

**ECE 457 Spring 2003  
Practice Exam Final Solutions**

1. a) False. For example, the modulated signal can be  $x_c(t) = A_c \cos(2\pi f_c t + \frac{\pi}{2} t)$ . This can be a PM signal with  $k_p = \frac{\pi}{2}$ ,  $m(t) = t$  or it can be a FM signal with  $k_f = \frac{\pi}{2}$ ,  $m(t) = 1$ .

b) True.  $SNR_{BB} = \frac{P_R}{N_0 W} = \frac{P_T 10^{-5}}{10^{-12} \times 5 \times 10^3} = 10^2 \Rightarrow P_T = 0.05W$ .

c) False. The output of a discriminator is  $K_D \frac{1}{2\pi} \frac{d\phi(t)}{dt} = \frac{2}{2\pi} (20t) = \frac{20t}{\pi}$ .

d) True. If there is a phase shift in the demodulator carrier, the output of the demodulator will be:

$$A_c m(t) \cos(2\pi f_c t) \frac{2}{A_c} \cos(2\pi f_c t + \phi) = m(t) \cos(\phi) + m(t) \cos(4\pi f_c t + \phi) \Rightarrow LPF \Rightarrow m(t) \cos(\phi)$$

e) False.  $SNR_{AM} = E \frac{P_T}{N_0 W}$ ,  $SNR_{SSB} = \frac{P_T}{N_0 W}$ . Since efficiency is less than 1, to achieve the same SNR for AM and SSB, the transmitted power for AM should be greater.

2. a) The peak phase deviation in this case is equal to  $\frac{f_d A_m}{f_m} = \frac{10}{25} = 0.4$ . Note that by definition this is the modulation index,  $\beta$ . When  $\beta < 1$ , that corresponds to narrowband FM. Therefore, the bandwidth is  $2\lfloor \beta + 1 \rfloor f_m = 2\lfloor 1.4 \rfloor 25 = 50kHz$ .

b)

$$SNR_{FM} = 3 \left( \frac{f_d}{W} \right)^2 \overline{m_n^2(t)} \frac{P_T}{N_0 W} = 3 \left( \frac{10}{25} \right)^2 \left( \frac{1}{2} \right) \frac{P_T}{(6 \times 10^{-5})(25 \times 10^3)} = 10^4 \Rightarrow P_T = 62.5kW$$

Note that the message power is 0.5, since it's a sinusoid. The message bandwidth is the same as  $f_m$  for the case of a sinusoid.

c) With deemphasis filter,

$$SNR = \left( \frac{f_d}{f_3} \right)^2 \overline{m_n^2(t)} \frac{P_T}{N_0 W} = \left( \frac{10}{5} \right)^2 \left( \frac{1}{2} \right) \frac{P_T}{(6 \times 10^{-5})(25 \times 10^3)} = 10^4 \Rightarrow P_T = 7.5kW$$

Note that  $f_3$  is the cutoff frequency of the filter (3dB frequency), and in this case it is equal to 5kHz. With deemphasis filtering, the power consumption is less, about 8.3 times less than the standard FM.

3. a)

$$H_1(f) = \begin{cases} 1, & f_c \leq |f| \\ 0, & \text{otherwise} \end{cases}$$

$$H_2(f) = \begin{cases} 1, & |f| \leq f_c \\ 0, & \text{otherwise} \end{cases}$$

and  $f_1 = f_2 = f_c$ .

$$H_3(f) = \begin{cases} 2, & |f| \leq B_1 \\ 0, & \text{otherwise} \end{cases}$$

$$H_4(f) = \begin{cases} 2, & |f| \leq B_2 \\ 0, & \text{otherwise} \end{cases}$$

b) Even without noise, the practical implementation of this system design will have some distortion. Signals with frequency components at DC will be distorted because a practical filter will not be able to sharply separate the upper and lower sidebands. The signals given in this question fall in this category.

$$S_{n_1}(f) = \begin{cases} \frac{N_0}{2}, & |f| \leq B_1 \\ 0, & \text{otherwise} \end{cases}$$

c)

$$S_{n_2}(f) = \begin{cases} \frac{N_0}{2}, & |f| \leq B_2 \\ 0, & \text{otherwise} \end{cases}$$

$$\text{The SNR in the upper branch is: } \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\int_{-B_1}^{B_1} |M_1(f)|^2 df}{\left(\frac{N_0}{2}\right)2B_1} = \frac{2B_1}{N_0 B_1} = \frac{2}{N_0}$$

$$\text{The SNR in the lower branch is: } \frac{\text{Signal Power}}{\text{Noise Power}} = \frac{\int_{-B_2}^{B_2} |M_2(f)|^2 df}{\left(\frac{N_0}{2}\right)2B_2} = \frac{(2B_2/3)}{N_0 B_2} = \frac{2}{3N_0}$$

Note that for the signal power in the lower branch, we have to take the square of a triangle waveform to get the power spectral density and integrate that over the bandwidth.

4. a) Modulation index: Rewrite the given AM signal:

$$s(t) = 20 \cos(300\pi t) + 12 \cos(300\pi t) \cos(20\pi t)$$

$$= 20 \cos(300\pi t) \left(1 + \frac{12}{20} \cos(20\pi t)\right)$$

Therefore, the modulation index is  $12/20=0.6$ .

b) Efficiency:  $\frac{a^2 \overline{m^2_n(t)}}{1 + a^2 \overline{m^2_n(t)}} = \frac{(0.6)^2(0.5)}{1 + (0.6)^2(0.5)} = 0.153$

c) SNR:  $(0.153) \frac{P_R}{N_0 W} = (0.153) \frac{10^{-4} \times (236)}{(2 \times 10^{-10})(10)} = 1.81 \times 10^6 \Rightarrow 62.57 \text{ dB}$

Note the power of the AM modulated signal can be found as:  $\frac{A_c^2}{2} (1 + a^2 \overline{m^2_n(t)})$

d) SNR for DSB is computed as:  $\frac{10^{-4} \times P_T}{(2 \times 10^{-10})(10)} = 1.81 \times 10^6 \Rightarrow P_T = 36.2 \text{ W}$

Note that this is less than power transmitted for AM 236W.