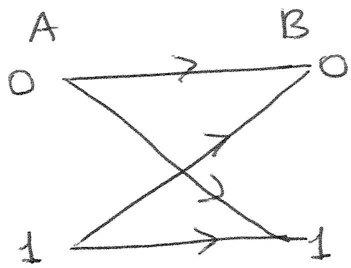
ECE 457

HW #8

Solutions

1)



$P(A=1)=0.4$, $P(A=0)=0.6$

$P(B=0 \mid A=0)=P(B=1 \mid A=1)=0.8$

a) $$P[A=1 \mid B=1]=\frac{P[B=1 \mid A=1]\,P[A=1]}{P[B=1]}$$

$$=\frac{(0.8)(0.4)}{P[B=1]}=\frac{0.32}{0.44}=0.727 \,//$$

$$P[B=1]=P[B=1 \cap A=1]+P[B=1 \cap A=0]$$
$$=P[B=1 \mid A=1]\,P[A=1]+P[B=1 \mid A=0]\,P[A=0]$$
$$=(0.8)(0.4)+(0.2)(0.6)=0.44$$

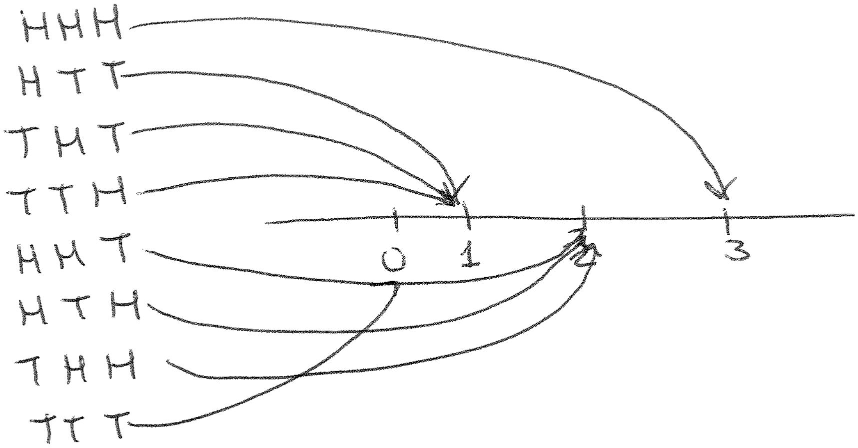
b) $$P[A=1 \mid B=0]=\frac{P[B=0 \mid A=1]\,P[A=1]}{P[B=0]}$$

$$=\frac{(0.2)(0.4)}{1-0.44}=0.143 \,//$$

c) $$P[A=0 \mid B=0]=\frac{P[B=0 \mid A=0]\,P(A=0)}{P(B=0)}$$

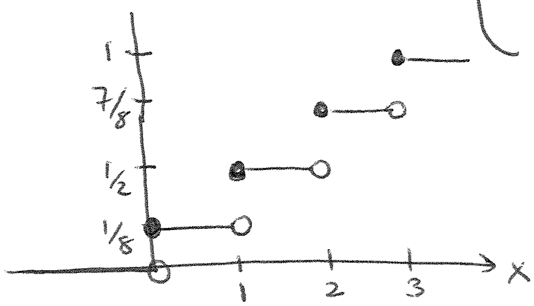
$$=\frac{(0.8)(0.6)}{1-0.44}=0.857 \,//$$

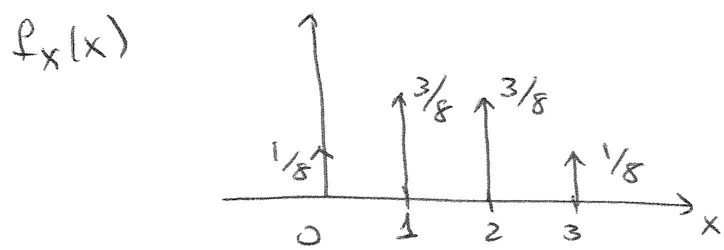
② 4.9



$P[X=0]=1/8$
$P[X=1]=3/8$
$P[X=2]=3/8$
$P[X=3]=1/8$

$$F_X(x)=P[X\le x]=\begin{cases}0, & x<0\\ 1/8, & 0\le x<1\\ 1/2, & 1\le x<2\\ 7/8, & 2\le x<3\\ 1 & x\ge 3\end{cases}$$





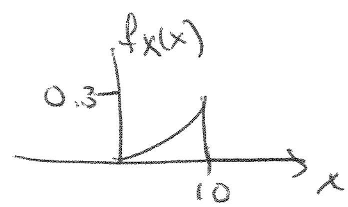
③ 4.10 $F_X(x)=\begin{cases}0, & x\le 0\\ Ax^3, & 0\le x\le 10\\ B, & x>10\end{cases}$

a) $B=1$ since $F_X(\infty)=1$

$A(10)^3=1$ since $F_X(x)$ is continuous.

