ECE 366 HW 7
Solutions
Fall 2008

① 7.4-3

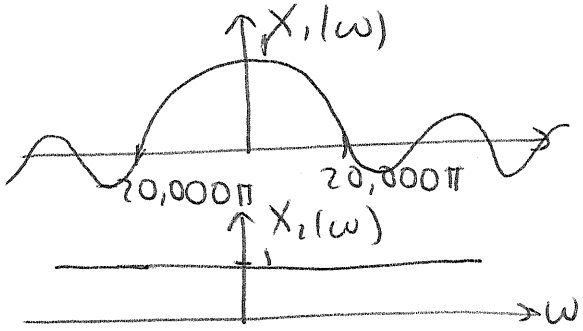
$x_1(t) = 10^4 \text{rect}(10^4 t)$

$x_2(t) = \delta(t)$

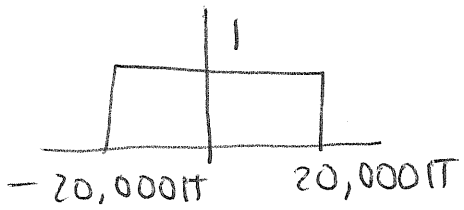
$H_1(\omega) = \text{rect}\left(\frac{\omega}{40{,}000\pi}\right)$ $H_2(\omega) = \text{rect}\left(\frac{\omega}{20{,}000\pi}\right)$

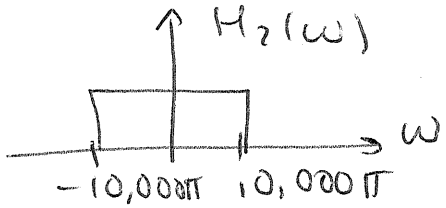
a) $X_1(\omega) = \text{sinc}\left(\frac{\omega}{20{,}000}\right)$

$X_2(\omega) = 1$

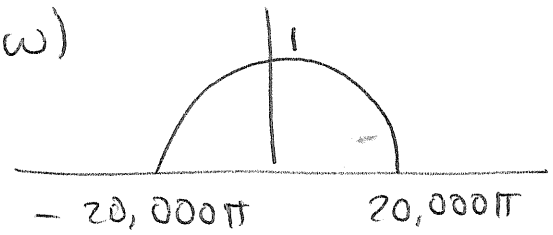


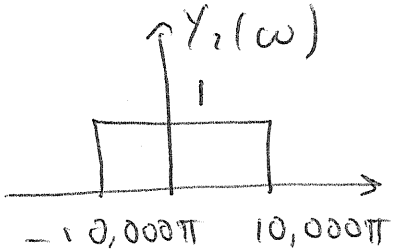
b) $H_1(\omega) =$





c) $Y_1(\omega)$





d) BW of $y_1(t)$ $20{,}000\pi$

BW of $y_2(t)$ $10{,}000\pi$

BW of $y(t)$ $\left(Y(\omega) = \frac{1}{2\pi} Y_1(\omega) * Y_2(\omega) \right)$

