ECE 366 HW2
Solutions

(1) a) $x(t) = 10\sin(5t)\cos(10t)$

$$= 5[\sin(-5t) + \sin(15t)]$$

$$= -5\sin(5t) + 5\sin(15t)$$

$T_{01} = 2\pi/5$ $\quad \frac{T_{01}}{T_{02}} = 3$

$T_{02} = 2\pi/15$

$\Rightarrow$ periodic $\Rightarrow$ power signal

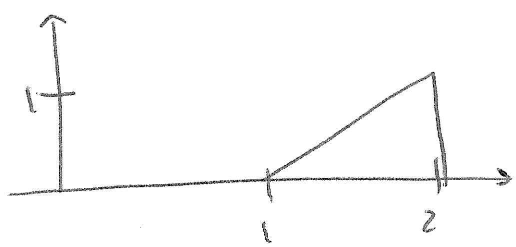
$$\boxed{T_0 = \frac{2\pi}{5}}$$

$$P = \frac{1}{T_0}\int_{T_0} |x(t)|^2 dt$$

$$= \frac{5}{2\pi}\left[\int_{T_0}\left[25\sin^2(5t) + 25\sin^2(15t) - 50\sin(5t)\sin(15t)\right]dt\right]$$

$$= \frac{25}{2} + \frac{25}{2} = \frac{50}{2} = 25 //$$

b)



finite duration, finite amplitude
$\Rightarrow$ energy signal

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

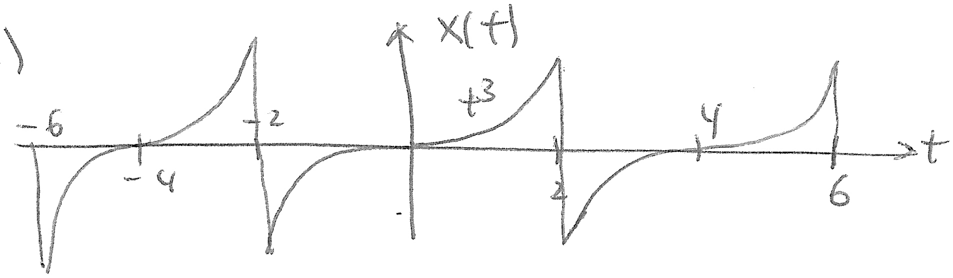
$$= \int_1^2 (t-1)^2 dt$$

$$= \int_1^2 (t^2 - 2t + 1)\,dt$$

$$t^3/3 - t^2 + t \Big|_1^2$$

$$= 8/3 - 4 + 2 - 1/3 + \cancel{1} - \cancel{1}$$

$$= 7/3 - 2 = 1/3 //$$

c) 

periodic
→ power

$$P_x = \frac{1}{4}\int_{-2}^{2} t^6\,dt = \frac{1}{4}\,\frac{t^7}{7}\Big|_{-2}^{2}$$

$$= \frac{1}{4}\left[\frac{128}{7} + \frac{128}{7}\right]$$

$$= \frac{1}{4}\left(\frac{256}{7}\right) = \frac{64}{7}$$

d) $x(t) = t e^{-|t|}$

$$E_x = \int_{-\infty}^{\infty} \left(t e^{-|t|}\right)^2 dt = \int_{-\infty}^{0} t^2 e^{2t}\,dt + \int_{0}^{\infty} t^2 e^{-2t}\,dt$$

$u = t^2$
$dv = e^{2t}\,dt$

$$\frac{t^2 e^{2t}}{2} - \frac{2}{2}\int t e^{2t}\,dt\Big|_{-\infty}^{0} + \frac{-t^2 e^{-2t}}{2} + \int t e^{-2t}\,dt\Big|_{0}^{\infty}$$

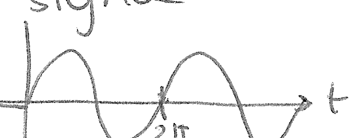
$$= \frac{e^{2t}}{8}\left(4t^2 - 4t + 2\right)\Big|_{-\infty}^{0} + \frac{e^{-2t}}{-8}\left(4t^2 + 4t + 2\right)\Big|_{0}^{\infty}$$

$$= \frac{t^2 e^{2t}}{2} - \frac{t e^{2t}}{2} + \frac{1}{4}e^{2t}\Big|_{-\infty}^{0} - \frac{e^{-2t}t^2}{2} - \frac{t e^{-2t}}{2} - \frac{e^{-2t}}{4}\Big|_{0}^{\infty}$$

