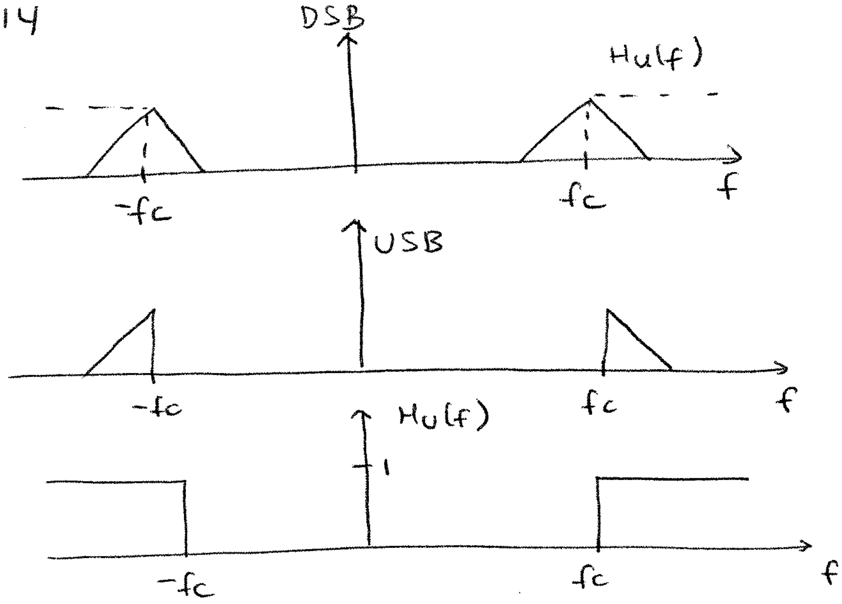
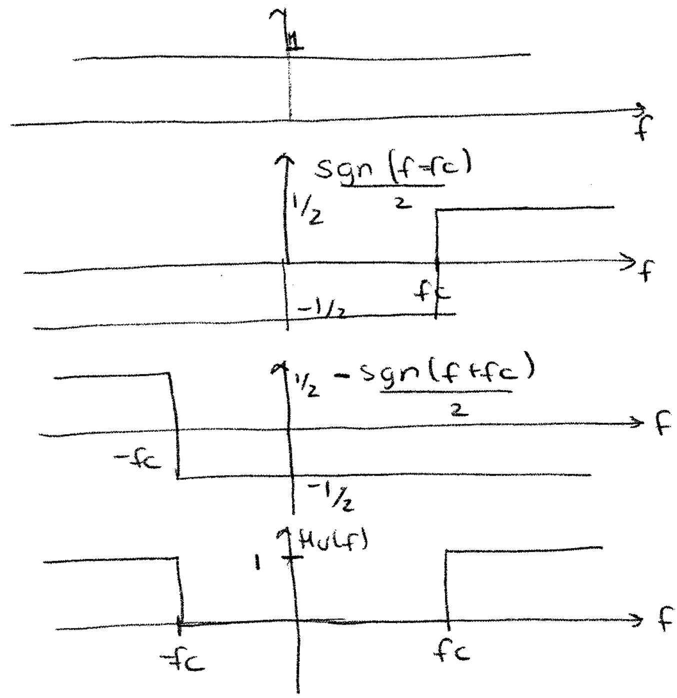
# ECE 457 HW #4

(1) 3.14




$$H_u(f) = 1 + \frac{\mathrm{sgn}(f-f_c)}{2} - \frac{\mathrm{sgn}(f+f_c)}{2}$$




$$X_c(f) = X_{DSB}(f)\,H_u(f)$$

$$= \left[\frac{1}{2}A_c M(f+f_c) + \frac{1}{2}A_c M(f-f_c)\right]\left[1 + \frac{\mathrm{sgn}(f-f_c)}{2} - \frac{\mathrm{sgn}(f+f_c)}{2}\right]$$

$$= \frac{1}{2}A_c\left[M(f+f_c) + M(f-f_c)\right] + \frac{1}{4}A_c\left[M(f+f_c)\,\mathrm{sgn}(f-f_c) + M(f-f_c)\,\mathrm{sgn}(f-f_c)\right]$$

$$- \frac{1}{4}A_c\left[M(f+f_c)\,\mathrm{sgn}(f+f_c) + M(f-f_c)\,\mathrm{sgn}(f+f_c)\right]$$