$30{,}000\pi$

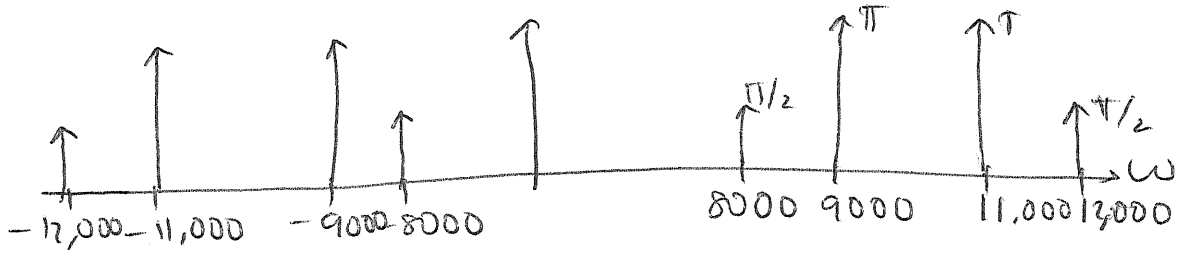
② 7.7-1 ii

$$m(t) = 2\cos 1000t + \cos(2000t)$$

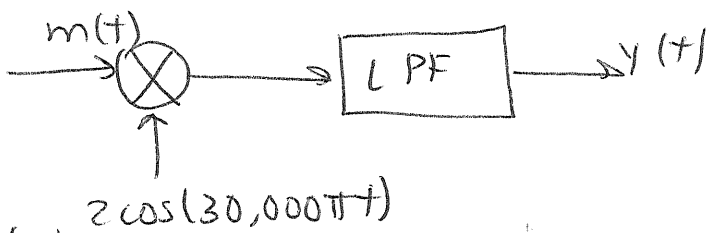
a) $M(\omega) = 2\pi[\delta(\omega-1000) + \delta(\omega+1000)]$

$$+\pi[\delta(\omega-2000) + \delta(\omega+2000)]$$

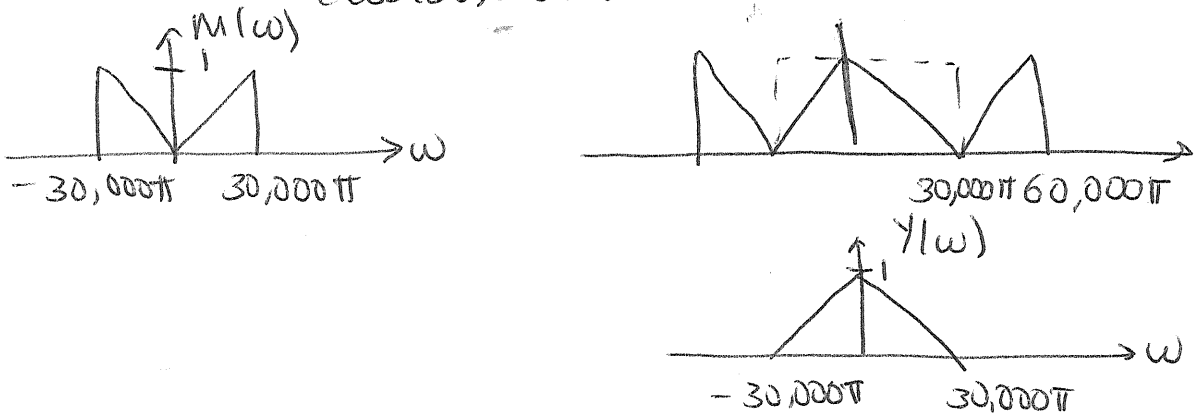
b) $m(t)\cos(10{,}000t) \rightarrow \frac{1}{2}[M(\omega-10{,}000) + M(\omega+10{,}000)]$



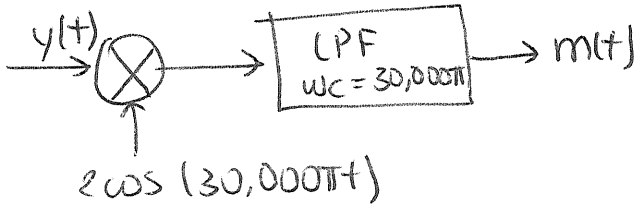
③ 7.7-5



a)



b)



(4) 7.6-6

$$x(t) = \frac{2a}{t^2+a^2} \longrightarrow X(\omega) = 2\pi e^{-a|\omega|}$$

$$E_x = \frac{(2\pi)}{2\pi}\int_{-\infty}^{\infty}\left(e^{-a|\omega|}\right)^2 d\omega = \frac{(4\pi^2)}{2\pi}\cdot 2\int_0^{\infty} e^{-2a\omega}\, d\omega$$

$$= 4\pi\int_0^{\infty} e^{-2a\omega}\, d\omega = 4\pi\,\frac{e^{-2a\omega}}{-2a}\Big|_0^{\infty} = 4\pi\left(\frac{1}{2a}\right) = \frac{2\pi}{a}$$

