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Student ID: \_\_\_\_\_

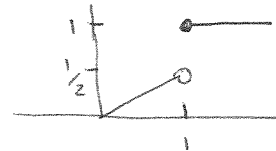
**ECE 457 EXAM 2**  
**April 8, 2005**

- No textbooks, notes or HW solutions.
- One page of hand-written notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [30] Answer the following questions briefly.

a) [10] A random variable X is defined by its cumulative distribution function:

$$F_X(x) = \begin{cases} 0, & x < 0 \\ \frac{1}{2}x, & 0 \leq x < 1 \\ K, & x \geq 1 \end{cases}$$



i) (2) Find the value of K.

$$K = 1$$

ii) (3) Find  $P(0.5 < X \leq 1)$ .

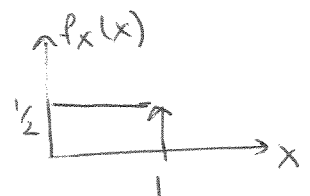
$$F_X(1) - F_X(0.5) = 1 - \frac{1}{4} = \frac{3}{4}$$

iii) (2) Find  $P(X > 2)$ .

$$1 - F_X(2) = 0.$$

iv) (3) Find the probability density function,  $f_X(x)$ .

$$f_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2} & 0 \leq x \leq 1 \\ 0 & x > 1 \end{cases}$$



b) [5] Which of the following statements is false for a voltage controlled oscillator?

- i) It can be used for implementing a FM transmitter.
- ii) If the input is a constant, then the output will be a constant frequency sinusoid.
- iii) It can be implemented using a differentiator followed by an envelope detector.

c) [5] If the input to a frequency discriminator is  $x_c(t) = 10 \cos(2\pi f_c t + 10t^2)$ , what is the output?

Note: Assume that the frequency discriminator is built using a differentiator followed by an envelope detector and a DC block.

$100t^2$        $200t$        $20t$        $10t^2$

$$\phi(t) = 10t^2 = 2\pi f_d \int m(z) dz \Rightarrow 20t = 2\pi f_d m(t)$$

d) [5] The random variable  $Y = 2X + 5$ . If  $X$  is a random variable with mean 2 and variance 4, what is the variance of  $Y$ ?

- 8      16      97      81

e) [5] The noise in a channel at time  $t$ ,  $n(t)$ , is modeled as a Gaussian random variable with mean 1 and variance 4. What's the probability that the noise is negative in terms of the Q-function?

$Q(0.25)$        $1-Q(0.5)$        $Q(0.5)$        $1-Q(0.25)$

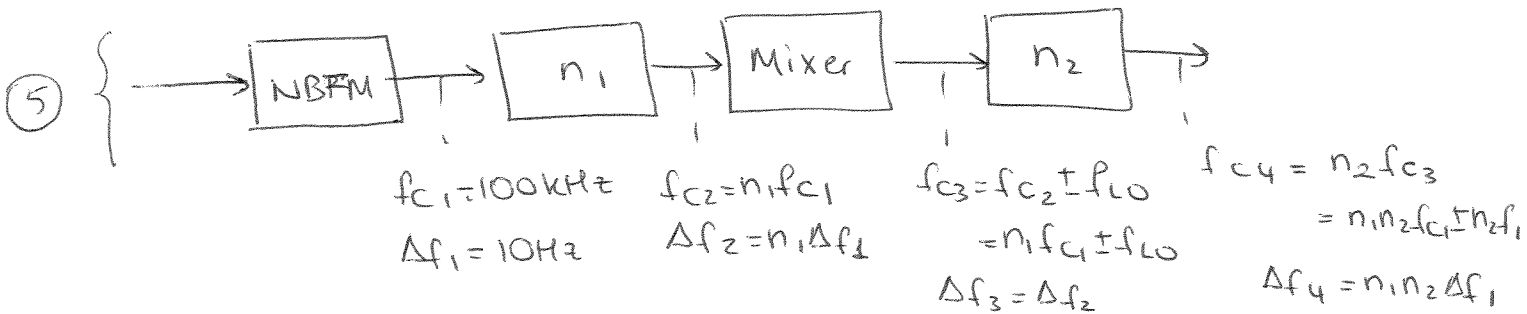
Note:  $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt$

$$\begin{aligned}
 P(n(t) < 0) &= 1 - Q\left(\frac{0-1}{2}\right) \\
 &= 1 - Q(-0.5) \\
 &= 1 - (1 - Q(0.5)) \\
 &= Q(0.5)
 \end{aligned}$$