Note: $M(f+f_c)\,\mathrm{sgn}(f-f_c) = -M(f+f_c)$ and $M(f-f_c)\,\mathrm{sgn}(f+f_c) = M(f-f_c)$

$$X_c(f) = \frac{1}{2} A_c \left[M(f+f_c) + M(f-f_c)\right] - \frac{1}{4} A_c \left[M(f+f_c) + M(f-f_c)\right]$$

$$+ \frac{1}{4} A_c \left[M(f-f_c)\,\mathrm{sgn}(f-f_c) - M(f+f_c)\,\mathrm{sgn}(f+f_c)\right]$$

$$= \frac{1}{4} A_c \left[M(f+f_c) + M(f-f_c)\right] + \frac{1}{4} A_c \left[M(f-f_c)\,\mathrm{sgn}(f-f_c) - M(f+f_c)\,\mathrm{sgn}(f+f_c)\right]$$

$$\downarrow \mathcal{F}^{-1} \qquad \downarrow \mathcal{F}^{-1}$$

$$\frac{1}{-j} \hat{m}(t) e^{j2\pi f_c t} \qquad \frac{-1}{j} \hat{m}(t) e^{-j2\pi f_c t}$$

Note: $\hat{m}(t) \xrightarrow{\mathcal{F}} -j\,\mathrm{sgn}(f) M(f)$

$\hat{m}(t) e^{j2\pi f_c t} \xrightarrow{\mathcal{F}} -j\,\mathrm{sgn}(f-f_c) M(f-f_c)$

$\hat{m}(t) e^{-j2\pi f_c t} \xrightarrow{\mathcal{F}} -j\,\mathrm{sgn}(f+f_c) M(f+f_c)$

$$x_c(t) = \frac{1}{2} A_c m(t) \cos(2\pi f_c t) + \frac{1}{4} A_c \left[ -\frac{\hat{m}(t) e^{j2\pi f_c t}}{j} + \frac{\hat{m}(t) e^{-j2\pi f_c t}}{j} \right]$$

$$= \frac{1}{2} A_c m(t) \cos(2\pi f_c t) - \frac{1}{2} A_c \hat{m}(t) \sin(2\pi f_c t) //$$

(2)

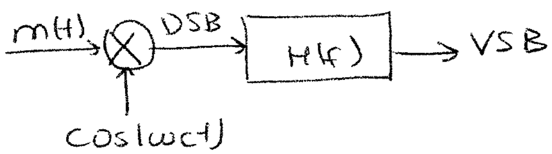
( ) 3.16 VSB filter is $H(f_c - f_1) = \epsilon e^{j\phi}$

$H(f_c + f_1) = (1-\epsilon)e^{j\theta_1}$

$H(f_c + f_2) = e^{j\theta_2}$

$m(t) = A\cos(\omega_1 t) + B\cos(\omega_2 t)$

VSB modulation:




$e_{DSB}(t) = \frac{1}{2} A\cos(\omega_c+\omega_1)t + \frac{1}{2} A\cos(\omega_c-\omega_1)t + \frac{1}{2} B\cos(\omega_c+\omega_2)t$

$+ \frac{1}{2} B\cos(\omega_c-\omega_2)t$ (eqn. 3.65)

$x_c(t) = \frac{1}{2} A\epsilon\cos[(\omega_c-\omega_1)t+\phi] + \frac{1}{2} A(1-\epsilon)\cos[(\omega_c+\omega_1)t+\theta_1]$

$+ \frac{1}{2} B\cos[(\omega_c+\omega_2)t+\theta_2]$

Demodulation: Multiply by $4\cos(\omega_c t)$ and then pass through LPF

$\rightarrow 2A\epsilon\cos(\omega_c t)\cos[(\omega_c-\omega_1)t+\phi] + 2A(1-\epsilon)\cos[(\omega_c+\omega_1)t+\theta_1]\cos(\omega_c t)$

