ECE 366 HW #3

Fall 2008

Solutions

① a) $\frac{\sin t}{t^2+2}\,\delta(t) = 0$

b) $\frac{\sin\left[\frac{\pi}{2}(t-2)\right]}{t^2+4}\,\delta(1-t) = \frac{\sin\left[-\pi/2\right]}{5}\,\delta(1-t)$

$= \frac{-1}{5}\,\delta(1-t)$

c) $\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau = x(t)$

d) $\int_{-\infty}^{\infty} \sin(\pi t)\delta(2t-3)\,dt = \frac{1}{2}\int_{-\infty}^{\infty} \sin(\pi t)\,\delta(t-3/2)\,dt$

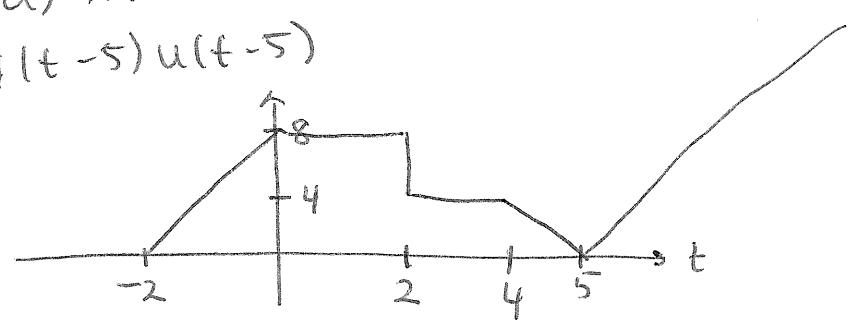
$= \frac{1}{2}\sin\left(\frac{3\pi}{2}\right) = -1/2$

e) $\int_{-\infty}^{t} e^{-\tau}\delta(\tau)\,d\tau = \begin{cases} 0 & \text{if } t<0 \\ e^{-0} & \text{if } t \geq 0 \end{cases} = u(t)$

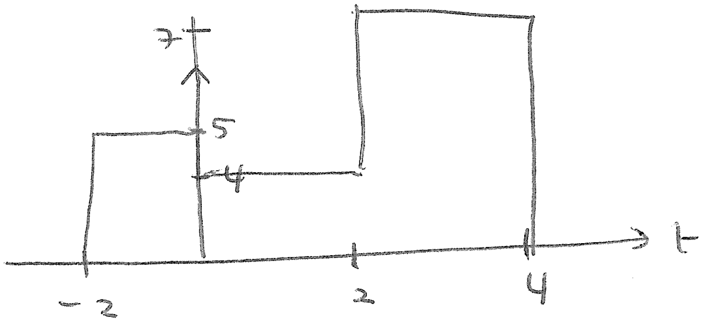
f) $\int_{-\infty}^{\infty} e^{x-1}\cos\left[\frac{\pi}{2}(x-5)\right]\delta(x-3)\,dx$

$= e^2\cos\left[\frac{\pi}{2}(-2)\right] = e^2\cos[-\pi] = -e^2$

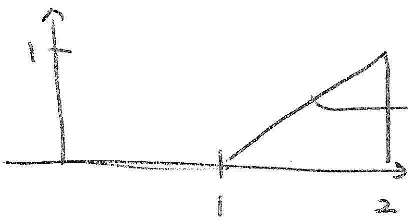
② a) $x(t) = 4(t+2)u(t+2) - 4tu(t) - 4u(t-2) - 4(t-4)u(t-4)$
$+ 4(t-5)u(t-5)$



b) $x(t) = 5u(t+2) - u(t) + 3u(t-2) - 7u(t-4)$



(3) a) P. 1.1-2d



$(t-1)[u(t-1) - u(t-2)]$

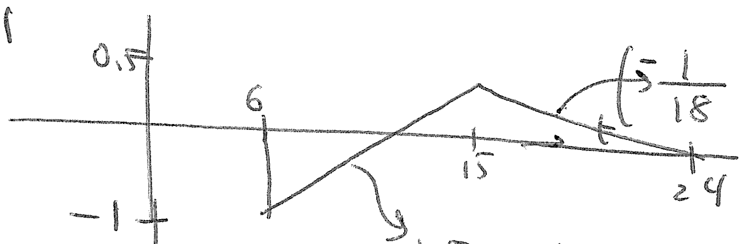
$= (t-1)u(t-1) - (t-1)u(t-2)$

$= r(t-1) - (t-2+1)u(t-2)$

$= r(t-1) - (t-2)u(t-2) - u(t-2)$

$= r(t-1) - r(t-2) - u(t-2)$

b) P 1.2-1



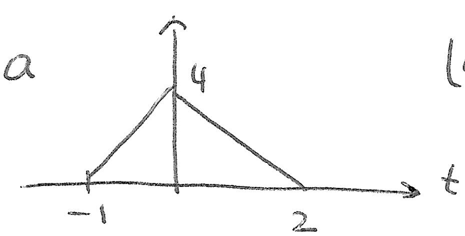
$\left(-\frac{1}{18}t + \frac{4}{3}\right)[u(t-15) - u(t-24)]$

