

①

2.5-1 a, b

$$(D^2 + 7D + 12)y(t) = (D + 2)x(t)$$

$$y(0) = 0$$

$$\dot{y}(0) = 1$$

a) $x(t) = u(t)$

ZIRAssume $x(t) = 0$

$$(D^2 + 7D + 12)y(t) = 0$$

$$\Rightarrow s^2 + 7s + 12 = 0$$

$$(s + 3)(s + 4) = 0$$

$$s_1 = -3 \quad s_2 = -4$$

$$y_N(t) = C_1 e^{-3t} + C_2 e^{-4t}$$

$$y_F(t) = 0$$

$$y_{ZIR}(t) = C_1 e^{-3t} + C_2 e^{-4t}$$

$$\Rightarrow \begin{cases} C_1 + C_2 = 0 \\ -3C_1 - 4C_2 = 1 \end{cases}$$

$$\underline{-C_2 = 1} \Rightarrow \begin{cases} C_2 = -1 \\ C_1 = 1 \end{cases}$$

$$y_{ZIR}(t) = (e^{-3t} - e^{-4t})u(t)$$

ZSR

$$y_N(t) = C_1 e^{-3t} + C_2 e^{-4t}$$

$$y_F(t) = Ku(t) \Rightarrow 12Ku(t) = 2u(t)$$

$$K = 1/6$$

$$y_{ZSR}(t) = C_1 e^{-3t} + C_2 e^{-4t} + 1/6 u(t)$$

$$y(0) = C_1 + C_2 + 1/6 = 0 \quad \Rightarrow \quad C_1 + 2/3 = 0 \Rightarrow C_1 = -2/3$$

$$\dot{y}(0) = -3C_1 - 4C_2 = 0$$

$$C_2 = 1/2$$

$$y_{ZSR}(t) = \left[-2/3 e^{-3t} + 1/2 e^{-4t} + 1/6 \right] u(t)$$

b)

$$x(t) = e^{-t} u(t)$$

$$y_{zIR}(t) = (e^{-3t} - e^{-4t}) u(t) \quad (\text{same as part (a)})$$

since it
doesn't depend on)
 $x(t)$.