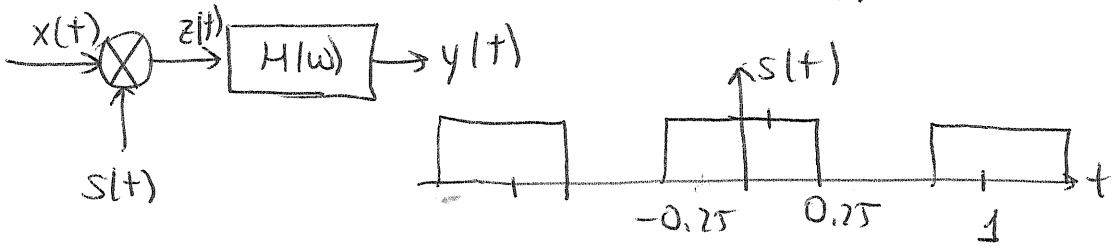
$$2\cdot 4\pi\int_0^{B} e^{-2a\omega}\, d\omega = \left(\frac{2\pi}{a}\right)0.99$$

$$\frac{e^{-2a\omega}}{-2a}\Big|_0^{B} = \frac{e^{-2aB}}{-2a} + \frac{1}{2a} = \frac{0.99}{2}$$

$$\rightarrow e^{-2aB} = 0.01$$

$$B = \frac{2.3026}{a}\ //$$

(5)



a) $g(t) = \text{rect}\left(\frac{t}{0.5}\right) = \text{rect}(2t)$

$\omega_0 = 2\pi$

$G(\omega) = 0.5\,\text{sinc}\left(\frac{\omega}{4}\right)$

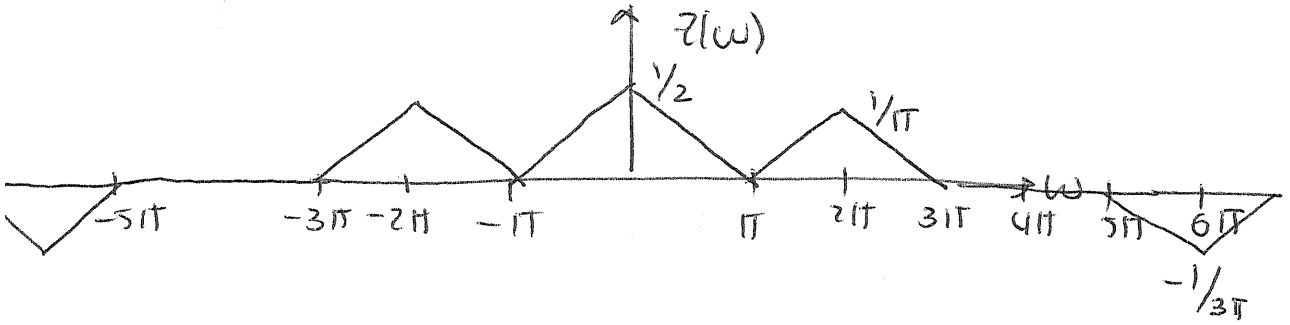
$$S(\omega) = 2\pi\sum_{k=-\infty}^{\infty} 0.5\,\text{sinc}\left(k\cdot\frac{2\pi}{4}\right)\delta(\omega - k2\pi)$$

$$= \pi\sum_{k=-\infty}^{\infty}\text{sinc}\left(\frac{\pi k}{2}\right)\delta(\omega - k2\pi)$$

b) $Z(\omega) = \frac{1}{2\pi} X(\omega) * S(\omega)$

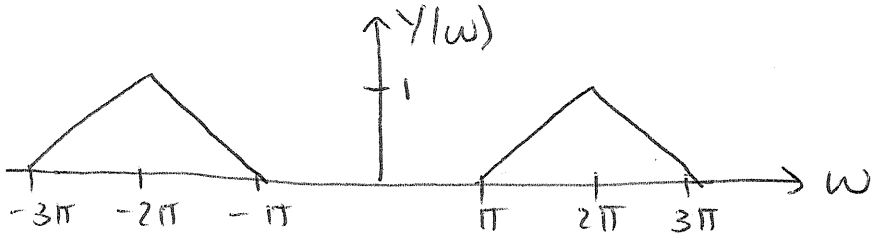
$= \frac{1}{2} X(\omega) * \sum \text{sinc}\left(\frac{\pi k}{2}\right) \delta(\omega - k2\pi)$