=

$= 0.5$ Energy signal

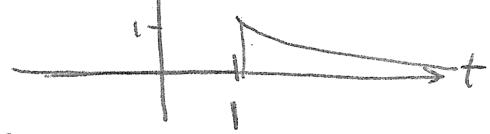
e) $\sin(t)u(t)$ power signal (constant envelope)



$$\lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \left(\sin(t)u(t)\right)^2 dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int^{T/2} \sin^2(t)\,dt = \lim_{T\to\infty} \frac{1}{T}\int^{T/2} \frac{1}{2} - \frac{\cos(2t)}{2}\,dt = \frac{1}{2}$$

f) $x(t) = \begin{cases} t^{-1/2}, & t \geq 1 \\ 0, & t < 1 \end{cases}$ $\quad \frac{1}{\sqrt{t}}$



$$\mathcal{E}_x = \int_1^{\infty} \frac{1}{t}\, dt = \ln t \Big|_1^{\infty} \to \infty$$ not an energy signal

not a power signal <u>Neither</u>

② <u>1.3-5</u> $x(t)$ $\mathcal{E}_x = 1.0417$

a) $y_1(t) = \frac{1}{3} x(2t)$ $\mathcal{E}_y = \int_{-\infty}^{\infty} y_1^2(t)\, dt$

$$= \frac{1}{9} \int_{-\infty}^{\infty} x^2(2t)\, dt$$

$$= \frac{1}{18} \int_{-\infty}^{\infty} x^2(\tau)\, d\tau = \frac{1.0417}{18}$$

$$= 0.0579$$

b) $T_0 = 4$ $\quad P_{y_2} = \frac{1}{4} \underbrace{\int_{T_0} y_2^2(t)\, dt}_{\mathcal{E}_x} = \frac{1.0417}{4} = 0.2604$

c) $y_3(t) = \frac{1}{3} y_2(2t)$

$T_0 = 2$

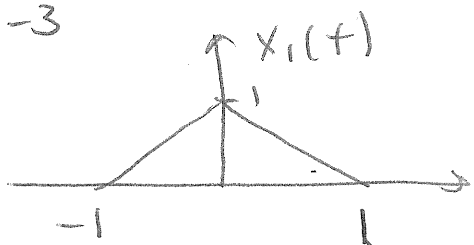
$$P_{y_3} = \frac{1}{2} \int_{T_{y_3}} y_3^2(t)\, dt$$

$$= \frac{1}{2} \int_{T_{y_3}} \frac{1}{9} y_2^2(2t)\, dt$$

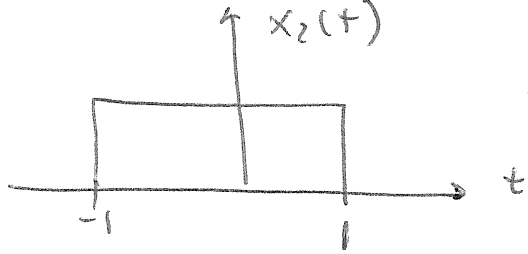
$$= \frac{1}{36} \int_{T_{y_2}} y_2^2(\tau)\, d\tau = \frac{\mathcal{E}_x}{36}$$