 y_{zSR}

$$y_N(t) = C_1 e^{-3t} + C_2 e^{-4t}$$

$$y_F(t) = K e^{-t} u(t)$$

$$\Rightarrow K e^{-t} + -7K e^{-t} + 12K e^{-t} = -e^{-t} + 2e^{-t}$$

$$6K e^{-t} = e^{-t}$$

$$K = 1/6$$

$$y_{zSR}(t) = (C_1 e^{-3t} + C_2 e^{-4t} + 1/6 e^{-t}) u(t)$$

$$y_{zSR}(0) = C_1 + C_2 + 1/6 = 0$$

$$-3C_1 - 4C_2 - 1/6 = 0 \Rightarrow C_1 + 1/2 = 0$$

$$C_1 = -1/2$$

$$C_2 = 1/3$$

$$\left[-1/2 e^{-3t} + 1/3 e^{-4t} + 1/6 e^{-t} \right] u(t)$$

② 2.6-1

$$a) (D^2 + 8D + 12) y(t) = (D - 1) x(t)$$

$$(s + 2)(s + 6) \quad s_1 = -2$$

$$s_2 = -6$$

asymptotically stable \rightarrow BIBO stable

$$b) D(D^2 + 3D + 2) y(t) = (D + 5) x(t)$$

$$s_1 = 0$$

$$s_2 = -1$$

$$s_3 = -2$$

\rightarrow marginally stable

since one root on

JW axis

\rightarrow BIBO

unstable

$$c) D^2(D^2+2)y(t) = x(t)$$

$$s_1 = s_2 = 0$$

$$s_3 = \sqrt{2}j, s_4 = -\sqrt{2}j$$

~~marginally stable~~ ^{unstable} → BIBO unstable

$$d) (D+1)(D^2-6D+5)y(t) = (3D+1)x(t)$$

$$s_1 = -1$$

$$s_2 = 1 \quad \text{unstable} \rightarrow \text{BIBO unstable}$$

$$s_3 = 5$$

$$\textcircled{3} \quad H(s) = \frac{s+5}{s^2+7s+5}$$

$$a) x(t) = 4 \quad s=0 \quad y(t) = 4 H(0) = 4 \frac{5}{5} = 4$$

$$b) x(t) = e^{-2t} \quad s=-2 \quad y(t) = H(-2)e^{-2t} = \frac{3}{5} e^{-2t}$$

$$c) x(t) = 5 \sin(4t) \quad s = 4j$$

$$y(t) = 5 |H(4j)| \sin(4t + \angle H(4j))$$

$$\frac{4j+5}{-16+5+8j} = \frac{5+4j}{-11+8j}$$

$$|H(4j)| = \frac{\sqrt{41}}{\sqrt{185}} = 0.4708 \quad \frac{121}{64} \frac{64}{55}$$

$$\angle H(4j) = \tan^{-1}\left(\frac{4}{5}\right) - \tan^{-1}\left(\frac{8}{-11}\right) = 38.66 - 143.97 = -105.31^\circ$$

$$y(t) = 2.3540 \sin(4t - 105.31^\circ)$$

```
n=[0 1 5];
d=[1 2 5];
%part a
h=polyval(n,0)/polyval(d,0);
ymag=4*abs(h)
yphase=angle(h)*180/pi
%part b;
h=polyval(n,-2)/polyval(d,-2);
ymag=1*abs(h)
yphase=angle(h)*180/pi
%part c;
h=polyval(n,4*j)/polyval(d,4*j);
ymag=5*abs(h)
yphase=angle(h)*180/pi
%part d
h=polyval(n,j*3)/polyval(d,j*3);
ymag=4*abs(h)
yphase=angle(h)*180/pi
```

ymag =

4

yphase =

0

ymag =

0.6000

yphase =

0

ymag =

2.3538

yphase =

-105.3128

ymag =

3.2344

yphase =

-92.7263

$$d) x(t) = 4e^{j3t} \quad s = j3$$

$$y(t) = 4 \left(\frac{3j+5}{-9+6j+5} \right) e^{j3t}$$

$$= 4 \left(\frac{5+3j}{-4+6j} \right) e^{j3t}$$

$$= 4 \left(\frac{\sqrt{34}}{\sqrt{52}} \right) e^{j \left(\tan^{-1}(3/5) - \tan^{-1}(6/-4) \right)} e^{j3t}$$

$$= 3.2344 e^{-j92.72^\circ} e^{j3t}$$

(4) 6.3-3 a, b, c

$$x(t) = 3\cos t + \sin(5t - \pi/6) - 2\cos(8t - \pi/3)$$

$$= 3 \left(\frac{e^{jt} + e^{-jt}}{2} \right) + \left(\frac{e^{j(5t - \pi/6)} - e^{-j(5t - \pi/6)}}{2j} \right)$$

