ECE 366 HW #8

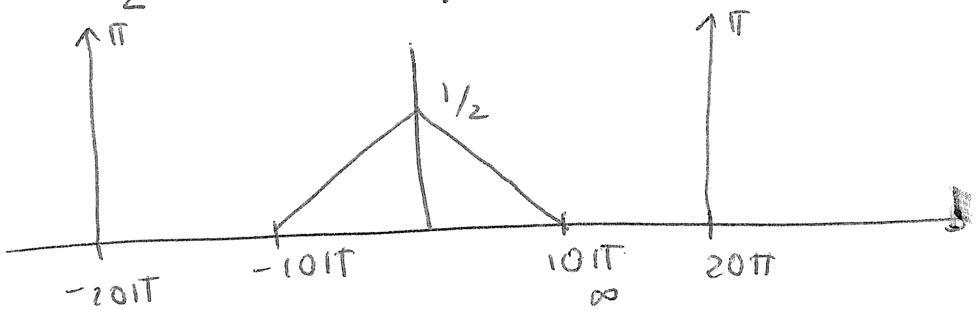
Fall 2008

solutions

1) 8.1-7

a) $x(t) = 5\,\text{sinc}^2(5\pi t) + \cos(20\pi t)$

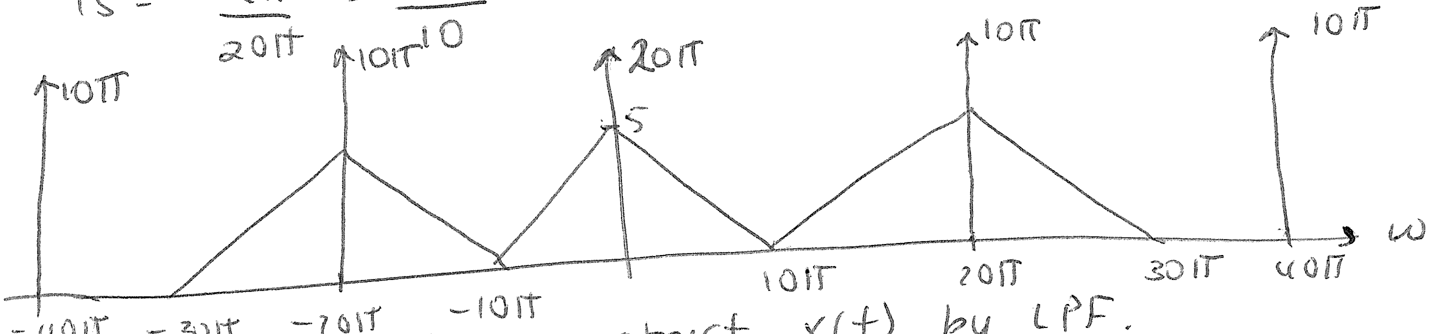
$$X(\omega) = \frac{1}{2}\,\text{tri}\left(\frac{\omega}{20\pi}\right) + \pi\left[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)\right]$$



$\omega_s = 20\pi$ rad/s.