$$= 0.0289$$

(3) 1.2-3

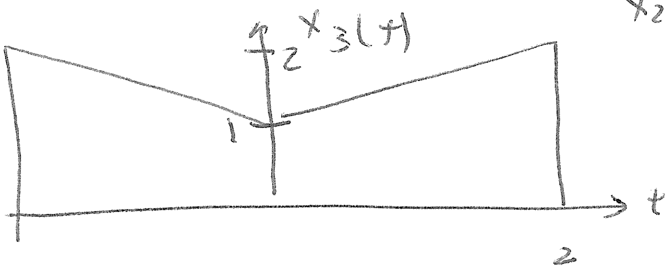


$$x_1(t) = x(t+1) + x(-t+1)$$



$$x_2(t) = \frac{dx(t+1)}{dt} + \frac{dx(t)}{dt}$$

OR
$$x_2(t) = x\left(\frac{t+1}{2}\right) + x\left(\frac{1-t}{2}\right)$$

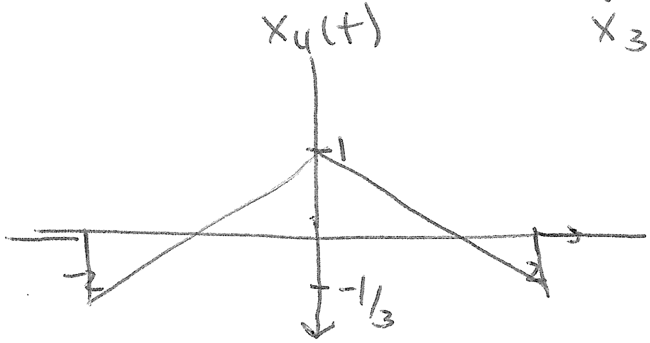


$$x_2\left(\frac{t}{2}\right) + x\left(\frac{t}{2}\right) + x\left(\frac{-t}{2}\right)$$

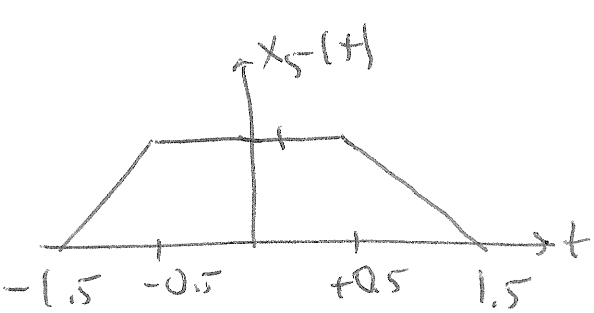
$$= \frac{d}{dt}\left[x\left(\frac{t}{2}+1\right) + x\left(\frac{t}{2}\right)\right] + x\left(\frac{t}{2}\right) + x\left(-\frac{t}{2}\right)$$

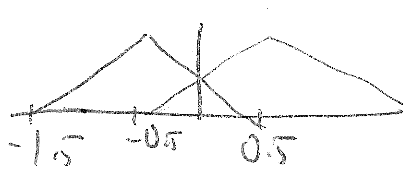
$$= \frac{1}{2}\frac{d}{dt}x\left(\frac{t}{2}+1\right) \ \frac{1}{2}\frac{dx(t/2)}{dt} + x\left(t/2\right) + x\left(-t/2\right)\Big]$$

OR
$$x_3(t) = x\left(\frac{t+2}{4}\right) + x\left(\frac{2-t}{4}\right) + x\left(t/2\right)$$
$$+ x\left(-t/2\right)$$



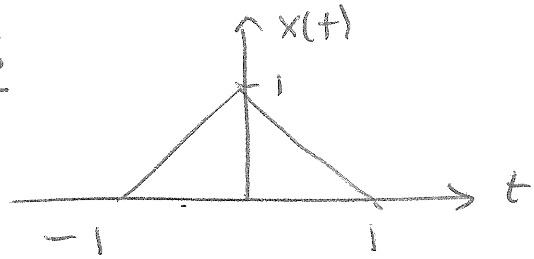
$$\frac{4}{3}x_1\left(\frac{t}{2}\right) - \frac{1}{3} = \frac{4}{3}\left[x\left(\frac{t}{2}+1\right) + x\left(\frac{-t}{2}+1\right)\right]$$
$$-1/3.$$





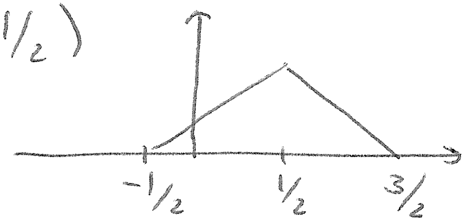
$$x_1(t+0.5) + x_1(t-0.5)$$
$$= x(t+1.5) + x(t+0.5)$$
$$+ x(-t+0.5) + x(-t+1.5)$$

