ECE 366 EXAM 1
October 12, 2007

- No textbooks, notes or HW solutions.
- You are allowed to use one page of notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [30] Answer the following short answer questions. Show work to get partial credit.

a) [6] Determine whether the system with the input-output relationship
\[ y(t) = \frac{x(t)}{x(t-1)} \]

is linear, time-invariant and causal.
- Nonlinear
- Time invariant
- Causal

b) [4] Your boss gives you a black box. You input \( x(t) = \cos(t) \) and the system outputs \( y(t) = \cos(2t) \). Could the system be an LTI system? Justify your answer.

\[ \text{Not LTI} \]
c) [5] Determine whether the system with the input-output relationship

\[ y(t) = \left( \frac{x(t)}{t} \right) u(t - 1) \]

is BIBO stable.

\[ |y(t)| \leq \left| \frac{x(t)}{t} \right| |u(t - 1)| \leq \frac{M u(t - 1)}{t + 1} \geq 0 \quad \text{as } t \to \infty. \]

\[ y(t) \text{ is BIBO stable.} \]

\[ d) [5] \text{ Evaluate } \int_{-\infty}^{\infty} e^{-\tau} \delta(t) d\tau = u(t) \]

\[ \delta(0) \quad \text{if } t < 0 \quad \Rightarrow 0. \]

\[ \int_{t}^{\infty} \delta(t) d\tau = 1 \quad \Rightarrow u(t) \]

\[ e) [5] \text{ Determine whether the signal } x(t) = u(t) + u(t - 1) \text{ is a power or energy signal.} \]

\[ \text{power} \]

\[ \text{bounded by a constant envelope.} \]

\[ f) [5] \text{ Sketch the signal } x(t) = tu(t) - tu(t - 1) + 0.5u(t - 1) - 0.5u(t - 2) \]

\[ x(t) = t \left( u(t) - u(t - 1) \right) \]
2. [35] A periodic signal is given as
\[ x(t) = 2 + 3 \cos(2t) + 2 \sin(3t + \frac{\pi}{6}) - \cos(7t + \frac{5\pi}{6}) \]

a) [5] Determine the fundamental frequency of this signal in rad/sec.
b) [10] Find the exponential Fourier series representation for this signal and determine the Fourier series coefficients, \( C_k \).
c) [10] Sketch the amplitude and the phase spectra for \( x(t) \).
d) [10] If the given signal goes through a LTI system with transfer function
\[ H(s) = \frac{2}{s + 1} \], what is the output signal \( y(t) \)?

\[ a) \quad T_{0,1} = \frac{2\pi}{2} = \pi, \quad T_{0,2} = \frac{2\pi}{3} \]
\[ T_{0,3} = \frac{2\pi}{7} \]
\[ \frac{T_{0,1}}{T_{0,2}} = \frac{3}{2}, \quad \frac{T_{0,1}}{T_{0,3}} = \frac{7}{2} \]
\[ T_0 = \frac{2\pi}{(2)\pi} = \frac{2\pi}{1 \text{ rad/sec}} \]

\[ b) \quad x(t) = 2 + 3 \left( \frac{e^{j2t} + e^{-j2t}}{2} \right) + 2 \left( \frac{e^{j(3t + \frac{\pi}{6})} - e^{-j(3t + \frac{\pi}{6})}}{2j} \right) \]
\[ - \left( \frac{e^{j(7t + \frac{5\pi}{6})} + e^{-j(7t + \frac{5\pi}{6})}}{2} \right) \]

\[ C_0 = 2 \]
\[ C_1 = C_{-1} = 0 \]
\[ C_2 = \frac{3}{2}, \quad C_{-2} = \frac{3}{2} \]
\[ C_3 = \frac{2}{2j} e^{j\pi/6} = -j \frac{e^{j\pi/6}}{2} = \frac{1}{2} e^{j3\pi/6} e^{-j\pi/2} = \frac{1}{2} e^{j\pi/6} e^{-j\pi/3} \]
\[ C_{-3} = e^{j\pi/3} \]
\[ C_7 = -\frac{j}{2} e^{j5\pi/6} = \frac{1}{2} e^{j5\pi/6}, \quad C_{-7} = \frac{1}{2} e^{-j5\pi/6} \]
\[ C_{-1} = \frac{1}{2} e^{j\pi/6} \]

\[ C_k \]

\[ \Theta_k \]

\[ \begin{align*}
  &\frac{1}{2} & 1 & 2 & 3 & 4 & 5 & k \\
  &-3 & -2 & -1 & 0 & 1 & 2 & k
\end{align*} \]
Extra Page for Question 2:

(d) \[ H(0) = 2 \]

\[ H(2j) = \frac{2}{2j + 1} \quad \Rightarrow \quad |H(2j)| = \frac{2}{\sqrt{5}}, \quad \tan^{-1}(2) \]

\[ H(3j) = \frac{2}{3j + 1} \quad |H(3j)| = \frac{2}{\sqrt{10}}, \quad \tan^{-1}(3) \]

\[ H(7j) = \frac{2}{7j + 1} \quad |H(7j)| = \frac{2}{\sqrt{50}}, \quad \tan^{-1}(7) \]

\[ y(t) = (1 + 2.68 \cos(2t - 63.43\degree) + 1.26 \sin(3t - 41.56\degree)) - 0.28 \cos\left(7t + \frac{5\pi}{6} - 81.86\degree\right) - 0.78 \cos(7t + 65.13\degree) \]
3. [35] Consider the interconnection of LTI systems in the following figure:

\[ h(t) = [(h_1(t) * h_5(t)) + (h_2(t) * h_4(t) * h_3(t)) + (h_3(t) * h_1(t)) + (h_4(t) * h_2(t) * h_5(t))] \]

a) [10] Show that the equivalent impulse response is

\[ h(t) = h_1(t) * h_5(t) + h_1(t) * h_2(t) * h_3(t) + h_1(t) * h_4(t) * h_5(t) \]

Hint: You can let \( x(t) = \delta(t) \) and solve for the output to derive the overall impulse response of the system.

b) [5] Let \( h_3(t) = h_4(t) = h_5(t) = u(t) \) and \( h_1(t) = h_2(t) = 5\delta(t) \). Find the impulse response of the system.

c) [6] Determine whether this system is causal and stable. Justify your answers.

d) [14] An input signal, \( x(t) = u(t+1) - u(t-2) \), is applied to this system. Given an expression for \( y(t) \) and sketch it.

\[ y(t) = x(t) * h(t) \]

\[ = [u(t+1) - u(t-2)] * 5s u(t) \]

\[ = 5s \int_{t-2}^{t+1} u(t-2) u(t+1) dt - 5s \int_{t-2}^{t+1} u(t-1) u(t+1) dt \]

\[ = 5s \left[ \int_{t-2}^{t+1} u(t-2) dt - \int_{t-2}^{t+1} u(t-1) dt \right] \]
Extra Page for Question 3:

\[ 55 \begin{pmatrix} 1 & 1 \end{pmatrix} u \begin{pmatrix} 1 & 1 \end{pmatrix} \]

\[ 55 \begin{pmatrix} 1 & -2 \end{pmatrix} u \begin{pmatrix} 1 & -2 \end{pmatrix} \]

\[ 16 \begin{pmatrix} 1 & 1 \end{pmatrix} \]

\[ 55 (u) = 55 (2) 55 \]