

ECE 802-601 Homework 3
Spring 2008
Due February 26, 2008

1. [10] Let ϕ and ψ be the Haar scaling and wavelet functions, respectively. Let V_j and W_j be the spaces generated by $\phi(2^j x - k), k \in \mathbb{Z}$ and $\psi(2^j x - k), k \in \mathbb{Z}$, respectively. Consider the function defined on $0 \leq x < 1$ given by

$$f(x) = \begin{cases} 2 & 0 \leq x < 1/4 \\ -3 & 1/4 \leq x < 1/2 \\ 1 & 1/2 \leq x < 3/4 \\ 3 & 3/4 \leq x < 1 \end{cases}$$

- Express f first in terms of the basis for V_2 and then decompose f into its component parts in W_1, W_0 , and V_0 .
2. [30] Let $f(t) = e^{-t^2/10}(\sin(2t) + 2\cos(4t) + 0.4\sin(t)\sin(50t))$. Discretize this signal over the interval $0 \leq t \leq 1$ as $f[n] = f(n/2^J), n = 0, 1, \dots, 2^J - 1$, where J is the largest scale. Use $J=8$ as the highest scale.
- Implement a Haar decomposition algorithm and plot the approximations in each level, $f_{j-1} \in V_{j-1}$ for $j = 8, 7, \dots, 1$ and compare with the original signal.
 - Plot the wavelet coefficient for each level.
 - Filter the wavelet coefficients computed in part b by setting to zero any wavelet coefficient whose absolute value is less than 0.1. Then reconstruct the signal from these filtered coefficients and plot the reconstructed signal and compute the mean square error between the reconstructed and the original signals.
 - Experiment with different threshold values. Plot the percentage of wavelet coefficients that have been filtered out versus the threshold. Plot the mean square error versus the threshold.

3. [20] Let V_j be the space of all finite energy signals that are continuous and piecewise linear, with possible corners occurring only at the dyadic points, $k/2^j$. This system of subspaces forms the basis for linear spline multiresolution analysis.

a. Show that $\{V_j\}_{j \in \mathbb{Z}}$ satisfies the following properties:

- $V_j \subset V_{j+1}$
- The function $f(t)$ belongs to V_j if and only if the function $f(2t)$ belongs to V_{j+1} .

b. Let $\phi(t) = \begin{cases} t+1, & -1 \leq t \leq 0 \\ 1-t, & 0 \leq t \leq 1 \\ 0, & \text{otherwise} \end{cases}$. Show that $\{\phi(t-k)\}_{k \in \mathbb{Z}}$ is an

orthogonal basis for V_0 . Find the scaling relation (equation) for this scaling function.

- c. Construct the wavelet basis that is orthogonal to the scaling function. Plot the wavelet function.

4. [20] The DWT of a signal can be written as $f(t) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} d(j,k) \psi_{j,k}(t)$.

- a. Is DWT shift invariant? Under what conditions will it be shift invariant?
 b. Using the 'chirp' command in MATLAB generate a signal of length 256. $Y = \text{CHIRP}([0:1/255:1], 5, 1, 25)$; Generate a shifted version of this signal. Save both signals and look at their corresponding wavelet transforms (wavelet coefficients) in 'wavemenu'. Compare the DWT coefficients for the two signals.

5. [20] In this question, we will try to derive the sinc wavelet. First, we will start with a sequence of embedded spaces. Let V_0 be the space of functions limited to $[-\pi, \pi]$ and spanned by the orthogonal basis $\phi(t-k)$ where $\phi(t) = \frac{\sin(\pi t)}{\pi t}$. Thus V_1 is the space of functions bandlimited to $[-2\pi, 2\pi]$.

- a. Define W_0 .
 b. Express the equivalent of the dilation equation $\phi(t) = \sum_n h(n) \phi(2t-n)$ in the frequency domain.
 c. Derive $h(n)$, the scaling filter for the sinc scaling functions. Hint: First, solve for $H(\omega)$.
 d. Find the wavelet function corresponding to the sinc scaling function.