Name: Solutions

Student ID:

ECE 366 EXAM 1
October 12, 2007

- No textbooks, notes or HW solutions.
- You are allowed to use one page of notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [30] Answer the following short answer questions. Show work to get partial credit.

a) [6] Determine whether the system with the input-output relationship $y(t) = \dfrac{x(t)}{x(t-1)}$ is linear, time-invariant and causal.

- Nonlinear
- Time invariant
- causal.

b) [4] Your boss gives you a black box. You input $x(t) = \cos(t)$ and the system outputs $y(t) = \cos(2t)$. Could the system be an LTI system? Justify your answer.

Not LTI

c) [5]Determine whether the system with the input-output relationship $y(t) = \left(\frac{x(t)}{t}\right)u(t-1)$ is BIBO stable.

if $|x(t)| \leq M$ $\quad |y(t)| \leq \left|\frac{x(t)}{t}\right| |u(t-1)| \leq \frac{M}{|t|} \rightarrow 0$ $\quad t \geq 1$

as $t \rightarrow \infty$.

BIBO stable.

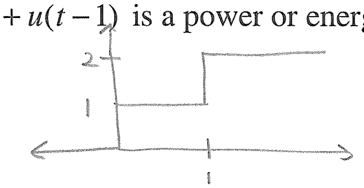
d) [5] Evaluate $\int_{-\infty}^{t} e^{-\tau}\delta(\tau)d\tau$ $= \bar{e} u(t)$

$\delta(t)$ if $t < 0 \rightarrow 0$.

if $t > 0 \rightarrow 1$ $\quad \Rightarrow u(t)$

e) [5] Determine whether the signal $x(t) = u(t) + u(t-1)$ is a power or energy signal.

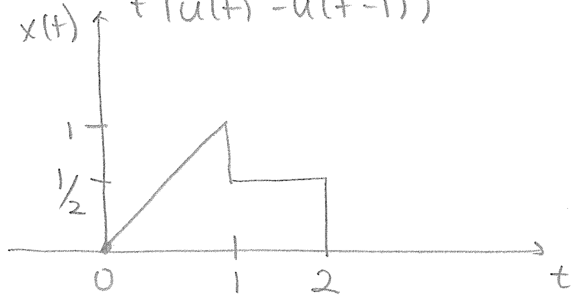
power



bounded by a constant envelope.

f) [5] Sketch the signal $x(t) = tu(t) - tu(t-1) + 0.5u(t-1) - 0.5u(t-2)$

$t(u(t) - u(t-1))$



2. [35] A periodic signal is given as

$$x(t) = 2 + 3\cos(2t) + 2\sin(3t + \frac{\pi}{6}) - \cos(7t + \frac{5\pi}{6})$$

a) [5] Determine the fundamental frequency of this signal in rad/sec.

b) [10] Find the exponential Fourier series representation for this signal and determine the Fourier series coefficients, C_k.

c) [10] Sketch the amplitude and the phase spectra for x(t).

d) [10] If the given signal goes through a LTI system with transfer function $H(s) = \frac{2}{s+1}$, what is the output signal y(t)?

a) $T_{0,1} = \frac{2\pi}{2} = \pi$, $T_{0,2} = \frac{2\pi}{3}$ $\quad T_{0,3} = \frac{2\pi}{7}$

$\frac{T_{0,1}}{T_{0,2}} = \frac{3}{2}$ $\quad \frac{T_{0,1}}{T_{0,3}} = \frac{7}{2}$ $\quad T_0 = (2)\pi = 2\pi$

$\boxed{\omega_0 = 1 \text{ rad/sec}}$

b) $x(t) = 2 + 3\left(\frac{e^{j2t} + e^{-j2t}}{2}\right) + 2\left(\frac{e^{j(3t+\pi/6)} - e^{-j(3t+\pi/6)}}{2j}\right)$

$- \left(\frac{e^{j(7t+5\pi/6)} + e^{-j(7t+5\pi/6)}}{2}\right)$

$C_0 = 2$

$C_1 = C_{-1} = 0$.

$C_2 = \frac{3}{2}$ $\quad C_{-2} = \frac{3}{2}$

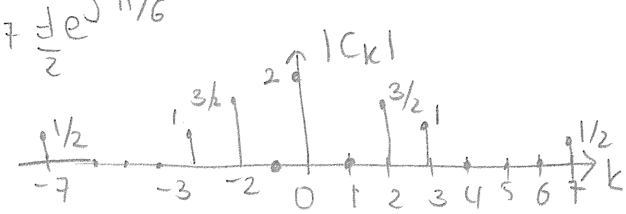
$C_3 = \frac{2}{2j} e^{j\pi/6} = -j e^{j\pi/6} = e^{j\pi/6} e^{-j\pi/2} = e^{-j2\pi/6} = e^{-j\pi/3}$

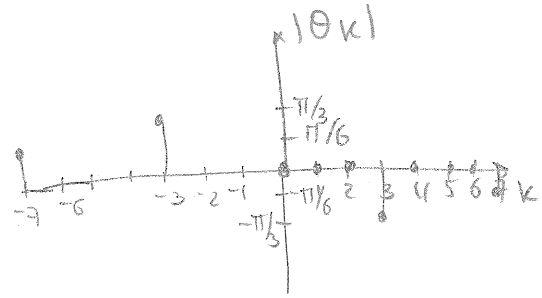
$C_{-3} = e^{j\pi/3}$

$C_7 = -\frac{1}{2} e^{j\frac{5\pi}{6}} = \frac{1}{2} e^{j5\pi/6} \cdot e^{-j\pi} = \frac{1}{2} e^{-j\pi/6}$

