b. Derive the corresponding wavelet function in the frequency domain.