Solutions to Practice Problems

1. a) 2W

   b) See Figure 3.3 a (Envelope detection)

   c) Derivative of the message

   d) A frequency discriminator generates an output voltage that's proportional to the frequency deviation of the input signal.

   e) VSB with carrier

2. See Figure 3.1 a) for the block diagram of a phase coherent demodulation system. The output of the multiplier will be:

$$A_c(1+am_n(t))\cos(w_c t).\frac{2}{A_c}\cos(w_c t) = 2(1+am_n(t))\cos^2(w_c t)$$
$$= (1+am_n(t))(1+\cos(2w_c t))$$

   The output of the lowpass filter is: $(1+am_n(t))$

   Therefore, the output of the phase coherent demodulation system is a shifted and scaled version of the message.

3. a) 50W
   b) 100 Hz
   c) 0.1 radians
   d) $\beta = 0.1$, therefore it's narrowband.
   e) Bandwidth for narrowband is approximately,2W. Since this is a tone modulated signal, bandwidth is $2kHz$. The amplitude spectrum will have a component at 1000 kHz with magnitude 5, another component at 1001 kHz with magnitude 0.25 and another component at 999 kHz with magnitude 0.25. A symmetric spectrum exists for negative frequencies.

4. The block diagram of Armstrong indirect FM modulator was given in class and consists of a NBFM generator, a multiplier, mixer, and a second multiplier. The multipliers are only available as doublers. $\Delta f = n_1 n_2 \Delta f_1$, $9 < \Delta f_1 < 10$, $9 < \dfrac{20{,}000}{n_1 n_2} < 10$ and $n_1 n_2$ has to be a power of 2. By trial and error, it can be found that $n_1 n_2 = 2048$. Let $n_1 = 64, n_2 = 32$. The output of the first multiplier has carrier frequency 12.8 MHz. The input to the second multiplier should have carrier frequency 3 MHz. Therefore, the local oscillator has frequency 9.8 MHz which is in the given range.

5. a) The output signal: $x_c(t) = \hat{m}(t)\cos(\omega_c t) + m(t)\sin(\omega_c t)$

   b) This is still a SSB signal with bandwidth equal to the message bandwidth. The spectrum of the signal is:

$$X_c(f) = -\frac{j}{2}\operatorname{sgn}(f - f_c)M(f - f_c) - \frac{j}{2}\operatorname{sgn}(f + f_c)M(f + f_c) - \frac{j}{2}M(f - fc) + \frac{j}{2}M(f + fc)$$

   If we draw this spectrum, it will have the same amplitude spectrum as USB signal, the only difference would be a 90 degree phase shift.

   c) You can demodulate this signal by carrier reinsertion. Unlike the classical carrier reinsertion, for this case you have to reinsert a 90 degree phase shifted version of the carrier, $K\sin(\omega_c t)$

   The output is: $\hat{m}(t)\cos(\omega_c t) + (m(t) + K)\sin(\omega_c t)$. When this is put into an envelope detector, if K is large: $R(t) = \sqrt{\hat{m}^2(t) + (m(t) + K)^2} \approx m(t) + K$