$= \frac{1}{2} \sum_{k=-\infty}^{\infty} \text{sinc}\left(\frac{\pi k}{2}\right) X(\omega - k2\pi)$



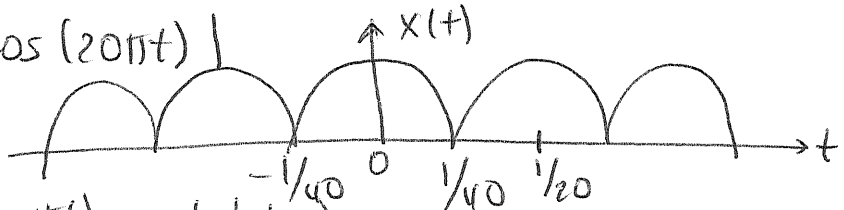
c) $y(t) = 2x(t)\cos(2\pi t)$

$\rightarrow Y(\omega) = X(\omega - 2\pi) + X(\omega + 2\pi)$



$H(\omega)$ is a BPF with gain π and cutoff freqs. π & 3π.

(6) $x(t) = |\cos(20\pi t)|$



$g(t) = \cos(20\pi t)\ \text{rect}\left(\frac{t}{1/20}\right)$

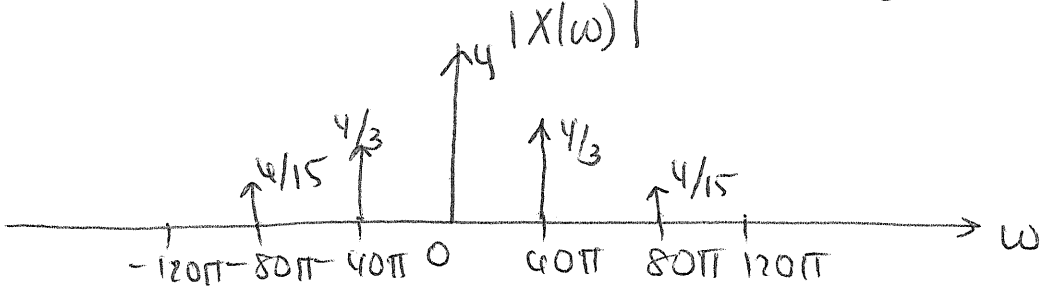
$G(\omega) = \frac{1}{2\pi} \pi[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)] * \frac{1}{20} \text{sinc}\left(\frac{\omega}{40}\right)$

$= \frac{1}{40}\left[\text{sinc}\left(\frac{\omega - 20\pi}{40}\right) + \text{sinc}\left(\frac{\omega + 20\pi}{40}\right)\right]$

$$\omega_0 = \frac{2\pi}{1/20} = 40\pi$$

$$X(\omega) = 40\pi \sum_{k=-\infty}^{\infty} \frac{1}{40}\left[\text{sinc}\left(\frac{k40\pi - 20\pi}{40}\right) + \text{sinc}\left(\frac{k40\pi + 20\pi}{40}\right)\right] \delta(\omega - k40\pi)$$

$$= \pi \sum_{k=-\infty}^{\infty}\left[\text{sinc}\left(k\pi - \frac{\pi}{2}\right) + \text{sinc}\left(k\pi + \frac{\pi}{2}\right)\right] \delta(\omega - k40\pi)$$



$$k=0 \quad \left(\text{sinc}\left(-\pi/2\right) + \text{sinc}\left(\pi/2\right)\right)\pi = \left(\frac{2}{\pi} + \frac{2}{\pi}\right)\pi = 4$$

$$k=1 \quad \left[\text{sinc}\left(\pi/2\right) + \text{sinc}\left(3\pi/2\right)\right]\pi = \left(\frac{2}{\pi} - \frac{2}{3\pi}\right)\pi = 4/3$$

$$k=2 \quad \left[\text{sinc}\left(\frac{3\pi}{2}\right) + \text{sinc}\left(\frac{5\pi}{2}\right)\right]\pi = \left(\frac{-2}{3\pi} + \frac{2}{5\pi}\right)\pi = \frac{-4}{15}$$

