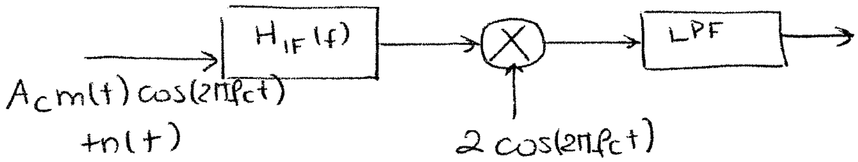
ECE 457

Practice Problems

Solutions

(1) **6.3**

$B_T > 2W$ and $B_D > W$



$(SNR)_{pre}$

Signal power: $\frac{A_c^2}{2}\overline{m^2(t)}$

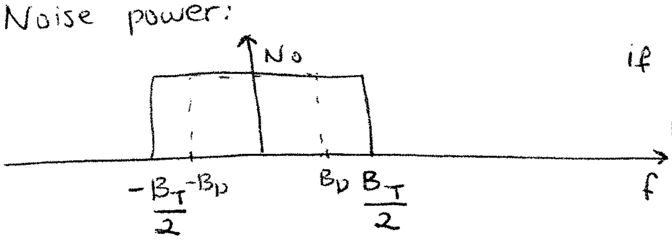
Noise power: $\overline{n_{bp}^2(t)} = 2\frac{N_0}{2}B_T = N_0 B_T$

$$(SNR)_{pre} = \frac{A_c^2\,\overline{m^2(t)}}{2N_0 B_T}$$

$(SNR)_{post}$

Signal power: $A_c^2\,\overline{m^2(t)}$

Noise power:



if $B_D < \frac{B_T}{2}$
Noise power: $2N_0B_D$

if $B_D > \frac{B_T}{2}$
Noise power: $N_0 B_T$

Two cases

$$(SNR)_{post} = \frac{A_c^2\,\overline{m^2(t)}}{2N_0B_D} \quad \text{or} \quad \frac{A_c^2\,\overline{m^2(t)}}{N_0 B_T}$$

Detection gain:
$$\frac{\dfrac{A_c^2\,\overline{m^2(t)}}{2N_0B_D}}{\dfrac{A_c^2\,\overline{m^2(t)}}{2N_0B_T}} = \boxed{\frac{B_T}{B_D} \text{ if } B_D < \frac{B_T}{2}}$$

OR
$$\frac{\dfrac{A_c^2\,\overline{m^2(t)}}{N_0B_T}}{A_c^2\overline{m^2(t)}/2N_0B_T} = \boxed{2 \text{ if } B_D > \frac{B_T}{2}}$$

(2) $P_T = 40kW$

attenuation 80dB

$\frac{N_0}{2} = 10^{-10}$ W/Hz.

$W = 10^4$ Hz

a) $(SNR)_{pre} = \frac{P_T \times 10^{-8}}{2N_0 W}$ for DSB

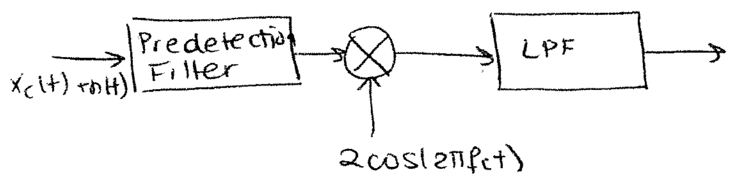
$= \frac{40 \times 10^3 \times 10^{-8}}{2 \times 2 \times 10^{-10} \times 10^4} = 10^2 \Rightarrow 20\,dB.$

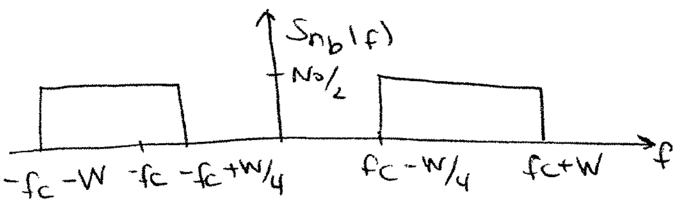
for SSB : $(SNR)_{pre} = \frac{P_T \times 10^{-8}}{N_0 W} = 2 \times 10^2 \Rightarrow 23\,dB.$