(4) 1.5-6



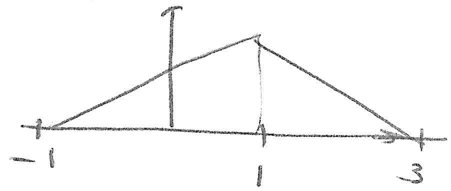
a) $v(t) = 3\,x\left(-\frac{1}{2}t - \frac{1}{2}\right)$

$x(t - 1/2)$

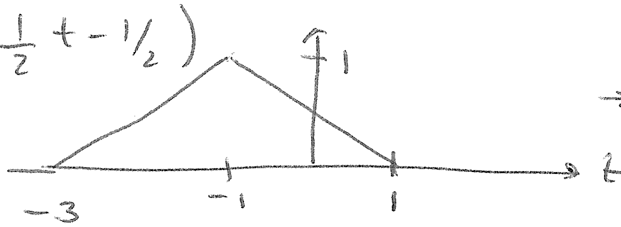


→

$x\left(\frac{1}{2}t - 1/2\right)$

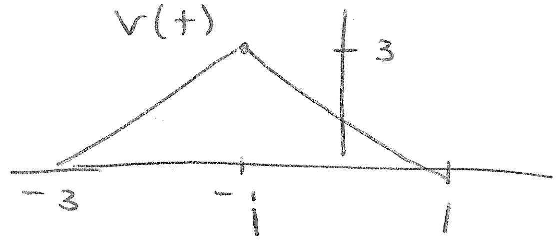


$x\left(-\frac{1}{2}t - 1/2\right)$



→

$v(t)$



b) $E_{v} = \int_{-3}^{1} v^2(t)\,dt = \int_{-3}^{-1}\left(\frac{3}{2}t + \frac{9}{2}\right)^2 dt + \int_{-1}^{1}\left(-\frac{3}{2}t + \frac{3}{2}\right)^2 dt$

$P_V = 0$

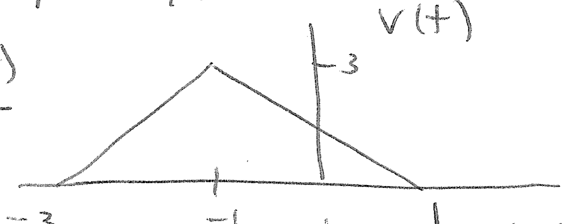
$= \int_{-3}^{-1}\left(\frac{9}{4}t^2 + \frac{81}{4} + \frac{27}{2}t\right)dt + \int_{-1}^{1}\left(\frac{9}{4}t^2 + \frac{9}{4} - \frac{9}{2}t\right)dt$

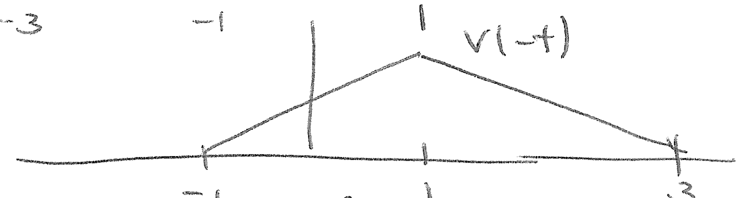
$\frac{9}{12}t^3 + \frac{81}{4}t + \frac{27}{4}t^2 \Big|_{-3}^{-1} + \frac{9}{12}t^3 + \frac{9}{4}t - \frac{9}{4}t^2 \Big|_{-1}^{1}$

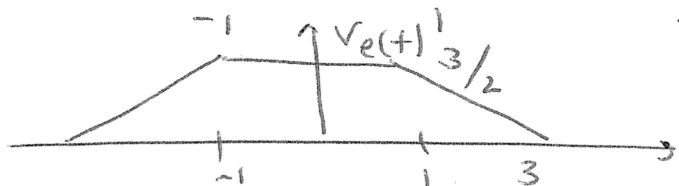
$= \cancel{-\frac{9}{12}} \cancel{-\frac{81}{4}} + \frac{27}{4} + \frac{(27)\cancel{(9)}3}{\cancel{12}4} + \frac{81(3)}{4} - \frac{27(9)}{4} + \cancel{\frac{9}{12}} + \cancel{\frac{9}{4}} - \cancel{\frac{9}{4}}$