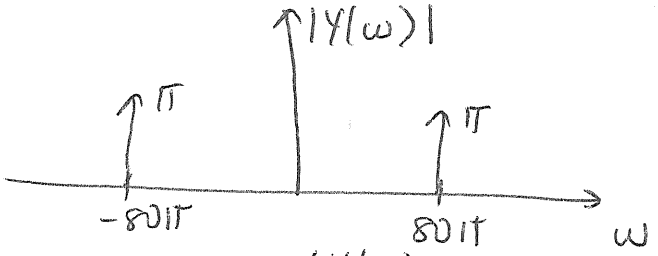
b) $$C_k = \frac{\pi\left(\text{sinc}\left(k\pi - \frac{\pi}{2}\right) + \text{sinc}\left(k\pi + \pi/2\right)\right)}{2\pi}$$

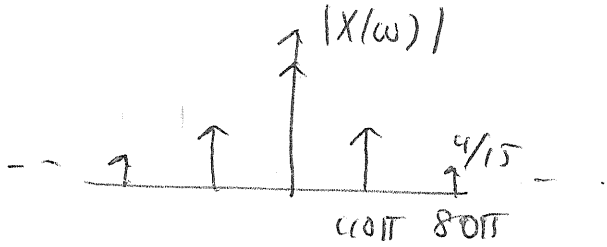
$$= \frac{1}{2}\left[\text{sinc}\left(k\pi - \pi/2\right) + \text{sinc}\left(k\pi + \pi/2\right)\right]$$

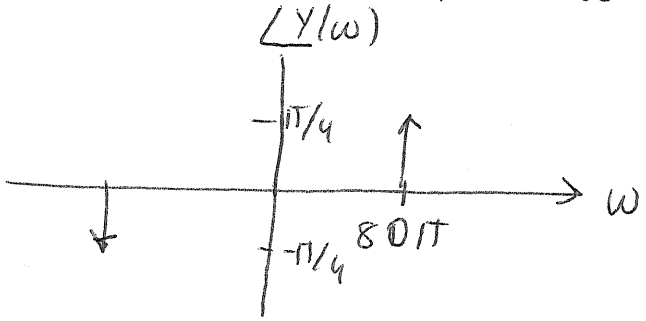
c) $y(t) = \cos\left(80\pi t + \pi/4\right)$ H(ω) should be a BPF without off

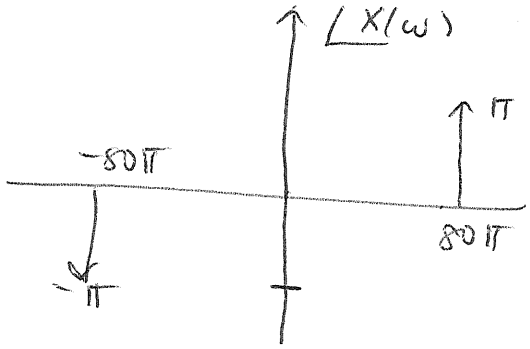
$40\pi < \omega_1 < 80\pi \quad + 80\pi < \omega_2 < 120\pi$

$$y(t) = \frac{e^{j80\pi t} \cdot e^{j\pi/4} + e^{-j80\pi t} e^{-j\pi/4}}{2} = e^{j\pi/4}\,\pi\,\delta(\omega - 80\pi) + e^{-j\pi/4}\,\pi\,\delta(\omega + 80\pi)$$

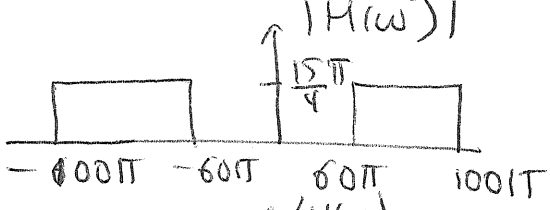








H(ω) should have gain $\frac{15}{4}\pi$ and phase



(not a unique answer)