Name: Solutions
Student ID:

ECE 366 EXAM 1
October 10, 2008

- No textbooks, notes or HW solutions.
- You are allowed to use one page of notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [40] Answer the following short answer questions. Show work to get partial credit.

a) [18] Determine whether the following systems are linear, time-invariant and causal.

System	Linear	Time-Invariant	Causal
$y(t) = \sin(x(t))$	No	Yes	Yes
$y(t) = \int_{-\infty}^{t} x(2\tau)d\tau$	Yes	No	No
$y(t) = x(t)u(t)$	Yes	No	Yes

$H(j) = \frac{1}{j+2}$

b) [7] For a LTI system with transfer function $H(s) = \dfrac{1}{s+2}$, what is the zero-state response if the input is $x(t) = 3 + \cos(t)$?

$|H(j)| = \frac{1}{\sqrt{5}}$

$y(t) = 3H(0) + |H(j)| \cos(t + \angle H(j))$

$\angle H(j) = -\tan^{-1}\left(\frac{1}{2}\right)$

$= 3/2 + \frac{1}{\sqrt{5}} \cos\left(t - \tan^{-1}(1/2)\right)$

d) [5] Evaluate $\int_{-\infty}^{2} \cos(2t-5)\delta(t-3)dt$

$\int_{-\infty}^{2} \cos(1)\, \delta(t-3)\, dt = 0$

e) [5] Determine whether the signal $x(t) = 2 + 3\cos(4\pi t + \pi/3) + 5\sin(2t)$ is periodic or not. If periodic, determine the period.

$T_{0,1} = \frac{2\pi}{4\pi} = 1/2$

$T_{0,2} = \frac{2\pi}{2} = \pi$ $\quad \frac{T_{0,1}}{T_0} = \frac{1}{2\pi}$

Not periodic

f) [5] Determine whether the signal $x(t) = \cos(t)u(t)$ is an energy or power signal.

Power signal.

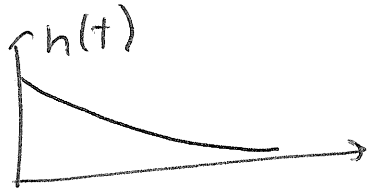
2. [30] Consider the cascade (in series) connection of two LTI systems with impulse responses $h_1(t), h_2(t)$, respectively. Assume that $h_1(t) = e^{-(t+2)}u(t+2)$ and $h_2(t) = \delta(t-2)$.

a) [8] Find and sketch the impulse response of the total system.

b) [8] Determine whether the system is memoryless, causal and stable.

c) [14] Assume that an input signal $x(t) = u(t) - u(t-2)$ is applied to this system, find the zero-state response $y(t)$. Use convolution to receive full credit.

a) $h(t) = e^{-t}u(t)$

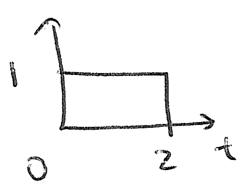


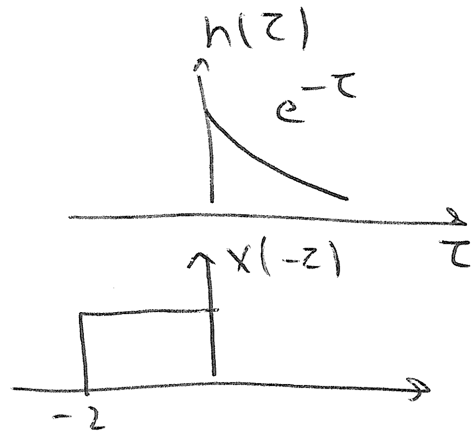
b) Not memoryless

causal

$\int_0^\infty e^{-t}\,dt = -e^{-t}\Big|_0^\infty = 1 < \infty$ stable

c)





for $t<0$ no overlap $y(t) = 0$

$0 \le t < 2$ partial overlap $\int_0^t e^{-\tau}\,d\tau = -e^{-\tau}\Big|_0^t$

$= -e^{-t} + 1$

$t \geq 2$ full overlap $\int_{t-2}^{t} e^{-\tau}\,d\tau = -e^{-\tau}\Big|_{t-2}^{t}$

$= -e^{-t} + e^{-(t-2)}$

Extra Page for Question 2:

$$y(t) = \left[1 - e^{-t}\right]\left[u(t) - u(t-2)\right] + \left[e^{-(t-2)} - e^{-t}\right]\left[u(t-2)\right]$$

$t - \tau - 2 \geq 0$

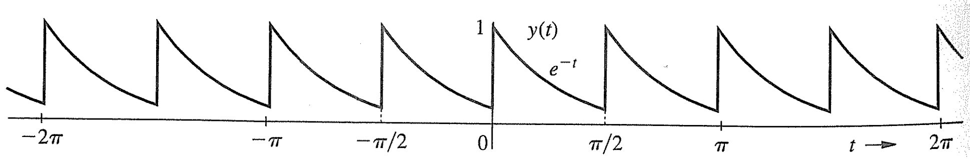
$t - 2 \geq \tau$

3. [30] For the following periodic signal:

   a) [3] What is the fundamental frequency and period of this signal?

   b) [15] Find the exponential Fourier series coefficients for this signal.

   c ~~b)~~ [12] Sketch the magnitude and phase spectrum for this signal for $k = -3, -2, -1, 0, 1, 2, 3$. Label your axes.



a) $T_0 = \pi/2$

$$\omega_0 = \frac{2\pi}{T_0} = 4 \text{ rad/sec}$$

b) $$C_k = \frac{1}{T_0}\int_{T_0} x(t) e^{-jk\omega_0 t}\, dt$$

$$= \frac{2}{\pi}\int_0^{\pi/2} e^{-t} e^{-jk4t}\, dt = \frac{2}{\pi} \left. \frac{e^{-(1+j4k)t}}{-(1+j4k)} \right|_0^{\pi/2}$$

$$= \frac{2}{\pi} \, \frac{e^{-(1+j4k)\pi/2} - 1}{-(1+j4k)} = \frac{2}{\pi} \, \frac{e^{-\pi/2}\left(e^{-j2\pi k}\right) - 1}{-(1+j4k)}, \quad e^{-j2\pi k} = 1$$

$$C_k = \frac{2}{\pi} \, \frac{1 - e^{-\pi/2}}{1 + j4k} = \frac{0.504}{1 + j4k}$$

c) $$|C_k| = \frac{0.504}{\sqrt{1 + 16k^2}}$$

$$\angle C_k = -\tan^{-1}(4k)$$