$+ \frac{\cancel{9}3}{\cancel{12}4} + \frac{9}{4} + \frac{9}{4} = \frac{48}{4} + \cancel{\frac{243}{4}} - \cancel{\frac{243}{4}} = 12$

c) $v_e(t) = \dfrac{v(t) + v(-t)}{2}$



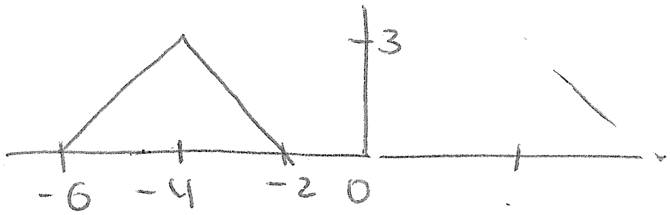




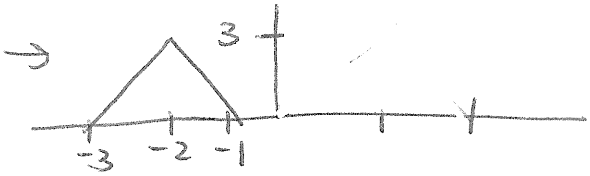
d) $a = 2 \quad b = 3$

$v(at+b) \quad v(2t+3)$

$v(t+3)$

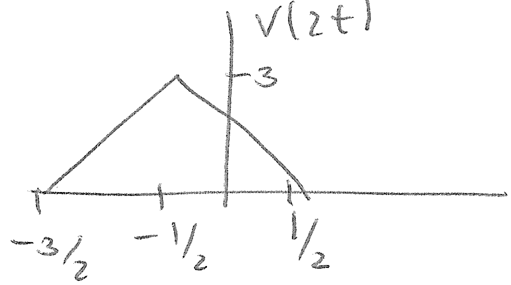


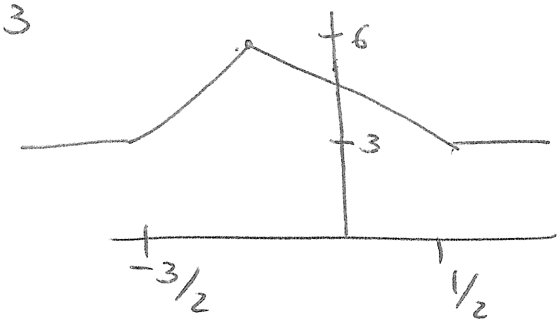
$\rightarrow$ $v(2t+3)$



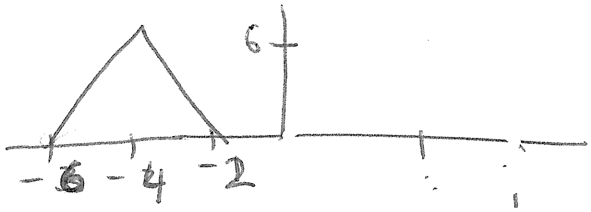
$v(at)+b = v(2t)+3$

$v(2t)$

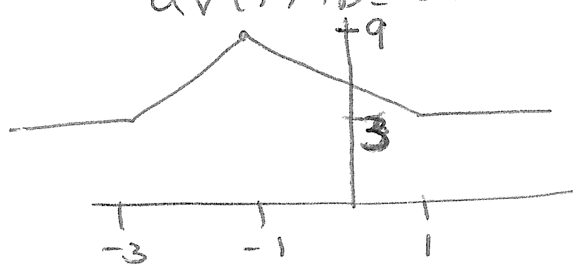




$a\,v(t+b) = 2v(t+3)$

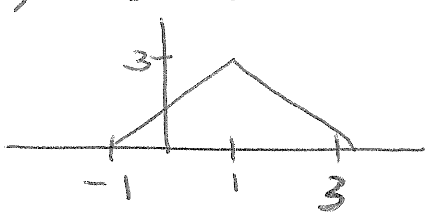


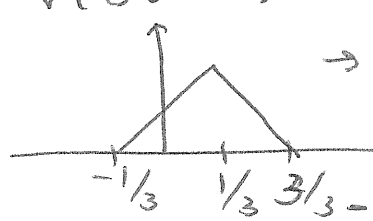
$a\,v(t)+b = 2v(t)+3$



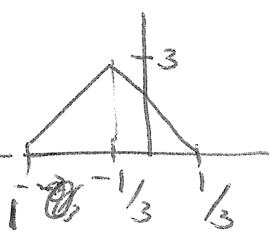
e) $a = -3,\ b = -2$

$v(-3t-2) \rightarrow v(t-2) \rightarrow v(3t-2) \quad v(-3t-2)$

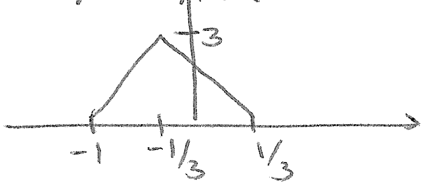




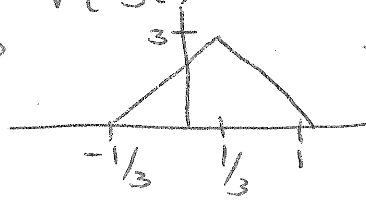
$\rightarrow$



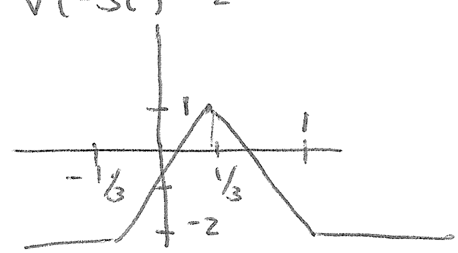
