ECE 366 HW # 6

Solutions

6.4-2

a) $x(t) = \cos(5t)\sin(3t)$

$$= \frac{1}{2}\left[\sin(-2t) + \sin(8t)\right]$$

$$= \frac{1}{2}\left[\sin(8t) - \sin(2t)\right]$$

$$= \frac{1}{2}\left[\frac{e^{j8t} - e^{-j8t}}{2j} - \frac{e^{j2t} - e^{-j2t}}{2j}\right]$$

$$= \frac{-1}{4j}e^{j2t} + \frac{1}{4j}e^{-j2t} + \frac{1}{4j}e^{j8t} - \frac{1}{4j}e^{-j8t}$$

$w_0 = 2$

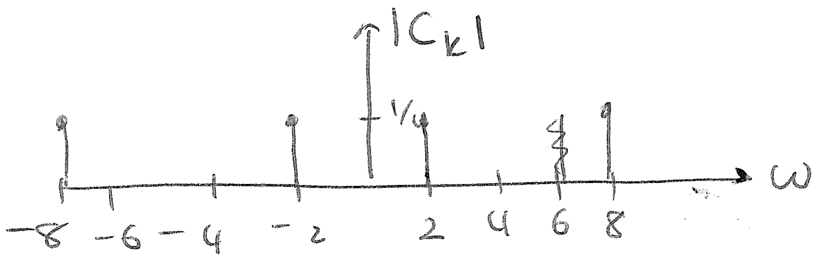
$$C_1 = -1/4j = j/4$$

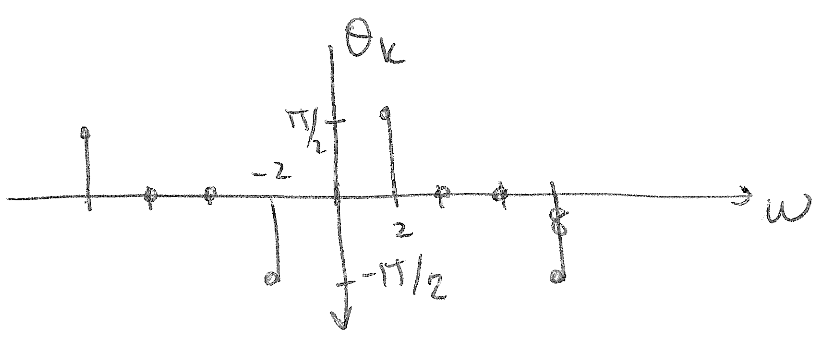
$$C_{-1} = 1/4j = -j/4$$

$$C_4 = \frac{1}{4j} = -j/4$$

$$C_{-4} = \frac{-1}{4j} = j/4$$

b)





c) $y(t) = 0$

② 7.1-8

if $x(t) \Longleftrightarrow X(\omega)$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$X(0) = X(\omega)\Big|_{\omega=0} = \int_{-\infty}^{\infty} x(t) dt$$

and
$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega$$

$$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) d\omega$$

$$\int_{-\infty}^{\infty} \operatorname{sinc}(x) dx = \mathcal{F}\{\operatorname{sinc}(x)\}\Big|_{\omega=0}$$
$$= \pi \operatorname{rect}\left(\frac{\omega}{2}\right)\Big|_{\omega=0} = \pi$$

$$\int_{-\infty}^{\infty} \operatorname{sinc}^2(x) dx = \mathcal{F}\{\operatorname{sinc}^2(x)\}\Big|_{\omega=0}$$
$$= \pi \operatorname{tri}\left(\frac{\omega}{4}\right)\Big|_{\omega=0} = \pi.$$

③ 7.2-4

a)
$$X(\omega) = \begin{cases} 1 \cdot e^{-j\omega t_0} & |\omega| \le \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t} d\omega = \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} e^{-j\omega t_0} e^{j\omega t} d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_0}^{\omega_0} e^{+j\omega(t-t_0)} d\omega = \frac{1}{2\pi} \frac{e^{j\omega(t-t_0)}}{j(t-t_0)} \Bigg|_{-\omega_0}^{\omega_0}$$

$$= \frac{1}{2\pi} \left[\frac{e^{j\omega_0 t} \cdot e^{-j\omega_0 t_0} - e^{-j\omega_0 t} e^{j\omega_0 t_0}}{j(t-t_0)} \right]$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega_0(t-t_0)} - e^{-j\omega_0(t-t_0)}}{j(t-t_0)\omega_0} \cdot \omega_0\right]$$