$T_s = \frac{2\pi}{20\pi} = \frac{1}{10}$

$$X_s(\omega) = 10 \sum_{n=-\infty}^{\infty} X(\omega - n20\pi)$$

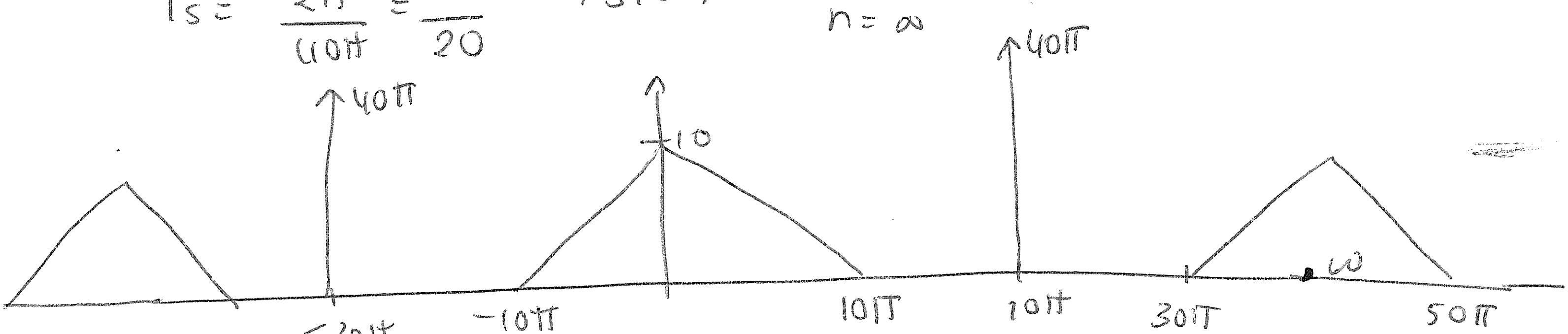


No, we can't reconstruct $x(t)$ by LPF.

b) $\omega_s = 40\pi$ rad/s

$T_s = \frac{2\pi}{40\pi} = \frac{1}{20}$

$$X_s(\omega) = 20 \sum_{n=\infty}^{\infty} X(\omega - 40\pi n)$$



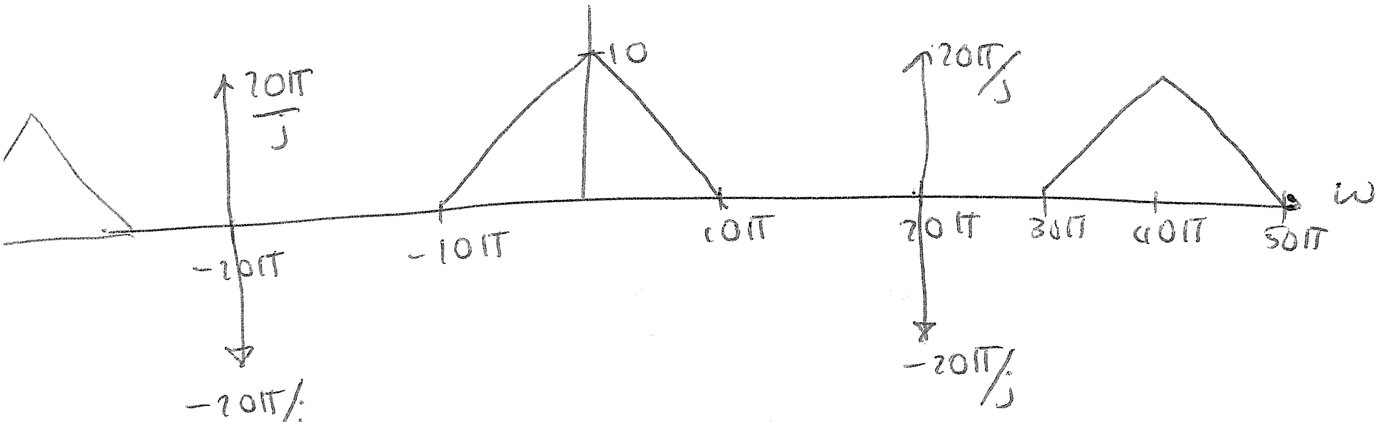
we can recover $x(t)$ up to a constant in the cosine term.

c) $x(t) = 5\,\text{sinc}^2(5\pi t) + \sin(10\pi t)$

$$X(\omega) = \frac{1}{2}\,\text{tri}\left(\frac{\omega}{20\pi}\right) + \frac{\pi}{j}\left[\delta(\omega - 20\pi) - \delta(\omega + 20\pi)\right]$$