$v(-3t)-2 \quad v(3t)$



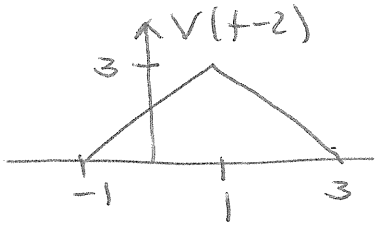
$\rightarrow$ $v(-3t)$

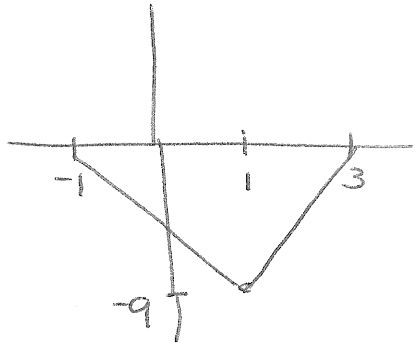


$\rightarrow$ $v(-3t)-2$

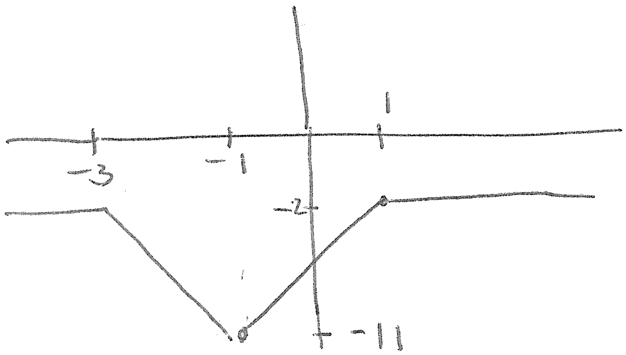


$-3\,v(t-2)$



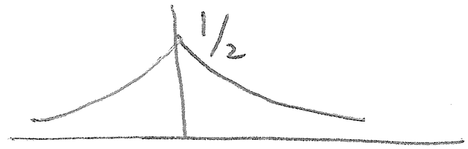


$-3\,v(t)-2$

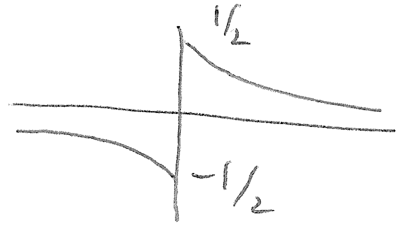


(5) a) $e^{-t}u(t)$

$$x_e(t) = \frac{e^{-t}u(t) + e^{t}u(-t)}{2}$$



$$x_o(t) = \frac{e^{-t}u(t) - e^{t}u(-t)}{2}$$

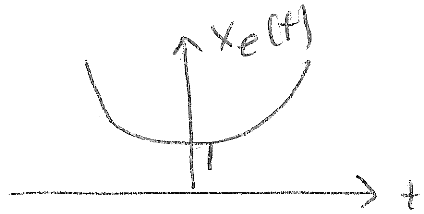


b) $(1+t)^2$

$$x_e(t) = \frac{(1+t)^2 + (1-t)^2}{2}$$

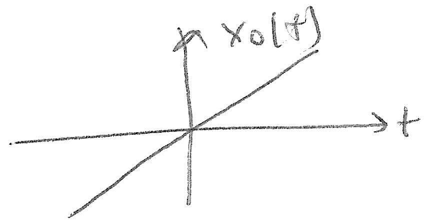
$$= \frac{1 + 2t + t^2 + 1 - 2t + t^2}{2}$$

$$= \frac{2 + 2t^2}{2} = 1 + t^2$$



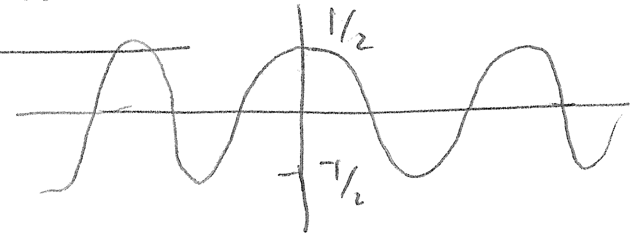
$$x_o(t) = \frac{\cancel{1} + 2t + \cancel{t^2} - \cancel{1} + 2t - \cancel{t^2}}{2}$$

$$= 2t$$

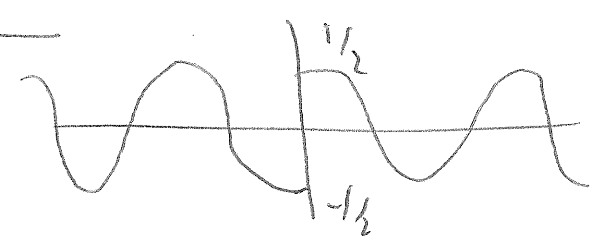


c) $\cos(\omega_0 t)u(t)$

$$x_e(t) = \frac{\cos(\omega_0 t)u(t) + \cos(-\omega_0 t)u(-t)}{2}$$



