Solutions to Practice Exam 2

1. a) The local oscillator frequency is $f_{LO} = f_c + f_{IF} = 1120 + 2500 = 3620kHz$. The image frequency should be $3620 + 2500 = 6120kHz$, such that $f_{image} - f_{LO} = f_{IF}$.

   b) The assumption that $\phi(t)$ is small provides the basis for using the PLL for demodulating FM signals. When $\phi(t)$ is small, we can make the assumption that $\sin(\phi(t)) \approx \phi(t)$. This approximation allows us to represent PLL by a linearized model, which enables us to perform frequency demodulation using PLL with output $v(t) \approx \frac{k_f}{k_v} m(t)$.

   c) Let $y(t) = n(t) * h(t)$. Therefore, $S_y(f) = |H(f)|^2 S_n(f) = rect(f)\frac{N_0}{2}$.

   $v(t) = y(t)\cos(2\pi(1)t)$.

   $$S_v(f) = \begin{cases} \frac{N_0}{8}, \frac{1}{2} \le |f| \le \frac{3}{2} \\ 0, \quad otherwise \end{cases}$$

   Note: Multiplying with $\cos(2\pi f_c t)$ in the time domain is equivalent to convolution in the frequency domain. Since we are computing power spectral densities, it is equivalent to convolution with $\frac{1}{4}[\delta(f - f_c) + \delta(f + f_c)]$.

2. a) $E[X_1 X_2] = E[X_1]E[X_2] = 4$, using independence of the random variables.

   b) $E[Y] = E[2X_1 + X_2] = 2E[X_1] + E[X_2] = 6$

   c)

   $\sigma_Y{}^2 = E[Y^2] - E^2[Y]$

   $E[Y^2] = E[(2X_1 + X_2)^2] = E[4X_1{}^2] + E[4X_1X_2] + E[X_2{}^2]$

   $= 4(16 + 4) + 4(4) + (16 + 4)$

   $= \cancel{48 + 16 + 12}$

   $= \cancel{76}$ 116

   $\sigma_Y{}^2 = \cancel{76 - 36 = 40}$

   d) Y is a Gaussian random variable since it is a linear transformation of Gaussian random variables. Therefore,

   $$f_Y(y) = \frac{1}{\sqrt{2\pi(\cancel{40})\,80}} \exp\left(-\frac{(y-6)^2}{\cancel{80}\,160}\right)$$

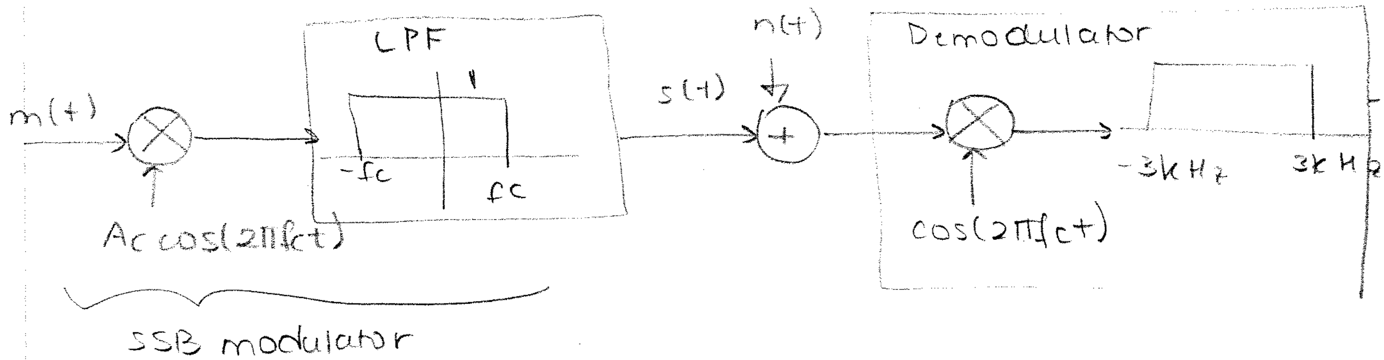
   e) $P[Y \le 2] = 1 - Q\left(\frac{2-6}{\sqrt{\cancel{40}\,80}}\right) = 1 - Q\left(\frac{-4}{\cancel{2\sqrt{10}}}\right) = Q\left(\frac{2}{\sqrt{10}}\right) \approx 0.27$

- Note: SNR is usually shown in terms of dB

$$SNR = 10\log_{10}\left(\frac{\text{Signal Power}}{\text{Noise Power}}\right)$$

Q3 Example: Consider SSB system shown below.

In general the transmit power (power of transmitted signal $s(t)$) is constrained by FCC or by a certain designed battery life.