$$- 2 \left(\frac{e^{j(8t - \pi/3)} + e^{-j(8t - \pi/3)}}{2} \right)$$

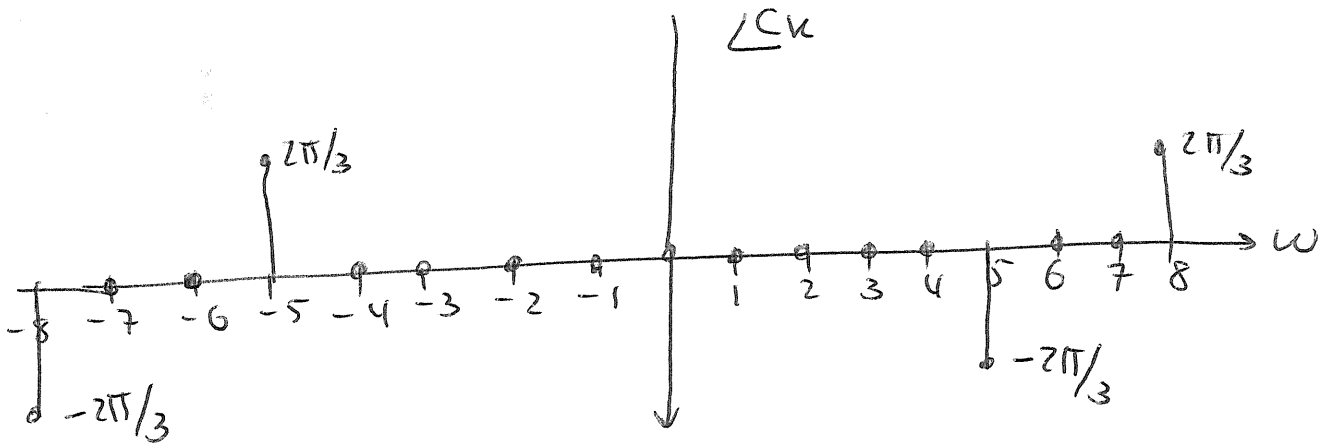
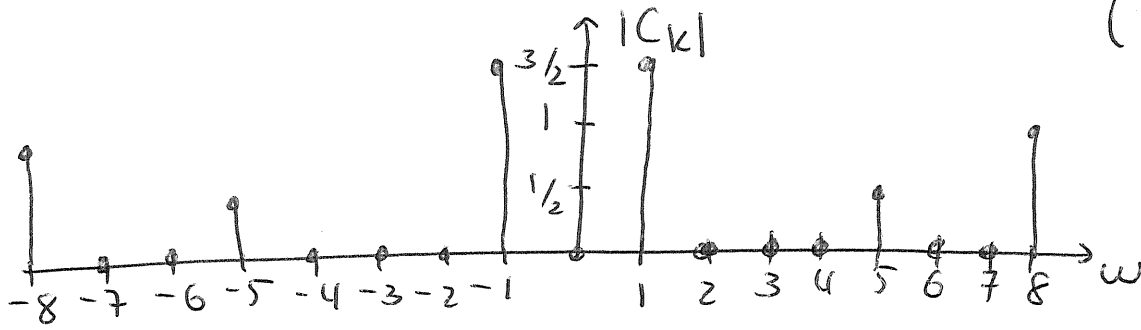
$$= \overset{e^{-j\pi}}{\uparrow} \ominus e^{+j\pi/3} e^{-j8t} \ominus \frac{1}{2j} \overset{e^{+j\pi/2}}{\rightarrow} e^{j\pi/6} e^{-j5t} + \frac{3}{2} e^{-jt} + \frac{3}{2} e^{jt}$$

$$+ \frac{1}{2j} \overset{e^{-j\pi/2}}{\rightarrow} e^{-j\pi/6} e^{j5t} - e^{-j\pi/3} e^{j8t}$$

$$= e^{-j\frac{2\pi}{3}} e^{-j8t} + \frac{1}{2} e^{j\frac{2\pi}{3}} e^{-j5t} + \frac{3}{2} e^{-jt} + \frac{3}{2} e^{jt} + \frac{1}{2} e^{-j\frac{2\pi}{3}} e^{j5t}$$

$$+ e^{j\frac{2\pi}{3}} e^{j8t}$$

(b)



c)

$$C_0 = 0$$

$$C_1 = \frac{3}{2} \quad C_{-1} = \frac{3}{2}$$

$$C_2 = C_3 = C_4 = 0$$

$$C_5 = \frac{1}{2} e^{-j2\pi/3} \quad C_{-5} = \frac{1}{2} e^{j2\pi/3}$$

$$C_8 = e^{j\pi/3} \quad C_{-8} = e^{-j2\pi/3}$$

$C_k = 0$ for all other k .

(5) 6.3-7(a)

$$x(t) = 2e^{j\pi t} + 2e^{j\frac{2\pi}{3}t} e^{j\pi t} + e^{j\pi/3} e^{j2\pi t} + 2e^{-j2\pi/3} e^{-j\pi t} + e^{-j\pi/3} e^{-j2\pi t}$$