$$x_o(t) = \frac{\cos(\omega_0 t)u(t) - \cos(-\omega_0 t)u(-t)}{2}$$



(6) a) $x(t) = 4 - 3\sin(12\pi t) + \sin(30\pi t)$

$T_{0,1} = \frac{2\pi}{12\pi} = 1/6$ $\quad T_{0,2} = \frac{2\pi}{30\pi} = 1/15$ periodic

$\frac{T_{01}}{T_{02}} = \frac{15}{6} = 5/2$ $\quad T_0 = 2(1/6) = 1/3$

$f_0 = 3$ Hz.

b) $x(t) = 4e^{j16\pi t} - 5e^{-7\pi t}$ $\rightarrow$ not periodic

c) $x(t) = 2\cos(8\pi t) + \cos^2(6\pi t)$

$= 2\cos(8\pi t) + \frac{1}{2} + \frac{1}{2}\cos(12\pi t)$

$T_{01} = \frac{1}{4} =$ $\quad T_{02} = \frac{1}{6}$ $\quad \frac{T_{01}}{T_{02}} = \frac{1/4}{1/6} = \frac{3}{2}$

$T_0 = \underset{\downarrow 2}{k_0}(T_{01}) = \frac{1}{2}$ periodic $f_0 = 2$ Hz

d) $x(t) = \cos(10\pi t)\cos(10t)$

$= \frac{1}{2}\left[\cos((10\pi - 10)t) + \cos((10\pi + 10)t)\right]$

$T_{01} = \frac{2\pi}{10\pi - 10}$ $\quad T_{02} = \frac{2\pi}{10\pi + 10}$ $\quad \frac{T_{01}}{T_{02}} = \frac{10\pi + 10}{10\pi - 10}$

$= \frac{\pi + 1}{\pi - 1}$

not periodic

not rational $\rightarrow$