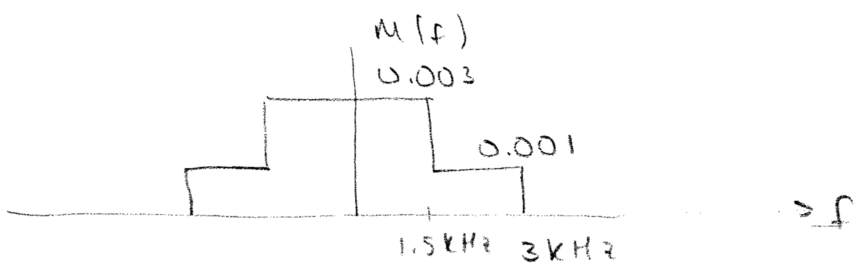


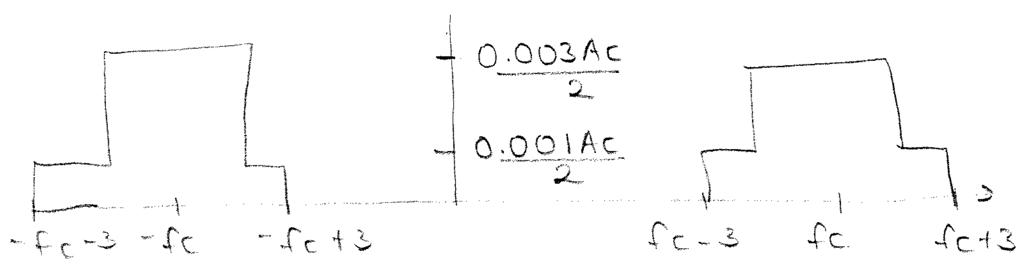
$m(t)$ has PSD $|M(f)|^2$

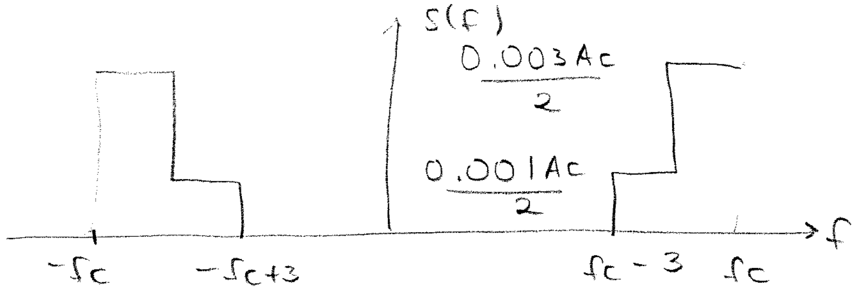
$$M(f) = \begin{cases} 0.003 & |f| \le 1.5\,kHz \\ 0.001 & 1.5\,kHz < |f| \le 3\,kHz \\ 0 & |f| > 3\,kHz \end{cases}$$

a) Find A_c such that transmitted power is 100mW

$S(f)$







Power $\int |s(t)|^2 dt = \int |S(f)|^2 df$

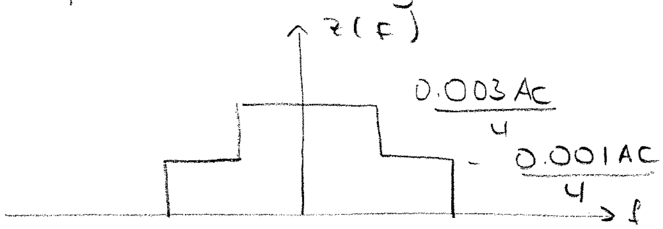
$$= 2\left[1500 \cdot \left(\frac{0.001Ac}{2}\right)^2 + 1500\left(\frac{0.003Ac}{2}\right)^2\right] = 100mW$$

$$= \frac{1}{2}\left[1500\,Ac^2 \cdot 10^{-6} + 1500\,Ac^2 \cdot 9 \cdot 10^{-6}\right] = 100 \cdot 10^{-3}$$

$$5(1500)Ac^2 = 10^5$$

$$Ac = 3.65$$

b) Power of $z(t)$ assuming there's no noise

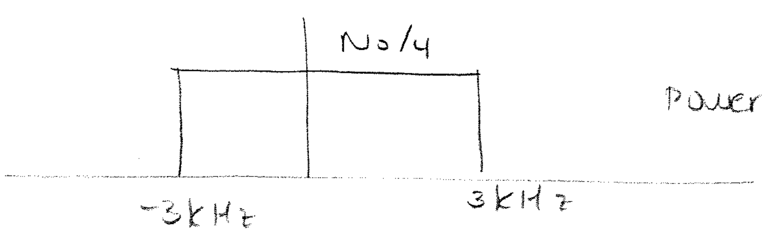


$$2\left[1500\left(\frac{0.003Ac}{4}\right)^2 + 1500\left(\frac{0.001Ac}{4}\right)^2\right] = \frac{1}{4} \cdot 100mW = \underline{\underline{25mW}}$$

c) $n(t)$ is AWGN $S_n(f) = \frac{0.5N_0}{2}$ $N_0 = 0.0001 mW/Hz$

PSD and power in demodul. output $z(t)$ due to noise $(m(t)=0)$

Note: $n(t)\cos(2\pi f_c t) \rightarrow \frac{1}{4} S_n(f-f_c) + \frac{1}{4} S_n(f+f_c)$



$$\text{Power} = \frac{(6000)}{4}(0.0001) \cdot 10^{-3}$$

$$= 0.00015\ W$$

d) SNR at the output.

$$\frac{25\,mW}{0.15\,mW} = 166.67 \approx 22.2\,dB //$$

e) Assume DSB (no filtering)

$A_c = ?$ $\quad P_T = 100\,mW$

$$4\left[1500\left(\frac{0.001\,A_c}{2}\right)^2 + 1500\left(\frac{0.003\,A_c}{2}\right)^2\right] = 100\,mW$$

$$10 \cdot 1500 \times 10^{-6}\,A_c^2 = 100 \times 10^{-3}$$

$$A_c = 2.58 //$$

power in $z(t)$

$$2\left[1500\left(\frac{0.003\,A_c}{2}\right)^2 + 1500\left(\frac{0.001\,A_c}{2}\right)^2\right]$$

50mW// (signal power)

noise power same $\quad 0.00015\,W \rightarrow 0.15\,mW$

$$SNR = \frac{50}{0.15} = 333.33 \rightarrow 25.2\,dB.$$

3dB increase in SNR.

If you have the same transmit power constraint, DSB has higher SNR, SSB is more spectrally efficient.