ECE 366 EXAM 2
November 16, 2007

- No textbooks, notes or HW solutions.
- One page of hand-written notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [28] Please answer the following questions briefly.

a) [8] For the following figure, match the time domain signals given in the left column with the magnitude spectra given in the right column.

A - 2
B - 3
C - 4
D - 1
b) [5] Given \( X(\omega) = \text{tri}(\omega/2) \), find \( \int_{-\infty}^{\infty} x(t)dt \).

\[
\int_{-\infty}^{\infty} x(t)dt = \left( \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \right) |_{\omega = 0} = X(0) = \text{tri}(0) = 1
\]

c) [5] For a signal with Fourier transform \( X(\omega) = \text{rect}\left(\frac{\omega}{1000}\right) \), what is the Nyquist sampling frequency in radians/sec for \( x(t) * \delta(t-1000) \)?

\[
x(t) * \delta(t-1000) = x(t-1000) \xrightarrow{F} X(\omega)e^{-j1000\omega}
\]

the highest freq. in \( X(\omega) \) 500 rad/sec.

\[
\omega_s = 1000 \text{ rad/sec}.
\]

d) [5] The continuous time signal \( x(t) = \cos(4\pi t) \) is sampled at \( \omega_s = 20 \text{ rad/sec} \). Is the resulting discrete-time signal periodic? If yes, give the period.

\[
x[n] = \cos \left( 4\pi \frac{n}{20} \right) = \cos \left( \frac{\pi}{5} n \right)
\]

\[
\Omega_0 = \frac{\pi}{5} = \left( \frac{\pi}{5} \right) 2\pi
\]

not periodic

e) [5] A periodic signal \( x(t) \) has a Fourier series representation \( x(t) = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi k t} \)

with \( C_0 = 1, C_1 = e^{j\pi/3}, C_{-1} = e^{-j\pi/3}, C_3 = C_{-3} = 2 \) and all other \( C_k = 0 \). Find the power of the signal \( x(t) \).

use Parseval's Thm.

\[
P_x = \sum_{k=-\infty}^{\infty} |C_k|^2
\]

\[
= 1 + 2 (|e^{j\pi/3}|^2) + 2 (2^2)
\]

\[
= 1 + 2 + 8 = 11 \text{ Watt}
\]
2. [36] Consider the following system in the figure. This system replaces analog multiplication with switching operation. In the figure shown below, \( x(t) \) is bandlimited with bandwidth equal to \( \pi \) rad/sec. \( s(t) \) is a periodic rectangular waveform with period equal to 1 sec.

a) [8] Find the Fourier transform of \( s(t) \).

b) [10] Find an expression for \( Z(\omega) \) in terms of \( X(\omega) \) and sketch \( Z(\omega) \).

c) [8] Given that the output \( y(t) = 2x(t)\cos(2\pi t) \), determine the type, gain and cutoff frequencies of the filter \( H(\omega) \).

d) [4] What type of modulation does this system correspond to?

e) [6] Draw the block diagram for the demodulator or the receiver that will recover \( x(t) \) from \( y(t) \). Please specify the characteristics of any filter you use in your design.

\[
\begin{align*}
\text{a)} & \quad T_0 = 1 \text{ sec} \quad s(t) \text{ is a periodic signal.} \quad \omega_0 = 2\pi \quad g(t) = \text{rect} \left( \frac{t}{0.5} \right) \quad \rightarrow G(\omega) = 0.5 \text{sinc} \left( \frac{\omega}{4} \right) \\
S(\omega) &= \frac{(2\pi)}{2} \sum_{k=-\infty}^{\infty} \text{sinc} \left( \frac{k \cdot 2\pi}{4} \right) \delta(\omega - k \cdot 2\pi) \\
S(\omega) &= \pi \sum_{k=-\infty}^{\infty} \text{sinc} \left( \frac{k \pi}{2} \right) \delta(\omega - k \cdot 2\pi) \\
\text{b)} & \quad z(t) = x(t)s(t) \quad \rightarrow Z(\omega) = \frac{1}{2\pi} X(\omega) * S(\omega) \\
Z(\omega) &= \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \text{sinc} \left( \frac{k \pi}{2} \right) X(\omega - k \cdot 2\pi) \\
&= \frac{1}{2} \sum_{k=-\infty}^{\infty} \text{sinc} \left( \frac{k \pi}{2} \right) X(\omega - k \cdot 2\pi) \\
&= \frac{1}{2} \text{sinc} \left( \frac{\pi}{2} \right) \frac{1}{\pi/2} = \frac{1}{\pi/2} = \frac{1}{\pi/2}
\end{align*}
\]
c) \[ y(t) = 2x(t) \cos(2\pi t) \]
\[ Y(\omega) = \left[ X(\omega - 2\pi) + X(\omega + 2\pi) \right] \]

\[ H(\omega) \text{ should be a bandpass filter with gain } 2 \]
\[ \omega \text{ between } \pi \text{ and } 3\pi \]

d) DSB.
e) Since this is DSB, we can use DSB demodulator
\[ y(t) \xrightarrow{\times} \text{LPF} \frac{\pi}{\pi} \xrightarrow{\text{LPF}} x(t) \]
\[ 2\cos(2\pi t) \]
3. [36] Signals $x_1(t) = 10^4 \text{rect}(10^4 t)$ and $x_2(t) = \delta(t)$ are applied at the inputs of the ideal lowpass filters $H_1(\omega) = \text{rect}(\omega/40,000\pi)$ and $H_2(\omega) = \text{rect}(\omega/20,000\pi)$, respectively. The outputs of these filters, $y_1(t)$ and $y_2(t)$, are multiplied to obtain the signal $y(t) = y_1(t)y_2(t)$.

a) [10] Sketch magnitude and phase spectra of $X_1(\omega)$ and $X_2(\omega)$. Label your axes.

b) [8] Sketch $H_1(\omega)$ and $H_2(\omega)$. Label your axes.

c) [12] Sketch $Y_1(\omega)$ and $Y_2(\omega)$. Label your axes.

d) [6] Find the bandwidths of $y_1(t), y_2(t)$ and $y(t)$.

Hint: You do not have to solve for $y(t)$ or $Y(\omega)$ explicitly.