$\frac{1.5}{9} = \frac{1}{6}$

$\left(\frac{1}{6}t - 2\right)[u(t-6) - u(t-15)]$

$\left(\frac{1}{6}t - 2\right)[u(t-6) - u(t-15)] + \left(\frac{-1}{18}t + \frac{4}{3}\right)[u(t-15) - u(t-24)]$

c) P1.4-2a



$(4t+4)[u(t+1) - u(t)] + (-2t+4)[u(t) - u(t-2)]$

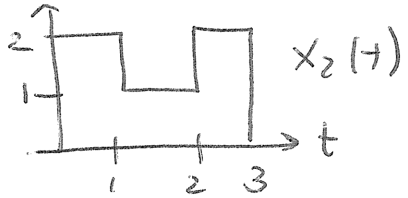
$4(t+1)\cancel{u(t+1)} - 4(t+1)u(t)$

$-2(t-2)u(t) + 2(t-\cancel{2})u(t-2)$

$= 4r(t+1) + 2r(t-2) - 4r(t) - 4\cancel{u}(t) - 2r(t) + \cancel{4u}(t)$

$= 4r(t+1) - 6r(t) + 2r(t-2)$

(4) 1.7-13



a) Is $x_2(t) = 2x_1(3t) - x_1(t-1)$? No

$x_2(t) = 2x_1(t) + x_1(t-1) + 2x_1(t-2)$

b) $y_2(t) = 2y_1(t) + y_1(t-1) + 2y_1(t-2)$

(5) a) $y(t) = \int_{-\infty}^{t} x(\tau)\,d\tau$

- Linear: $\int_{-\infty}^{t} (a_1 x_1(\tau) + a_2 x_2(\tau))\,d\tau$

$= a_1 \int_{-\infty}^{t} x_1(\tau) + a_2 \int_{-\infty}^{t} x_2(\tau)\,d\tau$

$= a_1 y_1(t) + a_2 y_2(t)$ ✓ Linear.

- Time-Invariance: $x(t) \xrightarrow{\text{Delay}} x(t-t_0) \xrightarrow{T} \int_{-\infty}^{t} x(\tau - t_0)\,d\tau$

$\int_{-\infty}^{t-t_0} x(\tau')\,d\tau$

$x(t) \xrightarrow{T} \int_{-\infty}^{t} x(\tau)\,d\tau \xrightarrow{\text{Delay}} \int_{-\infty}^{t-t_0} x(\tau)\,d\tau$

Time Invariant

- Not instantaneous (depends on the past)
- Causal
- $\frac{dy(t)}{dt} = x(t)$ invertible
- Stable? No $|y(t)| \le \int_{-\infty}^{t} |x(\tau)|\,d\tau$

$\le Mt \to \infty$

b) $y(t) = x(at) \quad a>1$

Linearity: $a_1 x_1(at) + a_2 x_2(at)$

Yes $\rightarrow$ Linear.

Time invariance: $x(t) \xrightarrow{\text{Delay}} x(t-t_0) \xrightarrow{T} x(at-t_0)$ $\neq$

$x(t) \xrightarrow{T} x(at) \xrightarrow{\text{Delay}} x(a(t-t_0))$

Time varying

Instantaneous: No $\quad$ y can depend on the past

Causal: Yes (does not depend on the future)

Invertible: $y(\frac{t}{a}) = x(t)$ Yes.

stable: if $|x(t)| \leq M \rightarrow |y(t)| = |x(at)| \leq M$

BIBO stable

c) $y(t) = x(t)\, t\, u(t)$

- $T[a_1 x_1(t) + a_2 x_2(t)] = (a_1 x_1(t) + a_2 x_2(t))\, t\, u(t)$

$= a_1 t\, x_1(t) u(t) + a_2 t\, x_2(t) u(t)$

$= a_1 y_1(t) + a_2 y_2(t)$

Linear

- Time invariance: $x(t) \xrightarrow{\text{Delay}} x(t-t_0) \xrightarrow{T} t\, x(t-t_0) u(t)$ $\neq$

$x(t) \xrightarrow{T} x(t)\, t\, u(t) \xrightarrow{\text{Delay}} x(t-t_0)(t-t_0) u(t-t_0)$

Time varying

- Instantaneous: Yes
- Causal: Yes
- Invertible: $x(t) = \dfrac{y(t)}{t\,u(t)}$ divide by zero for $t<0$ not well-defined

No.

- Stability: if $|x(t)| \leq M$

$\rightarrow |y(t)| = |x(t)|\, |t| \underbrace{|u(t)|}_{\leq 1} \leq M|t| \rightarrow \infty$

not BIBO stable.

d) $y(t) = e^{x(t)}$

Linearity: $T[a_1 x_1(t) + a_2 x_2(t)]$

$$= e^{(a_1 x_1(t) + a_2 x_2(t))} = e^{a_1 x_1(t)} \cdot e^{a_2 x_2(t)}$$

$$\neq a_1 y_1(t) + a_2 y_2(t)$$

Nonlinear

Time invariance: $x(t) \xrightarrow{\text{Delay}} x(t-t_0) \xrightarrow{T} e^{x(t-t_0)}$

$x(t) \xrightarrow{T} e^{x(t)} \xrightarrow{\text{Delay}} e^{x(t-t_0)}$

Time Invariant

- Instantaneous: Yes
- Causal: Yes
- Invertible: $x(t) = \ln y(t) \rightarrow$ Yes
- stable: if $|x(t)| \leq M \rightarrow |y(t)| = |e^{x(t)}|$

$$= e^{x(t)} \leq e^{M} < \infty$$

BIBO stable