$$= \frac{1}{\cancel{2}\pi} \frac{\cancel{2}\sin(\omega_0(t-t_0)) \cdot \omega_0}{(t-t_0)\omega_0}$$

$$= \frac{\omega_0}{\pi} \operatorname{sinc}(\omega_0(t-t_0))$$

b) $$X(\omega) = \begin{cases} e^{j\pi/2} & -\omega_0 \leq \omega < 0 \\ e^{-j\pi/2} & 0 \leq \omega < \omega_0 \\ 0 & \text{otherwise} \end{cases}$$

$$x(t) = \frac{1}{2\pi}\left[\int_{-\omega_0}^{0} e^{j\pi/2} e^{+j\omega t}\, d\omega + \int_{0}^{\omega_0} e^{-j\pi/2} e^{+j\omega t}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[e^{j\pi/2} \left.\frac{e^{j\omega t}}{jt}\right|_{-\omega_0}^{0} + e^{-j\pi/2} \left.\frac{e^{j\omega t}}{jt}\right|_{0}^{\omega_0}\right]$$

$$= \frac{1}{2\pi}\left[\underbrace{e^{j\pi/2}}_{j}\left(\frac{1}{jt} - \frac{e^{-j\omega_0 t}}{jt}\right) + \underbrace{e^{-j\pi/2}}_{-j}\left(\frac{e^{j\omega_0 t}}{jt} - \frac{1}{jt}\right)\right]$$

$$= \frac{1}{2\pi}\left[\frac{2}{t} - \frac{e^{-j\omega_0 t}}{t} - \frac{e^{j\omega_0 t}}{t}\right]$$

$$= \frac{1}{2\pi}\left[\frac{2}{t} - \frac{\cos(\omega_0 t)}{t}\right]$$

$$= \frac{1}{\pi t} - \frac{\cos(\omega_0 t)}{\pi t}$$

4

7.3-2

a) $x_1(t) = x(t+1) + x(1-t)$

$$X_1(\omega) = X(\omega)e^{j\omega} + X(-\omega)e^{-j\omega}$$

c) $x_3(t) = x\left(\frac{t+2}{4}\right) + x\left(\frac{2-t}{4}\right) + x\left(\frac{t}{2}\right) + x\left(\frac{-t}{2}\right)$

$$x\left(\frac{t+2}{4}\right) \longleftrightarrow 4X(4\omega)e^{j2\omega}$$

$$x\left(\frac{2-t}{4}\right) \longleftrightarrow 4X(-4\omega)e^{-j2\omega}$$

$$x\left(\frac{t}{2}\right) \longleftrightarrow 2X(2\omega) \qquad x\left(\frac{-t}{2}\right) \longleftrightarrow 2X(-2\omega)$$

$$x_3(t) \longleftrightarrow 4\left[X(4\omega)e^{j2\omega} + X(-4\omega)e^{-j2\omega} + 0.5X(2\omega) + 0.5X(-2\omega)\right]$$

d) $x_4(t) = \frac{4}{3}x\left(\frac{t+2}{2}\right) + \frac{4}{3}x\left(\frac{2-t}{2}\right) - \frac{1}{3}x\left(\frac{t+2}{4}\right) - \frac{1}{3}x\left(\frac{2-t}{4}\right)$

$$x\left(\frac{t+2}{2}\right) \longleftrightarrow 2X(2\omega)e^{j2\omega} \qquad x\left(\frac{t+2}{4}\right) \longleftrightarrow 4X(4\omega)e^{j2\omega}$$

$$x\left(\frac{2-t}{2}\right) \longleftrightarrow 2X(-2\omega)e^{-j2\omega} \qquad x\left(\frac{2-t}{4}\right) \longleftrightarrow 4X(-4\omega)e^{-j2\omega}$$

$$x_4(t) \longleftrightarrow \frac{8}{3}\left[X(2\omega)e^{j2\omega} + X(-2\omega)e^{-j2\omega}\right] - \frac{4}{3}\left[X(4\omega)e^{j2\omega} + X(-4\omega)e^{-j2\omega}\right.$$