$\omega_s = 40\pi$, $T_s = 1/20$

$$X_s(\omega) = 20 \sum X(\omega - n\,40\pi)$$

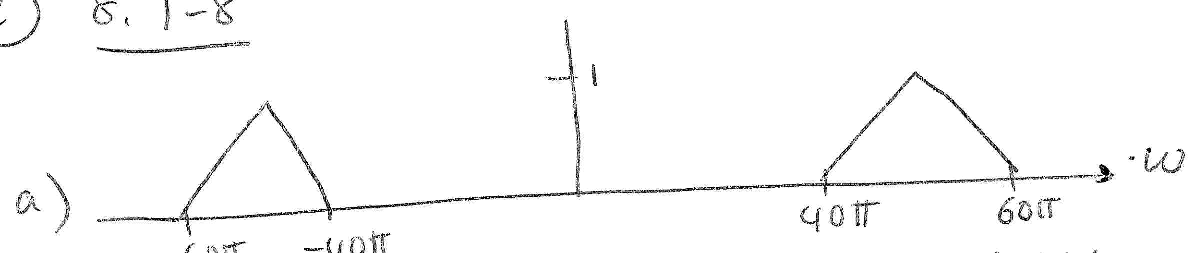


No you can't recover back x(t).

d) If $\omega_s = 42\pi$, then we can recover back x(t) by low pass filtering.

(2) 8.1-8

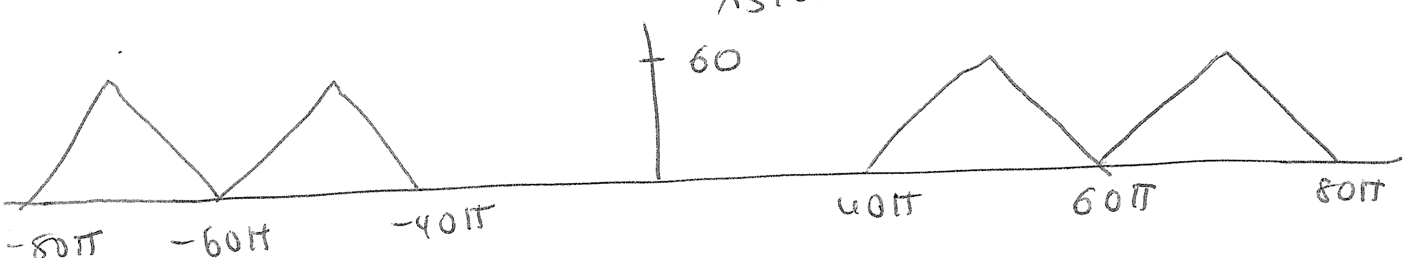
a)



$\omega_s = \cancel{60}\ 120\pi$.

$T_s = \frac{1}{60}$

$$X_s(\omega) = 60 \sum X(\omega - n\,120\pi)$$



Can reconstruct by LPF cutoff 60π.

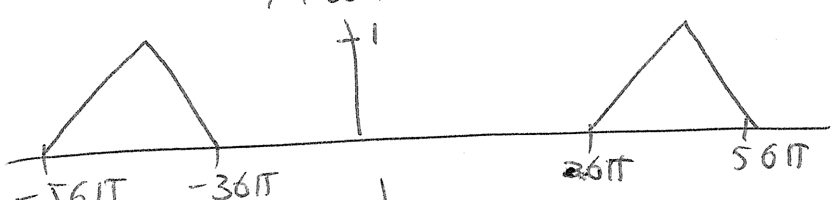
b) $\omega_s = 40\pi \quad T_s = 1/20$

$X_s(\omega) = 20 \sum X(\omega - n40\pi)$

We can reconstruct by using a BPF with cutoff between 40π & 60π.

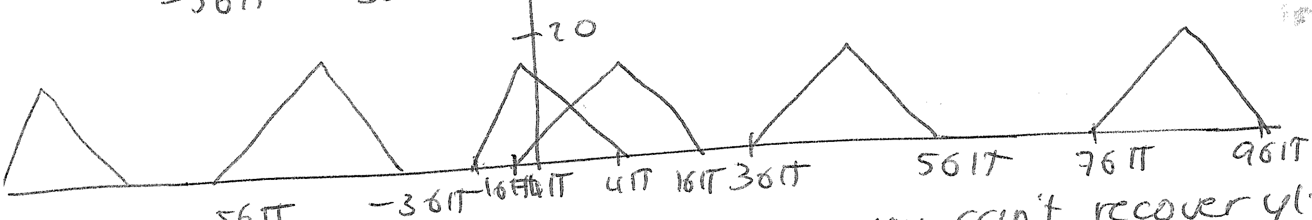
c)