$+ 2B\cos(\omega_c t)\cos[(\omega_c+\omega_2)t+\theta_2]$

using trigonometric identities:

$= A\epsilon[\cos(\omega_1 t-\phi) + \overbrace{\cos((2\omega_c-\omega_1)t+\phi)}^{\text{won't pass}}] + A(1-\epsilon)[\cos(\omega_1 t+\theta_1) + \underbrace{\cos((2\omega_c+\omega_1)t+\theta_1)}_{\text{won't pass}}]$

$+ B[\cos(\omega_2 t+\theta_2) + \underbrace{\cos((2\omega_c+\omega_2)t+\theta_2)}_{\text{won't pass}}]$

$\rightarrow \boxed{LPF} \rightarrow$

Output: $A\epsilon\cos(\omega_1 t-\phi) + A(1-\epsilon)\cos(\omega_1 t+\theta_1) + B\cos(\omega_2 t+\theta_2)$

For the output to be distortionless, output $= m(t-\tau)$ (delayed version of the message)

$\therefore \theta_1 = -\phi$

$= A\cos(\omega_1 t+\theta_1) + B\cos(\omega_2 t+\theta_2)$

$= A\cos\left(\omega_1\left(t+\frac{\theta_1}{\omega_1}\right)\right) + B\cos\left(\omega_2\left(t+\frac{\theta_2}{\omega_2}\right)\right)$

The ideal phase response is linear, since $\theta_1 = -\phi$, and it's odd symmetric around $f_c$.

Therefore, the delays should be equal $\frac{\theta_1}{\omega_1} = \frac{\theta_2}{\omega_2}$ $\Rightarrow \boxed{\theta_2 = \frac{\omega_2}{\omega_1}\theta_1}$

(3) $m(t) = \cos(20\pi t)$

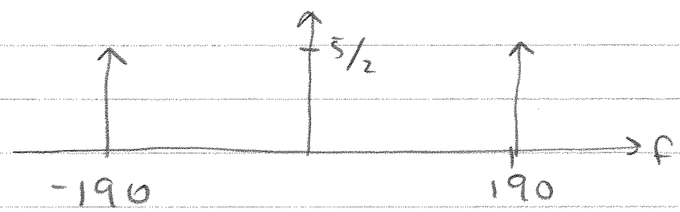
carrier $10\cos(400\pi t)$ LSB.

a) see plot.

