ECE 457 HW #6

solutions

① 3.27

$\phi(t) = \beta \sin(2\pi f_m t)$

$\beta = 10 \quad f_m = 20\,Hz \qquad f_c = 1000\,Hz$

Assume $A_c = 1$

$$x_c(t) = \sum_{n=-\infty}^{\infty} J_n(10) \cos\left[(2\pi(1000) + n(2\pi)(20))t\right]$$

$J_0(10) = -0.246$

$J_1(10) = 0.043$

$J_2(10) = 0.255$

$J_3(10) = 0.058$

$J_4(10) = -0.220$

$J_5(10) = -0.234$

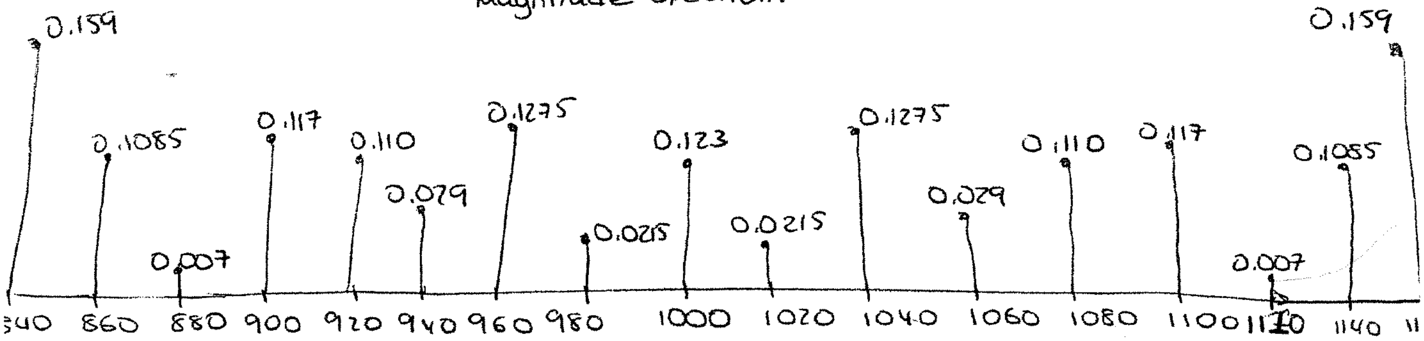
$J_6(10) = -0.014$

$J_7(10) = 0.217$

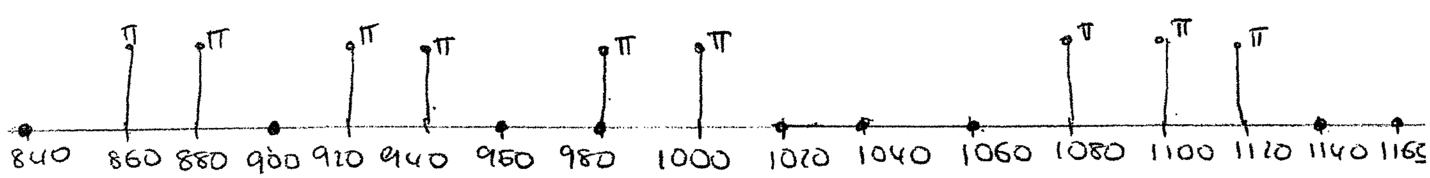
$J_8(10) = 0.318$

Single-sided spectrum: (same for negative frequences)

Magnitude Spectrum



Phase Spectrum
(odd symmetric for negative frequencies)



Q2

(2) 3.36

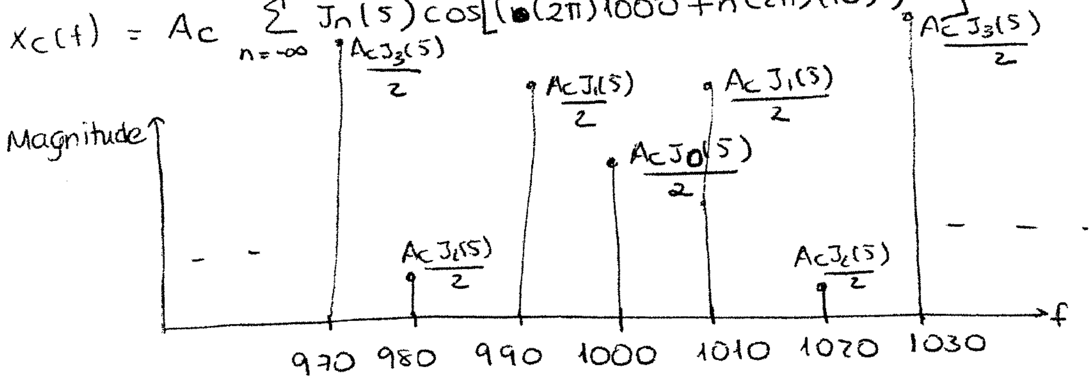
$f_c = 1000\,Hz$.