$Y(\omega)$



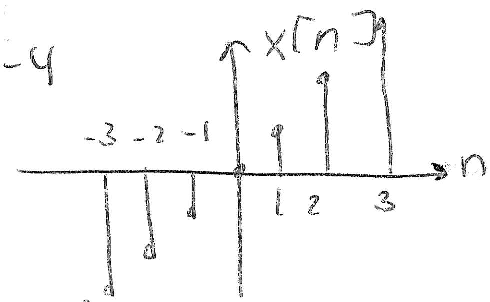
$\omega_s = 40\pi$

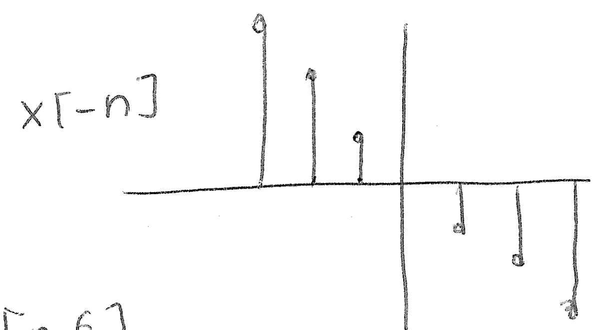
$T_s = \frac{1}{20}$

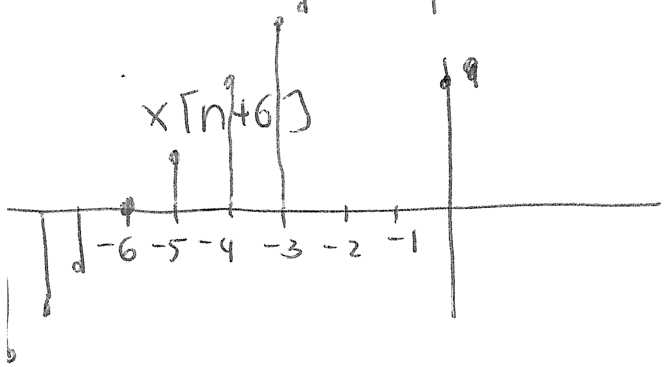


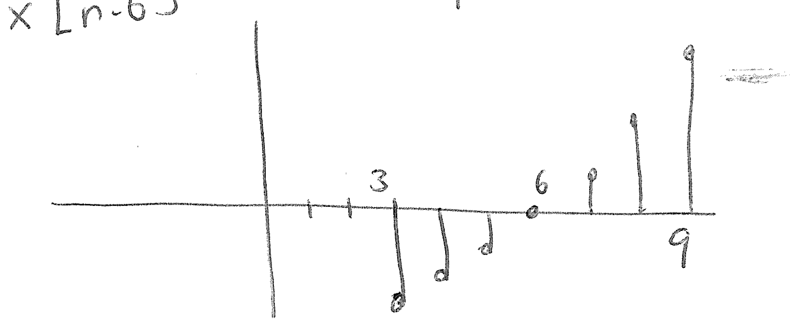
There will be overlaps, so you can't recover y(t).