2. [35] You are asked to design a FM transmitter (Armstrong) for an audio signal bandlimited to 10kHz with the following constraints.

- i) You have to use a narrowband FM generator with  $f_{c1} = 100\text{kHz}$  and peak frequency deviation  $\Delta f_1 = 10\text{Hz}$ .
- ii) The final carrier frequency is 98.1 MHz.
- iii) The final peak frequency deviation is 75kHz.
- iv) A variable frequency oscillator in the range of 10-11 MHz is available.
- v) Plenty of frequency doublers, triplers and quintuplers are available.

Give the block diagram of the system and specify the frequency multipliers, the local oscillator frequency and the carrier frequency and frequency deviation at the output of each block. Show your work to get full credit.



$f_{c4} = 98.1\text{MHz}$

$\Delta f_4 = 75\text{kHz}$

①  $\Delta f_4 = n_1 n_2 \Delta f_1$   
 $75000 = 10 n_1 n_2 \Rightarrow n_1 n_2 = 7500$  } ⑤

②  $f_{c4} = n_1 n_2 f_{c1} \pm n_2 f_{LO} = 98.1\text{MHz}$   
 $(7500)(100\text{K}) \pm n_2 f_{LO} = 98.1\text{M}$   
 $750 \pm n_2 f_{LO} = 98.1 \Rightarrow n_2 f_{LO} = 651.9$  } ⑤

③  $10 < f_{LO} < 11 \rightarrow 59.263 < n_2 < 65.19$  } ⑤

$n_2$  can be 60 or 64  
 if  $n_2 = 60, n_1 = 125$  /  $n_2 = 64, n_1 = 11.718$  } ⑤

④  $f_{LO} = \frac{651.9}{60} = 10.865\text{MHz}$  } ⑤

⑤  $f_{c2} = 12.5\text{MHz}$  |  $f_{c3} = 1.635\text{MHz}$  |  $f_{c4} = 98.1\text{MHz}$   
 $\Delta f_2 = 1.25\text{kHz}$  |  $\Delta f_3 = 1.25\text{kHz}$  |  $\Delta f_4 = 75\text{kHz}$  } ⑤

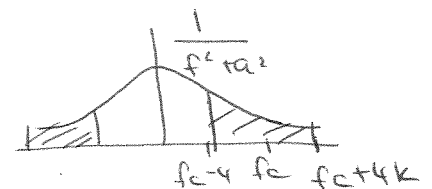
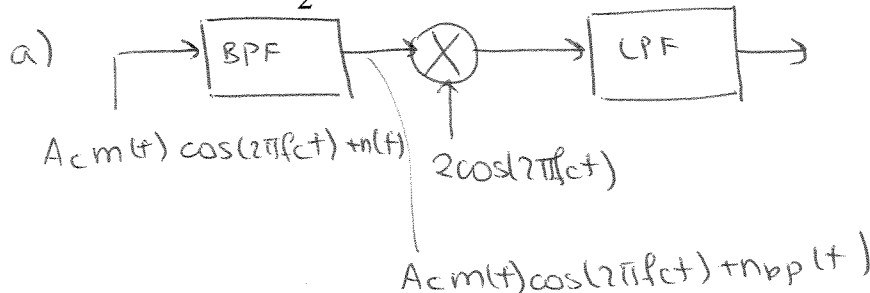
3. [35] A DSB system with carrier frequency,  $f_c = 500$  kHz transmits a message  $m(t)$  which has a bandwidth of 4 kHz. The modulated signal is transmitted over a channel with noise that has power spectral density,  $S_n(f) = \frac{1}{f^2 + a^2}$ , where  $a = 0.5 \times 10^6$ . The signal power at the receiver input is  $1 \mu$  Watts. The received DSB signal is bandpass filtered, multiplied by  $2 \cos(2\pi f_c t)$  and then low-pass filtered. (This is the coherent demodulator receiver structure we discussed in class.)