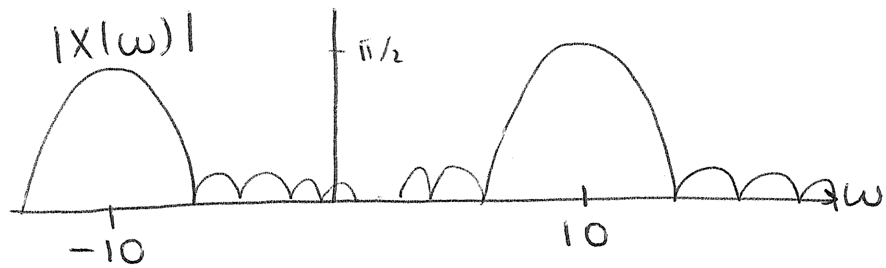
⑤ 7.3-6

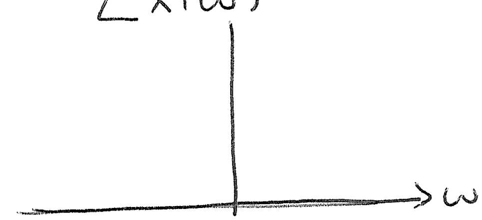
a) $x(t) = \text{tri}\left(\frac{t}{2\pi}\right)\cos(10t)$

$$\mathcal{F}\left\{\text{tri}\left(\frac{t}{2\pi}\right)\right\} = \pi\,\text{sinc}^2\left(\frac{\omega 2\pi}{4}\right)$$
$$= \pi\,\text{sinc}^2\left(\frac{\omega\pi}{2}\right)$$

$$X(\omega) = \frac{1}{2}\left[\pi\,\text{sinc}^2\left(\frac{(\omega-10)\pi}{2}\right) + \pi\,\text{sinc}^2\left(\frac{(\omega+10)\pi}{2}\right)\right]$$

by modulation property





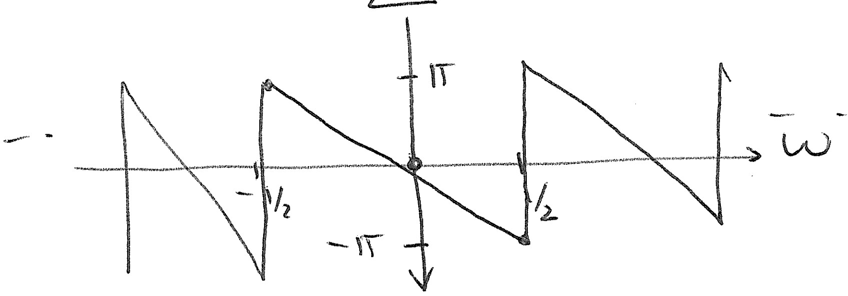
zero phase

b) $y(t) = x(t-2\pi)$

$$Y(\omega) = X(\omega)\,e^{-j\omega 2\pi}$$
$$= \frac{e^{-j\omega 2\pi}\pi}{2}\left[\text{sinc}^2\left(\frac{(\omega-10)\pi}{2}\right) + \text{sinc}^2\left(\frac{(\omega+10)\pi}{2}\right)\right]$$

$|Y(\omega)|$ is the same as $|X(\omega)|$

$\angle Y(\omega) = -2\pi\omega$



c) $x(t) = \text{rect}\left(\frac{t-2\pi}{2\pi}\right)\cos(10t)$

$$\mathcal{F}\left\{\text{rect}\left(\frac{t-2\pi}{2\pi}\right)\right\} = e^{-j\omega 2\pi}\, 2\pi\, \text{sinc}(\omega\pi)$$

$$X(\omega) = \frac{2\pi e^{-j2\pi\omega}}{2}\left[\text{sinc}(\pi(\omega+10)) + \text{sinc}(\pi(\omega-10))\right]$$

$$= \pi e^{-j2\pi\omega}\left[\text{sinc}(\pi(\omega+10)) + \text{sinc}(\pi(\omega-10))\right]$$

6) a) $e^{-|t|} \longleftrightarrow \frac{2}{\omega^2+1}$

$$\frac{2}{t^2+1} \longleftrightarrow 2\pi e^{-|-\omega|} = 2\pi e^{-|\omega|}$$

$$\frac{1}{2\pi(t^2+1)} \longleftrightarrow \frac{2\pi e^{-|\omega|}}{4\pi} = \frac{1}{2}e^{-|\omega|}$$

b) $\delta(t+T) + \delta(t-T)$

$$\cos(Tt) \longleftrightarrow \pi[\delta(\omega-T) + \delta(\omega+T)]$$

$$\pi[\delta(t-T) + \delta(t+T)] \longleftrightarrow 2\pi\cos(-T\omega)$$

$$= 2\pi\cos(T\omega)$$

$$[\delta(t-T) + \delta(t+T)] \longleftrightarrow 2\cos(T\omega)$$