$A=10^{-3}$

b) $f_X(x)=\dfrac{d}{dx}F_X(x)=\begin{cases}0 & x<0\\ 3\times10^{-3}x^2 & 0\le x\le 10\\ 0 & x>10\end{cases}$



c) $P(X > 7) = 1 - F_X(7)$

$$= 1 - (10^{-3})\, 7^3 = 0.657$$

d) $P(3 \leq X < 7) = F_X(7) - F_X(3)$

$$= 10^{-3}(7)^3 - (10)^{-3}(3)^3$$

$$= 0.316$$

(4) 4.23 $\quad f_X(x) = \frac{1}{2}\delta(x-4) + \frac{1}{8}[u(x-3) - u(x-7)]$

$$E[X] = \int_{-\infty}^{\infty} x f_X(x)\, dx$$

$$= \frac{1}{2}\int x\, \delta(x-4)\, dx + \frac{1}{8}\int_3^7 x\, dx$$

$$= 2 + \frac{1}{8}\left[\frac{x^2}{2}\Big|_3^7\right] = 2 + \frac{1}{8}\left[\frac{49}{2} - \frac{9}{2}\right] = 4.5 //$$

$$\mathrm{Var}[X] = E[X^2] - E^2[X]$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx$$

$$= \int \frac{x^2}{2}\delta(x-4)\, dx + \int_3^7 \frac{x^2}{8}\, dx$$

$$= 8 + \frac{x^3}{24}\Big|_3^7 = 8 + \left(\frac{(7)^3}{24} - \frac{27}{24}\right) = 21.17$$

$$21.17 - (4.5)^2 = 0.917 //$$

(5) $E[X]=3$

$Var[X]=4$

$E[Y]=-1$

$Var[Y]=2$

a) $Z = X - Y$

$E[Z] = E[X] - E[Y] = 3+1 = 4$ //

$Var[Z] = E[Z^2] - E^2[Z]$

$E[Z^2] = E[(X-Y)^2] = E[X^2 - 2XY + Y^2]$

$= \underbrace{E[X^2]}_{4+9} \underbrace{-2E[X]E[Y]}_{-2(3)(-1)} + \underbrace{E[Y^2]}_{2+1}$

$= 13 + 6 + 3 = 22$

$Var[Z] = 22 - 16 = 6$ //

b) $W = 2X + 3Y$

$E[W] = 2E[X] + 3E[Y]$

$= 6 - 3 = 3$ //

$E[W^2] = E[(2X+3Y)^2] = E[4X^2 + 12XY + 9Y^2]$

$= 4E[X^2] + 12E[X]E[Y] + 9E[Y^2]$

$= 4(13) + 12(-3) + 9(3)$

$= 43$

$Var[W] = 43 - 9 = 34$ //

c) $cov[Z,W] = E[ZW] - E[Z]E[W]$

$E[(X-Y)(2X+3Y)] = E[2X^2] + 3E[XY] - 2E[XY] + 3E[Y^2]$

$= 2(13) + 3(-3) + 3(3)$

$= 26$

$cov[Z,W] = 26 - (4)(3) = 14$ //

(6) X is Gauss. with $m_x = 4$, $\sigma^2 = 9$

a) $P(X>7) = Q\left(\frac{7-4}{3}\right) = Q(1) = 0.15866$ //

b) $P(0<X<9) = P(X<9) - P(X<0)$

$$= 1 - Q\left(\frac{9-4}{3}\right) - \left(1 - Q\left(\frac{0-4}{3}\right)\right)$$

$$= Q\left(\frac{-4}{3}\right) - Q\left(\frac{5}{3}\right)$$

$$= 1 - Q\left(\frac{4}{3}\right) - Q\left(\frac{5}{3}\right)$$

$$= 1 - Q(1.3) - Q(1.66)$$

$$= 1 - 0.09680 - 0.05480 = 0.8484$$

c) $P(X<4) = 1 - Q\left(\frac{4-4}{3}\right) = 1 - Q(0) = 0.5$

(7) ~~4~~.43

a) $P(|X| > \sigma) = 2Q(1) = 0.31732$

b) $P(|X| > 2\sigma) = 2Q(2) = 0.0455$

c) $P(|X| > 3\sigma) = 2Q(3) = 0.0027$ //