b) $x_c(t) = 5\cos(400\pi t)\cos(20\pi t) + 5\sin(20\pi t)\sin(400\pi t)$

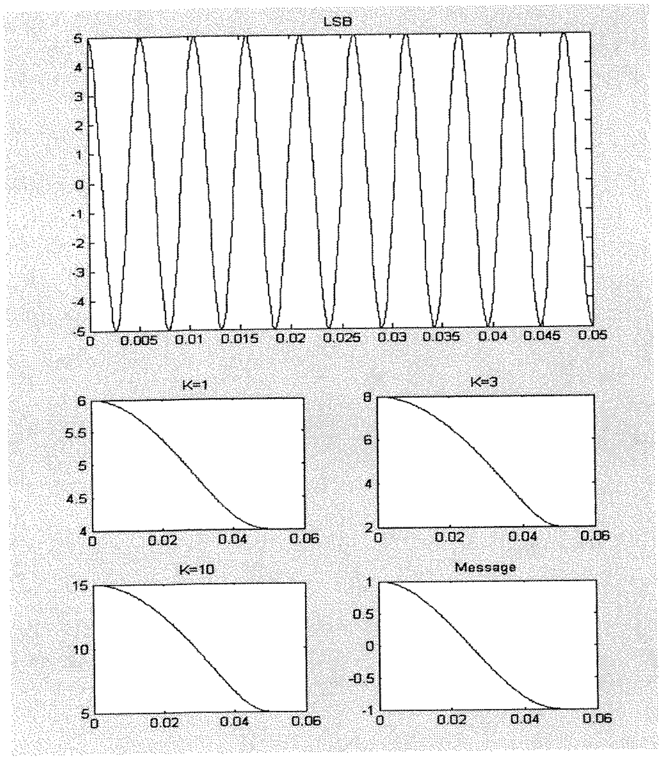
$= 5\cos((400\pi - 20\pi)t)$

$= 5\cos(380\pi t)$



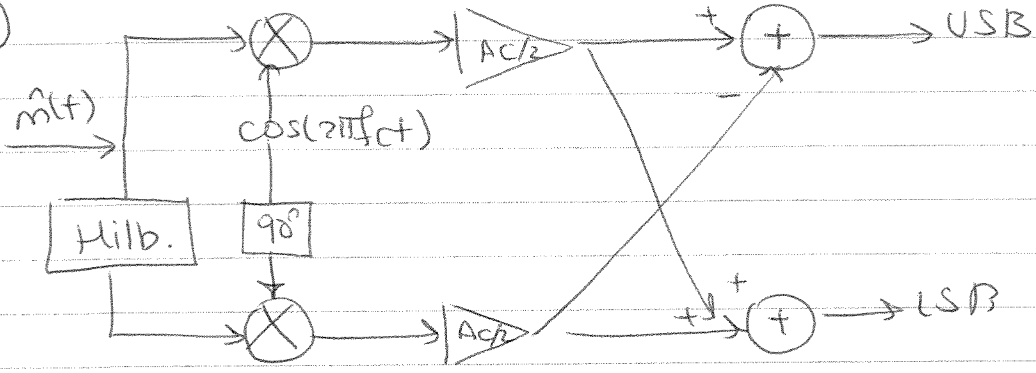

c) BW = 190 Hz

Power $= 2\left(\frac{5}{2}\right)^2 = 2\left(\frac{25}{4}\right) = \frac{25}{2} = 12.5\,W$ //




```
%hw4q1;
%02/11/04;
Ac=10;Am=1;wm=20*pi;wc=400*pi;
t=[0:.0001:.05];
%LSB signal;
xclsb=Am*Ac*0.5*(cos(20*pi*t).*cos(400*pi*t)+sin(20*pi*t).*sin(400*pi*t
));
figure;
plot(t,xclsb);
title('LSB');
%carrier reinsertion demodulation;
%add the carrier and find the actual envelope;
K=1;
env1=sqrt((0.5*Ac*cos(wm*t)+K).^(2)+(0.5*Ac*sin(wm*t)).^2);
K=3;
env2=sqrt((0.5*Ac*cos(wm*t)+K).^(2)+(0.5*Ac*sin(wm*t)).^2);
K=10;
env3=sqrt((0.5*Ac*cos(wm*t)+K).^(2)+(0.5*Ac*sin(wm*t)).^2);
figure
subplot(221);
plot(t,env1);
title('K=1');
subplot(222);
plot(t,env2);
title('K=3');
subplot(223);
plot(t,env3);
title('K=10');
subplot(224);
plot(t,cos(20*pi*t));
title('Message')
```

(4)




$\mathcal{H}\{\hat{m}(t)\} = -m(t)$

The output at the USB branch:

$$\frac{A_c}{2}\hat{m}(t)\cos(2\pi f_c t) + \frac{A_c}{2} m(t)\sin(2\pi f_c t)$$

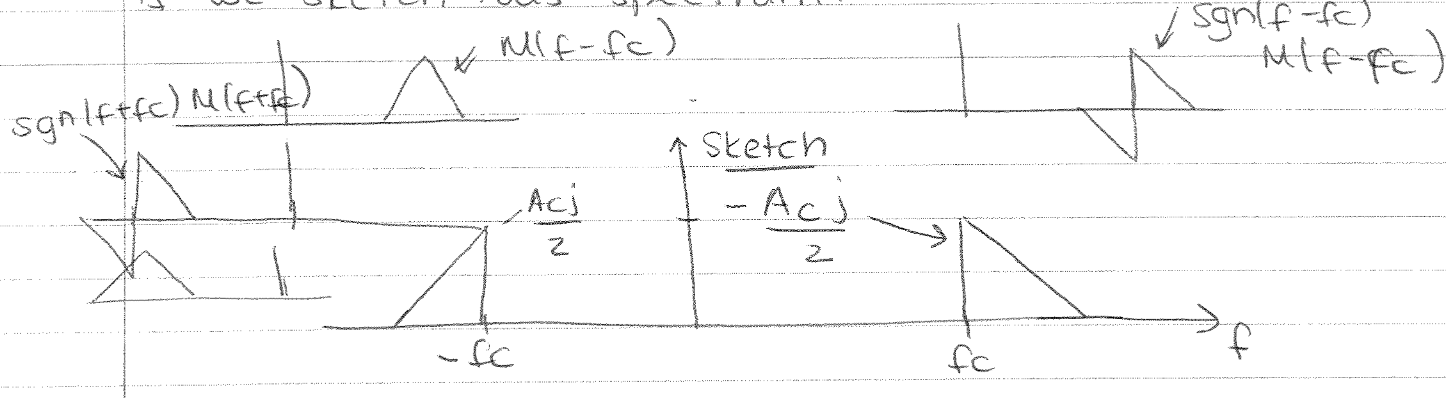
This is the original USB phase shifted.

Freq. domain

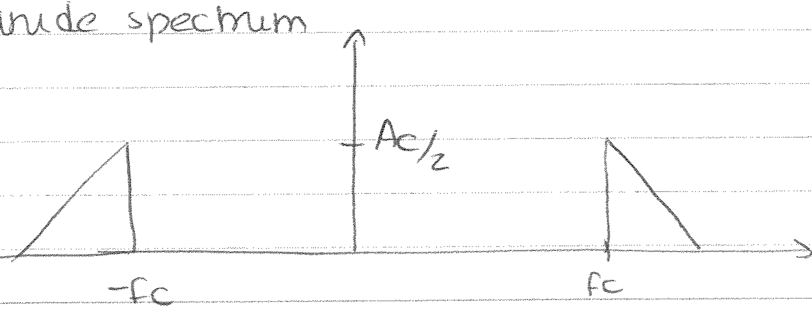
$$\frac{A_c}{2}\left[\frac{-j\,\mathrm{sgn}(f-f_c)M(f-f_c) - j\,\mathrm{sgn}(f+f_c)M(f+f_c)}{2}\right]$$

$$+\frac{A_c}{2}\left[\frac{-j\,M(f-f_c) + j\,M(f+f_c)}{2}\right]$$

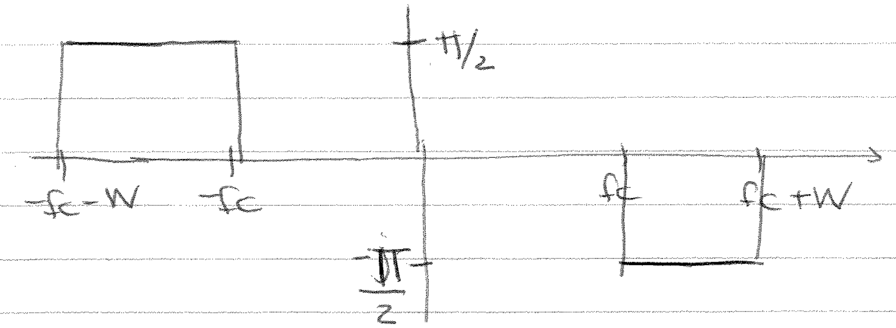
If we sketch this spectrum.




The amplitude spectrum



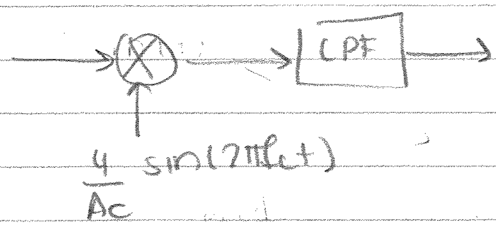
The phase spectrum




So this USB with $-\pi/2$ phase shift for $f>0$ and $\pi/2$ phase shift for $f<0$.