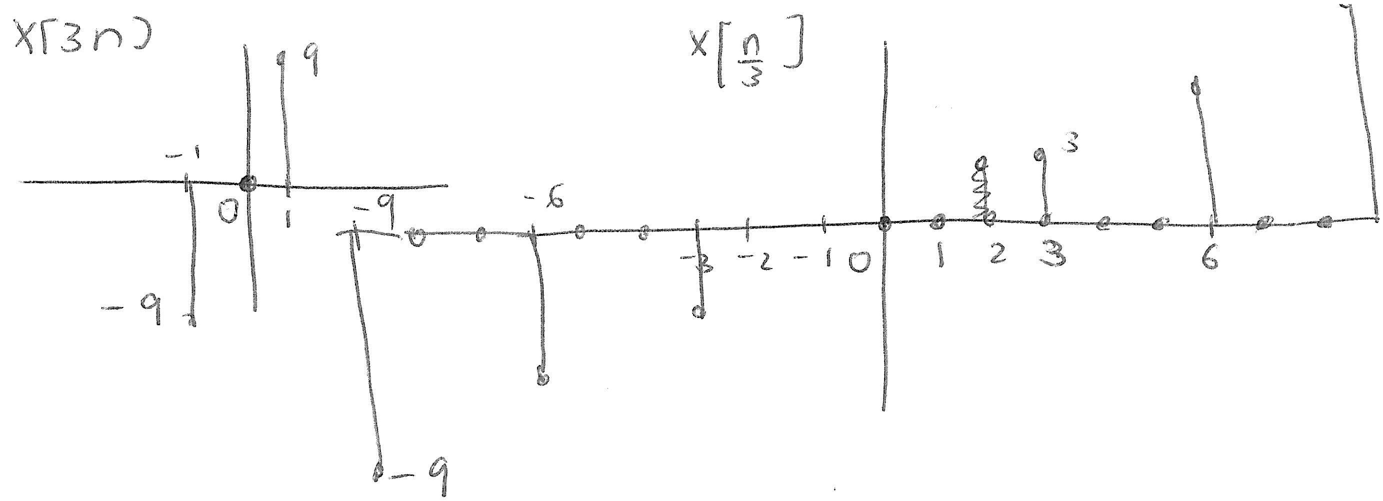
③ 3.2-4

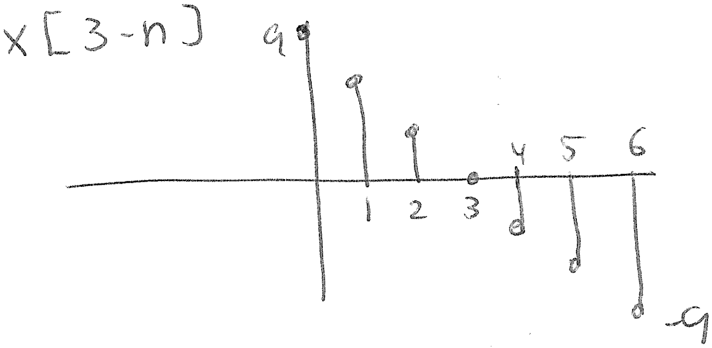




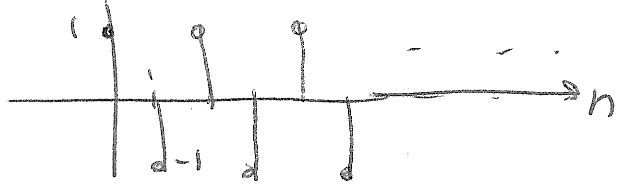








(4) a) $(-1)^n u[n]$



power signal

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} (-1)^{2n} u[n]$$

$$\lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} (-1)^{2n} \cancel{u[n]} = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=0}^{N} 1$$

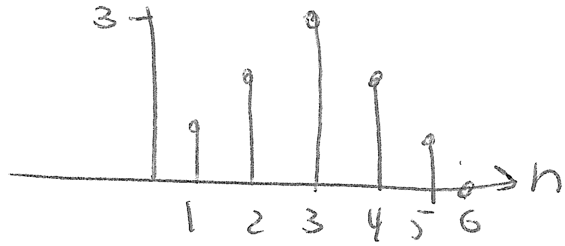
$$\lim_{N\to\infty} \frac{N+1}{2N+1} = \boxed{\frac{1}{2}}$$

~~if N is odd~~ $\cancel{\sum_{n=0}^{N} (-1)^n = 0}$

~~if N is even~~ $\cancel{\sum_{n=0}^{N} (-1)^n = 1}$

$\cancel{\lim_{N\to\infty} \frac{1}{2N+1}}$

b) P3.1-1 (b)