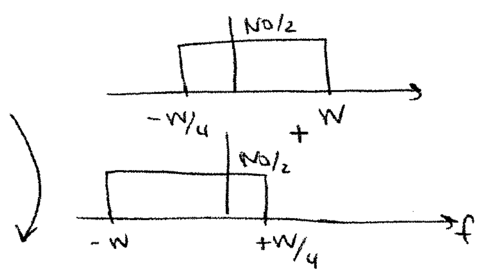
b) $(SNR)_o = (SNR)_{BB} = \frac{P_T \times 10^{-8}}{N_0 W} \Rightarrow 23dB$ for both DSB + SSB.

c)

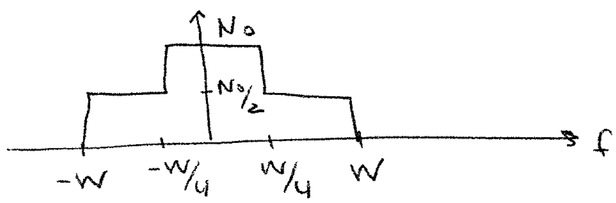
3







Output of the receiver:



b) $(SNR)_{out}$

Signal power at the output for SSB: $A_c^2\overline{m^2(t)}$

Noise power: $N_0\left(\frac{W}{2}\right)+\frac{N_0}{2}\left(\frac{3W}{2}\right)=\frac{N_0W}{2}+\frac{3N_0W}{4}$

$=\frac{5N_0W}{4}$

$$(SNR)_{out}=\frac{4}{5}\,\frac{A_c^2\overline{m^2(t)}}{N_0W}$$

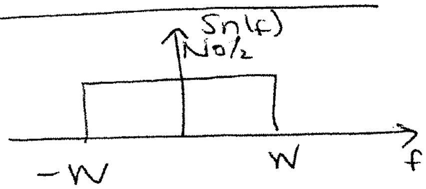
$(SNR)_{out}$ for conventional SSB system: $\frac{A_c^2\overline{m^2(t)}}{N_0W}$

Ratio: $\frac{4}{5}\approx -0.97\text{dB}$ //

4 $r(t) = s(t) + n(t)$

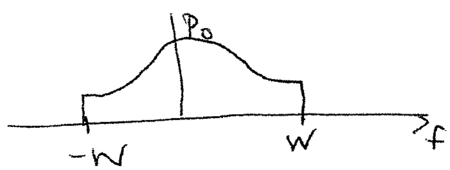
$$S_s(f) = \frac{P_o}{1 + (f/B)^2}$$

The noise at the output:



Noise power: $\left(\frac{N_o}{2}\right)(2W) = N_o W$

The signal at the output:



Signal power: $\int_{-W}^{W} \frac{P_o}{(1+(f/B)^2)} df$

$f' = f/B \quad df' = df/B$

$$= \int_{-W}^{W} \frac{P_o \cdot B}{1 + (f')^2} df' = P_o B \tan^{-1} f' \Big|_{-W/B}^{W/B}$$

$$= P_o B \left[\tan^{-1} \frac{W}{B} - \tan^{-1}\left(-\frac{W}{B}\right)\right]$$

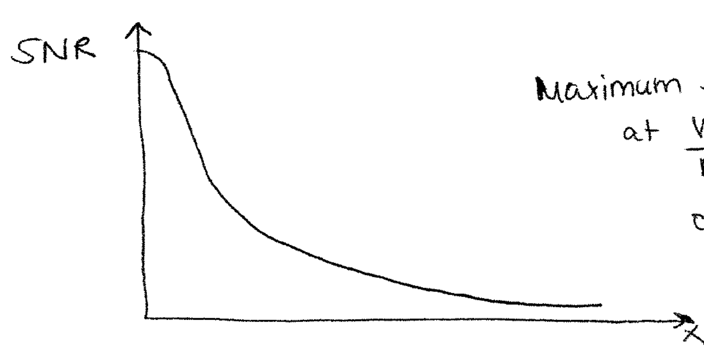
$$= 2 P_o B \tan^{-1}\left(\frac{W}{B}\right)$$

$\tan^{-1}\left(-\frac{W}{B}\right) = -\tan^{-1}\frac{W}{B}$

SNR: $\boxed{\frac{2 P_o B \tan^{-1}\left(\frac{W}{B}\right)}{N_o W}}$

let $\frac{W}{B} = x$

$$\frac{2P_o}{N_o x} \tan^{-1}(x)$$



Maximum SNR at $\frac{W}{B} = 0$ or $W = 0$ no noise will pass.