$\therefore$ we can demodulate this

if we multiply by $\frac{4}{A_c}\sin(2\pi f_c t)$ and then pass thru LPF




The input to LPF:

$$2\hat{m}(t)\cos(2\pi f_c t)\sin(2\pi f_c t)+2m(t)\sin(2\pi f_c t)\sin(2\pi f_c t)$$

$$\underbrace{\hat{m}(t)\sin(0)}_{=0} + \underbrace{\hat{m}(t)\sin(4\pi f_c t)} + m(t) - \underbrace{m(t)\sin(4\pi f_c t)}$$

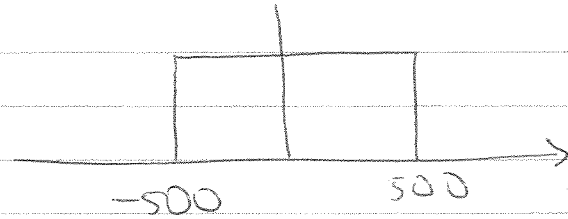
won't pass through LPF

We end up with $\underline{m(t)}$.

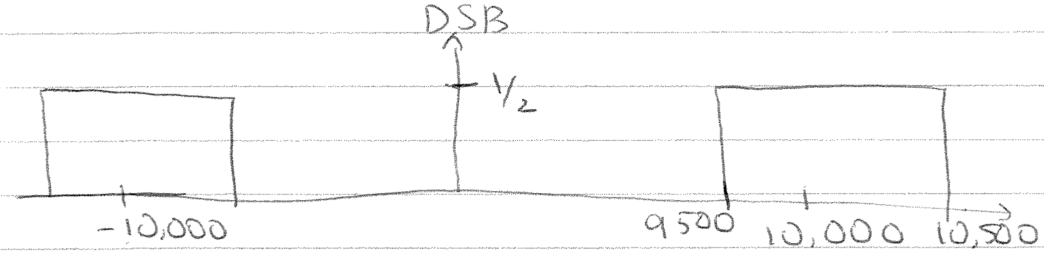
(5) $m(t) = B\,\mathrm{sinc}(Bt) \quad B = 1000$

modulated by $\cos(20{,}000\pi t)$

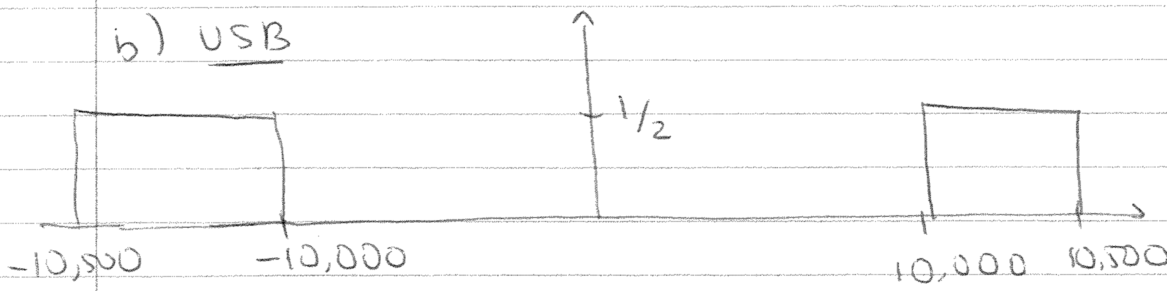
a) $M(f)$




$M(f) = \mathrm{rect}\left(\frac{f}{1000}\right)$




b) USB




c) USB in time domain

In freq. domain:

$$\frac{1}{2}\,\mathrm{rect}\left(\frac{f - 10{,}250}{500}\right) + \frac{1}{2}\,\mathrm{rect}\left(\frac{f + 10{,}250}{500}\right)$$

$$= \mathrm{rect}\left(\frac{f}{500}\right) * \left[\frac{1}{2}\,\delta(f - 10{,}250) + \frac{1}{2}\,\delta(f + 10{,}250)\right]$$

$$\Downarrow$$

$$500\,\mathrm{sinc}(500t) \cdot \cos(2\pi(10{,}250)\,t)$$

$$500\,\mathrm{sinc}(500t)\,\cos(20{,}500\pi t)$$