- [12] Find the pre-detection SNR, i.e. SNR at the output of the bandpass filter. Express your answer in dBs.
- [8] Find the output SNR. Express your answer in dBs.
- [5] Find the detection gain.
- [10] If the noise in the channel is white noise with power spectral density,  $\frac{N_0}{2}$ , equal to the peak of the power spectral density given in this question, what will the output SNR be (in dBs)? Compare your result with what you found in part (b) and explain the difference.

Hint:  $\int \frac{1}{1+x^2} dx = \tan^{-1}(x)$

$$\cos(x)\cos(y) = \frac{1}{2}(\cos(x-y) + \cos(x+y))$$

$$\sin(x)\cos(y) = \frac{1}{2}(\sin(x-y) + \sin(x+y))$$



③  $\rightarrow$  Signal power:  $1 \mu$ W

Noise power:  $\int S_{nb}(f) df = 2 \int_{496k}^{504k} \frac{1}{f^2 + a^2} df$  ④

$$= \frac{2}{a^2} \int_{496k}^{504k} \frac{1}{1 + (f/a)^2} df$$

let  $x = f/a$   $dx = df/a$

$$= \frac{2}{a} \int_{\frac{496k}{a}}^{\frac{504k}{a}} \frac{1}{1+x^2} dx$$

$$= \frac{2}{a} \left[ \tan^{-1}(x) \right]_{\frac{496k}{a}}^{\frac{504k}{a}}$$

Extra Page for Question 3:

$$= \frac{2}{0.5 \times 10^6} \left[ \tan^{-1} \left( \frac{504 \times 10^3}{0.5 \times 10^6} \right) - \tan^{-1} \left( \frac{496 \times 10^3}{0.5 \times 10^6} \right) \right]$$

$$= 4 \times 10^{-6} [0.78938 - 0.78138] = \underline{3.2 \times 10^{-8} \text{ W}} \quad (2)$$

$$\text{SNR} = \frac{1 \times 10^{-6}}{3.2 \times 10^{-8}} = 31.25 \Rightarrow 14.95 \text{ dB} \quad (1)$$

b)  $(\text{SNR})_0 \rightarrow$  Signal at the output is  $A_c m(t)$

$$\text{Signal power: } A_c^2 \overline{m^2(t)} = 2 \times 1 \times 10^{-6} \quad (3)$$

$$\text{Note: Transmitted power: } \frac{A_c^2}{2} \overline{m^2(t)} = 1 \times 10^{-6} \quad (3)$$

$$\text{Noise power: } \overline{n_c^2(t)} = \overline{n_{bp}^2(t)} = 3.2 \times 10^{-8} \quad (3)$$

$$(\text{SNR})_0 = \frac{2 \times 10^{-6}}{3.2 \times 10^{-8}} = 62.5 \Rightarrow 17.95 \text{ dB} \quad (2)$$

c) Detection Gain = 2.  $(5)$

$$d) \frac{N_0}{2} = \frac{1}{a^2} \Rightarrow N_0 = \frac{2}{0.25 \times 10^{12}} = 8 \times 10^{-12} \text{ W/Hz.} \quad (3)$$

$$(\text{SNR})_{\text{DSB}} = \frac{P_T}{N_0 W} = \frac{1 \times 10^{-6}}{8 \times 10^{-12} \times 4 \times 10^3} = 31.25 \Rightarrow 14.95 \text{ dB} \quad (2)$$

This is lower than what we found in part (b) since noise power is larger.  $(2)$