ECE 457

HW #11

Solutions

(1) SNR = 40 dB

BW of channel = 120 kHz

$W = 10\,kHz$

$\overline{m_n^2(t)} = 0.5$

$\frac{N_0}{2} = 10^{-8}\,W/Hz$.

$P_T = ?$

if 40 dB attenuation.

$SNR = 3(D)^2\,\overline{m_n^2(t)}\,(SNR)_{BB}$ (no attenuation)

$10^4 = 3(D)^2\,\overline{m_n^2(t)}\,\frac{10^{-4} \times P_T}{N_0 W}$ (with attenuation)

$2(D+1)W = 120\,kHz \Rightarrow D = 5$

$$10^4 = \frac{3(25)(0.5)\,10^{-4} \times P_T}{2 \times 10^{-8} \times 10 \times 10^3}$$

$P_T = 533.33\,W$.

(2) $W = 8\,kHz$

$\overline{m_n^2(t)} = 0.5$

channel BW 60 kHz.

attenuation 40 dB

$\frac{N_0}{2} = 10^{-12}\,W/Hz$.

a) $2(D+1)W = 60\,kHz. \Rightarrow D = 2.75$

$$10^4 = 3(D)^2(0.5)\frac{10^{-4} \times P_T}{2 \times 10^{-12} \times 8 \times 10^3} \Rightarrow P_T = 0.14\,W$$

b) SNR = 60dB

D is the same

$$10^6 = \frac{3(D)^2(0.5)\,10^{-4} \times P_T}{2\times10^{-12}\times 8\times10^3} \Rightarrow P_T = 14W.$$

c) $RC = 75\times10^{-6} \Rightarrow f_3 = \frac{1}{2\pi RC} = 2.122\,kHz$. $D = \frac{f_d |\max|m(t)||}{W}$

$f_d = (D)\,W$

$= (2.75)(8)$

$$SNR = \left(\frac{f_d}{f_3}\right)^2 \overline{m_n^2(t)}\ \frac{P_T\times10^{-4}}{N_0 W}$$

$$10^4 = \left(\frac{(2.75)(8)}{2.122}\right)^2 (0.5)\ \frac{P_T\times10^{-4}}{2\times10^{-12}\times8\times10^3}$$

$\underline{P_T = 0.0297\ W}$ (smaller power achieves the same SNR)

(3) <u>6.21</u>

Noise power with deemphasis: (1) $\frac{2N_0 f_3^2}{A_c^2}\left(\frac{W}{f_3} - \tan^{-1}\frac{W}{f_3}\right)$

Noise power without deemph: (2) $\frac{2}{3}\frac{N_0 W^3}{A_c^2}$

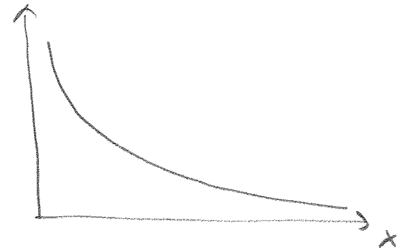
$$\frac{N_1}{N_2} = \frac{\frac{\cancel{2N_0} f_3^3}{\cancel{A_c^2}}\left(\frac{W}{f_3} - \tan^{-1}\frac{W}{f_3}\right)}{\frac{\cancel{2N_0} W^3}{3\cancel{A_c^2}}}$$

$$\frac{3\cdot f_3^3\left(\frac{W}{f_3} - \tan^{-1}\frac{W}{f_3}\right)}{W^3}$$

let $\frac{W}{f_3} = x$

Since x is large
$\tan^{-1} x \approx \pi/2$

$$\frac{N_1}{N_2} = \frac{3\,(x - \tan^{-1}x)}{x^3} \approx \frac{3x}{x^3} = \frac{3}{x^2}$$



$f_3 = 2.1\,\text{kHz}$
$W = 15\,\text{kHz}$ $\Rightarrow x = 7.14$

The ratio is $\frac{3}{x^2} = 0.0588$

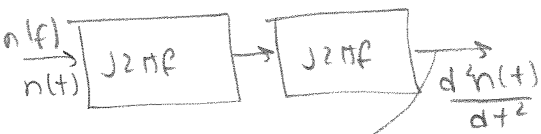
Improvement in dB 12.31 dB //

(4) 6.24

$\frac{A}{2\pi f_c}\sin(2\pi f_c t)$ $n(t)$

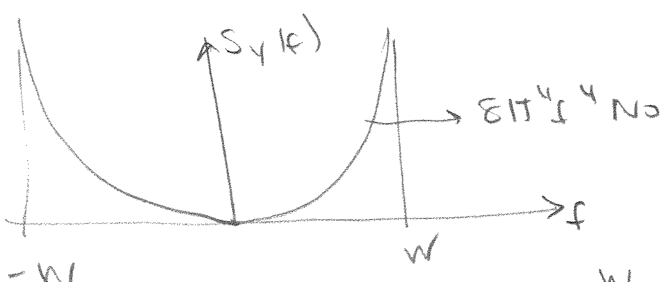
$x(t) = A\cos(2\pi f_c t)$

$\frac{-A}{4\pi^2 f_c^2}\cos(2\pi f_c t)$



$$y(t) = A\sin(2\pi f_c t) + \frac{d^2}{dt^2}n(t)$$

$$S_o(f) = \frac{N_o}{2}|2\pi f|^2|2\pi f|^2 = \frac{16\pi^4 f^4 N_o}{2} = 8\pi^4 f^4 N_o$$



Noise power: $2\int_0^W 8\pi^4 f^4 N_o\, df$

$$= 16\pi^4 N_o \left.\frac{f^5}{5}\right|_0^W = \frac{16\pi^4 N_o W^5}{5}$$

$$\text{SNR: } \frac{\text{Signal power}}{\text{Noise power}} = \frac{A^2/2}{\frac{16\pi^4 N_0 W^5}{5}} = \boxed{\frac{5A^2}{32\pi^4 N_0 W^5}}$$

(5) $W = 5\,\text{kHz}$

$\overline{m_n^2(t)} = 0.1\,W$

BW of channel $100\,\text{kHz} = 2(D+1)W$

$D = 9 = k_p \max|m(t)| \rightarrow k_p = 9$

Attenuation 80 dB.

$\frac{N_0}{2} = 0.5 \times 10^{-12}\ W/Hz.$

$P_T = 10\,kW$

attenuation

$$(SNR)_0 = k_p^2\, \overline{m_n^2(t)}\ (SNR)_{BB} \times 10^{-8}$$

$$= (81)(0.1)\frac{10 \times 10^3 \times 10^{-8}}{2 \times 0.5 \times 10^{-12} \times 5 \times 10^3}$$

$\boxed{52.1\,dB}$