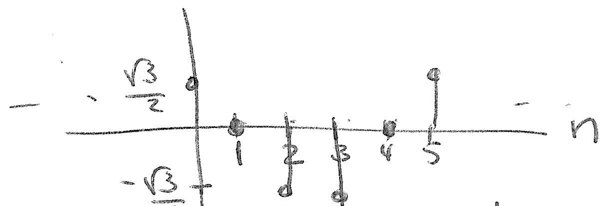


Energy signal

$(1)^2+(2)^2+(3)^2+(2)^2+(1)^2 = 1+4+9+4+1$

$= 19$ //

c) $\cos\left[\frac{\pi}{3}n + \frac{\pi}{6}\right]$



periodic → power signal

$N_0 = 5$

$\frac{1}{5}\left(\frac{3}{4}\cdot 3\right) = \frac{9}{20}$

5) a) $x(n) = e^{j5n\pi/7}$

$\Omega_0 = \frac{5\pi}{7} = \left(\frac{5}{14}\right)2\pi$ $\quad N_0 = 14$

b) $e^{j\frac{0.3n}{\pi}}$ $\quad \Omega_0 = \frac{0.3}{\pi} = \left(\frac{3}{20\pi^2}\right)2\pi$ $\quad$ Not periodic

c) $\sin(3.15\pi n)$ $\quad \Omega_0 = 3.15\pi = \left(\frac{315}{200}\right)2\pi$ $\quad \frac{315}{200} = \frac{63}{40}$

periodic $N_0 = 40$.

d). $e^{j5\pi n/7} + \sin(0.01\pi n)$

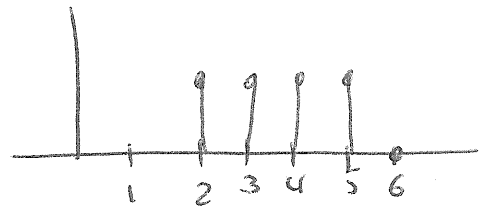
$\frac{5\pi}{7} = \left(\frac{5}{14}\right)2\pi$ $\quad \left(\frac{0.01}{2}\right)(2\pi) = \left(\frac{1}{200}\right)2\pi$

$N_{0,1} = 14$ $\quad N_{0,2} = 200$

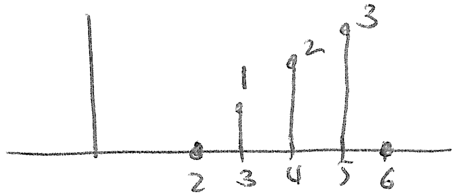
$N_0 = LCM(14, 200) = 1400.$

⑥ 3.3-3 a, c, e

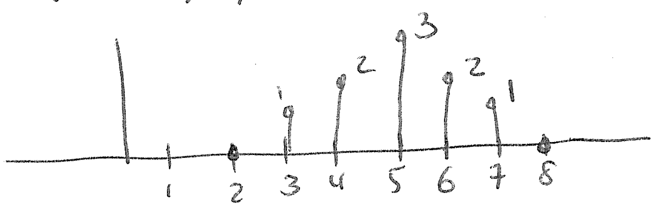
a) $u[n-2] - u[n-6]$



c) $(n-2)(u[n-2] - u[n-6])$



e) $(n-2)\{u[n-2] - u[n-6]\} + (-n+8)\{u[n-6] - u[n-9]\}$



⑦ 3.4-9 $\quad y[n] = \frac{1}{2} \sum_{k=-\infty}^{\infty} x[k](\delta[n-k] + \delta[n+k])$

a) This system takes the average of $x[n]$ and $x[-n]$ at each time n. (Finds the even part of a signal $x[n]$)

b) if $|x[n]| \le M \quad |y[n]| = \left| \frac{x[n] + x[-n]}{2} \right| \le M$

$\rightarrow$ stable.

c) Linear

d) Not memoryless can depend on future/past inputs

e) Not causal for $n<0$ $\quad y[n]$ depends on future inputs

f) $x[n] \xrightarrow{\text{Delay}} x[n-n_0] \xrightarrow{T} \frac{1}{2} \sum_{k=-\infty}^{\infty} x[k-n_0](\delta[n-k] + \delta[n+k])$

$$\frac{x[n-n_0] + x[-n-n_0]}{2}$$

$x[n] \xrightarrow{T} \frac{x[n] + x[-n]}{2} \xrightarrow{\text{Delay}} \frac{x[n-n_0] + x[-n+n_0]}{2}$

Not time invariant