$C_{-7} = \frac{1}{2} e^{j\pi/6}$

c)





Extra Page for Question 2:

d) $H(0) = 2$

$H(2j) = \frac{2}{2j+1} \Rightarrow |H(2j)| = \frac{2}{\sqrt{5}}$, $\angle H(2j) = -\tan^{-1}(2)$

$H(3j) = \frac{2}{3j+1}$ $|H(3j)| = \frac{2}{\sqrt{10}}$ $\angle H(3j) = -\tan^{-1}(3)$

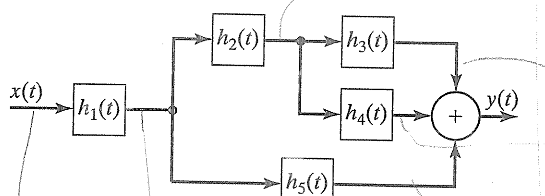
$H(7j) = \frac{2}{7j+1}$ $|H(7j)| = \frac{2}{\sqrt{50}}$ $\angle H(7j) = -\tan^{-1}(7)$

$y(t) = 4 + 2.68\cos(2t - 63.43^\circ) + 1.26\sin(3t - 41.56^\circ)$

$\underbrace{-0.28\cos(7t + \frac{5\pi}{6} - 81.86^\circ)}_{-0.28\cos(7t + 68.13^\circ)}$

3. [35] Consider the interconnection of LTI systems in the following figure:

$h_1(t) * h_2(t)$



$\to h_1(t) * h_2(t) * h_3(t)$

$\to h_1(t) * h_2(t) * h_4(t)$

$\to h_1(t) * h_5(t)$

$\delta(t)$ $\quad$ $\delta(t) * h_1(t) = h_1(t)$

a) [10] Show that the equivalent impulse response is
$h(t) = [(h_1(t) * h_5(t)) + (h_2(t) * h_4(t) * h_1(t)) + (h_2(t) * h_3(t) * h_1(t))]$
Hint: You can let $x(t) = \delta(t)$ and solve for the output to derive the overall impulse response of the system.
b) [5] Let $h_3(t) = h_4(t) = h_5(t) = u(t)$ and $h_1(t) = h_2(t) = 5\delta(t)$. Find the impulse response of the system.
c) [6] Determine whether this system is causal and stable. Justify your answers.
d) [14] An input signal, $x(t) = u(t+1) - u(t-2)$, is applied to this system. Given an expression for y(t) and sketch it.

a) $h(t) = h_1(t) * h_5(t) + h_1(t) * h_2(t) * h_3(t) + h_1(t) * h_2(t) * h_4(t)$

b) $h(t) = 5u(t) + 25u(t) + 25u(t) = 55u(t)$

c) causal

$\int |h(t)| dt = \int_0^\infty 55\, dt \to \infty$ not stable

d) $y(t) = x(t) * h(t)$

$= [u(t+1) - u(t-2)] * 55u(t)$

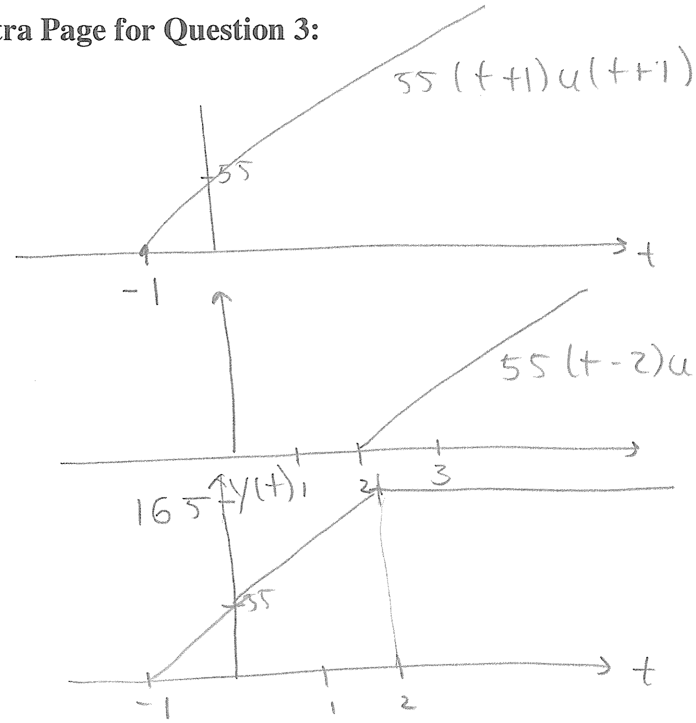
$= 55 \int_{-\infty}^{\infty} u(t-\tau)u(\tau+1) d\tau - 55 \int u(\tau-2) u(t-\tau) d\tau$

$= 55 \int_{-1}^{\infty} u(t-\tau) d\tau \xrightarrow{} t \geq -1 \quad - 55 \int_{2}^{\infty} u(t-\tau) d\tau \xrightarrow{} t \geq 2$

$= 55\left[\int_{-1}^{t} 1\, d\tau - \int_{2}^{t} d\tau\right]$

$= 55\left[(t+1)u(t+1) - (t-2)u(t-2)\right]$

Extra Page for Question 3:



55(4) -55 -(3)55