$f_d = 12.5$

$m(t) = 4\cos(2\pi(10)t)$

a) $\beta = \dfrac{A_m f_d}{f_m} = \dfrac{(4)(12.5)}{10} = 5$

b) $x_c(t) = A_c \sum_{n=-\infty}^{\infty} J_n(5)\cos[((2\pi)1000 + n(2\pi)(10))t]$



$J_0(5) = -0.178$

$J_1(5) = -0.328$

$J_2(5) = 0.047$

$J_3(5) = 0.365$

c) Not narrowband since $\beta > 1$

d) For PM $\Rightarrow$ $\beta = k_p A_m$

$5 = (k_p)(4) \Rightarrow k_p = 1.25$

Q3

3.40

$10 \cos(1000\pi t)$

$m(t) = 10\cos(20\pi t)$

$f_d = 8 Hz/V$

$\phi(t) = 2\pi f_d \int m(\tau)\, d\tau$

a) Peak freq. deviation: $f_d \max[|m(t)|]$

$= (8)(10) = 80 Hz.$

b) Peak phase deviation: $2\pi(8) \int 10\cos(20\pi \tau)\, d\tau$

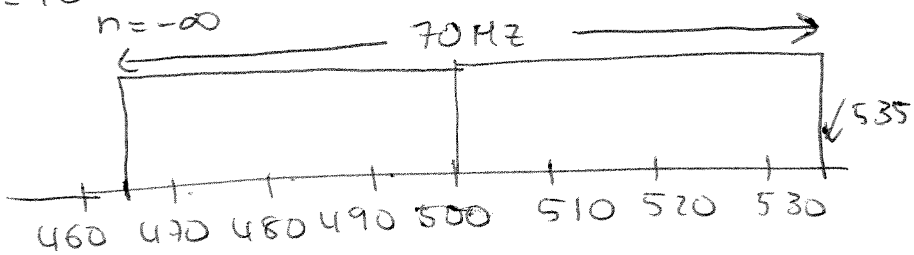
$= \frac{2\pi(8)(10)}{20\pi} \sin(20\pi t)$

= 8 radians

c) Modulation index = $\beta = 8$

d) Power at filter input = Total power = $\frac{A_c^2}{2} = \frac{(10)^2}{2} = 50W$

$x_c(t) = 10 \sum_{n=-\infty}^{\infty} J_n(8) \cos(2\pi(500 + n\,10)t)$



3 harmonics pass

Power at the output : $\frac{(10)^2}{2} \sum_{n=-3}^{3} J_n^2(8)$

$50\left[(0.172)^2 + 2(0.235)^2 + 2(0.113)^2 + 2(0.291)^2\right]$

$= 16.75 W.$

e) Spectrum at filter input:

$$10 \sum_{n=-\infty}^{\infty} J_n(8) \cos(2\pi(500 + n10)t)$$

$J_0(8) = 0.172$

$J_1(8) = 0.235$

$J_2(8) = -0.113$

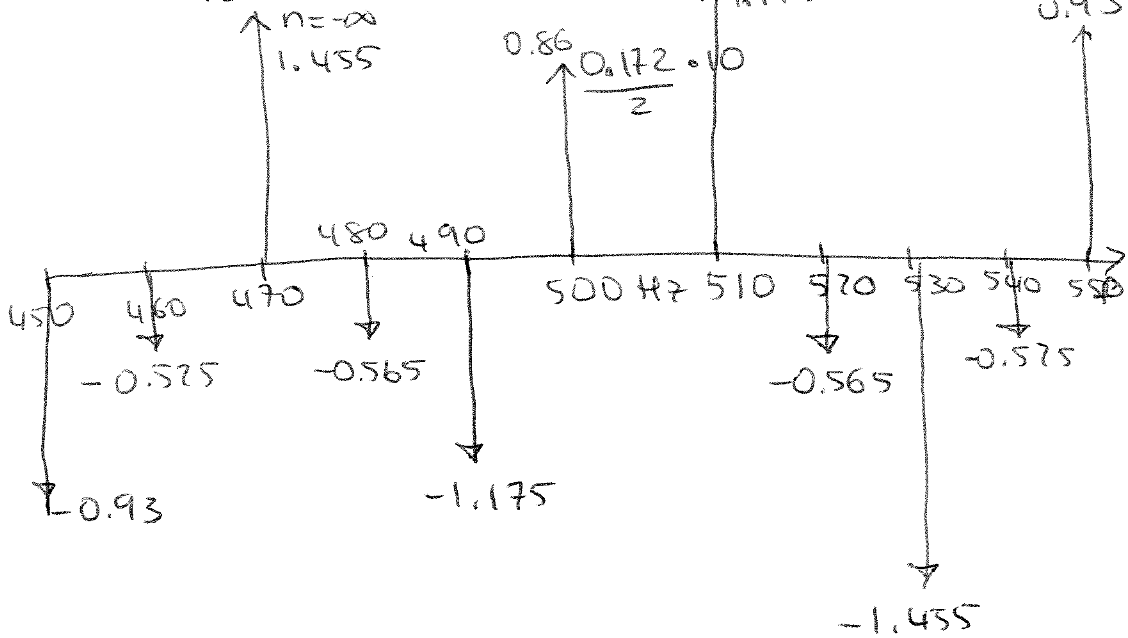
$J_3(8) = -0.291$

$J_4(8) = -0.105$

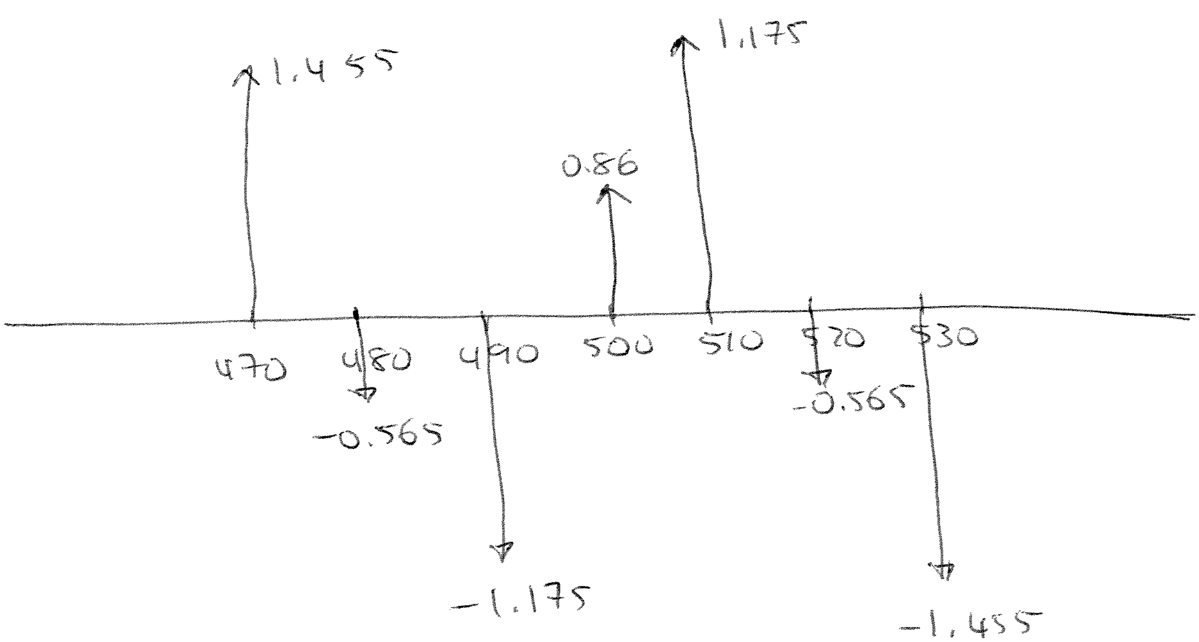
$J_5(8) = 0.186$

$J_6(8) = 0.338$

$J_7(8) = 0.321$



At the output of the filter



(4)

3.42

$f_m = 150\,Hz$

$\beta = 10$

BW for $P_r = 0.8$

$P_r = 0.8 \Rightarrow \sum_{n=-k}^{k} J_n^2(10) \geq 0.8 \Rightarrow k=9 \Rightarrow BW = 2kf_m$

$= (18)(150) = 2.7\,kHz$

$P_r = 0.9 \Rightarrow \sum_{n=-k}^{k} J_n^2(10) \geq 0.9 \Rightarrow k=10 \Rightarrow BW = 2kf_m$

$= (20)(150)$

$= 3\,kHz$

(use Table 3.2 on pg 131)

Q5

$x_c(t) = 100\cos(2\pi f_c t + 4\sin 2\pi f_m t)$

$f_c = 10\,MHz$, $f_m = 1000\,Hz$

a) $\beta \rightarrow$ modulation index = peak phase deviation

$\phi(t) = 4\sin 2\pi f_m t \Rightarrow \beta = 4$

$BW = 2\lfloor \beta + 1 \rfloor f_m$

$= (2)(5)(1000) = 10\,kHz.$

b) if f_m is doubled:

$BW = 2\lfloor \beta + 1 \rfloor f_m$

$= (2)(5)(2000)$

$= 20\,kHz.$