Name: Solutions
Student ID: ____________________

ECE 366 EXAM 2
November 17, 2006

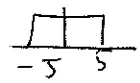
- No textbooks, notes or HW solutions.
- One page of hand-written notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [20] State whether the following statements are true or false. No partial credit will be given so you do not have to provide any explanation.

a) An audio signal with 10Hz bandwidth is transmitted over a channel using AM modulation. The minimum bandwidth of the channel should be 20Hz.

T

b) The Nyquist frequency of sampling for $x(t) = \sin c(5t)$ is 20 rad/sec.

$X(\omega) = \frac{\pi}{5} \text{rect}\left(\frac{\omega}{10}\right)$  $\omega_s = 10 \text{ rad/sec}$ F.

c) The signal $x[n] = \underbrace{\cos(3n)}_{\text{not periodic}} + \cos(\frac{8}{15}\pi n)$ is periodic.

F.

d) For a real and even periodic signal, the Fourier series coefficients, C_k, are always real.

T

e) The power of $u[n]$ is 1.

(1/2) F

f) The spectrum of a periodic signal is always periodic.

(it's discrete, not periodic) F

g) The ideal lowpass filter is not used in practice because it is not causal.

T

h) If a periodic signal $x(t)$ has Fourier series coefficients C_k, the Fourier series coefficients for the signal $x(-t)$ will also be equal to C_k.

F

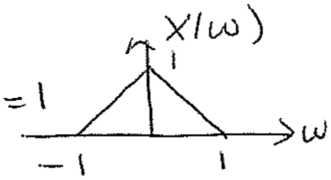
i) The inverse Fourier Transform of $e^{-2\omega}u(\omega)$ is $\frac{1}{2+jt}$.

Duality $\quad e^{-2t}u(t) \to \frac{1}{2+j\omega}$

$\frac{1}{2+jt} \longrightarrow 2\pi e^{2\omega}u(-\omega)$ F.

j) If $X(\omega) = tri(\omega/2)$, then $\int_{-\infty}^{\infty} x(t)dt = 1$.

$\int_{-\infty}^{\infty} x(t)\,dt = X(0) = 1$



T

2. [40] Consider the ideal sampling system given below for the continuous time signal with the spectrum given in Figure 1 (a). Let $\omega_1 = 8\pi, \omega_2 = 10\pi$.

a) [5] What is the Nyquist frequency of sampling for this signal?

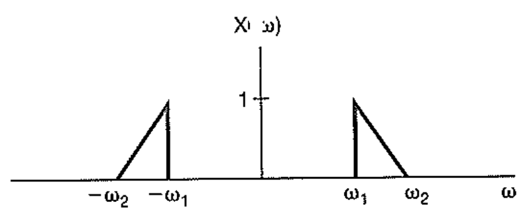
b) [6] Assume that this signal is sampled using the ideal sampler shown below with $T = 0.5$. Derive an expression for the spectrum of the sampled signal, $X_p(\omega)$, in terms of $X(\omega)$.

c) [10] Sketch the spectrum of the sampled signal, $X_p(\omega)$, you found in part (b). Draw at least the first 5 terms in the summation, i.e. $k = 0, \pm 1, \pm 2, \pm 3, \pm 4$. Make sure to label the frequency and the amplitude axes.

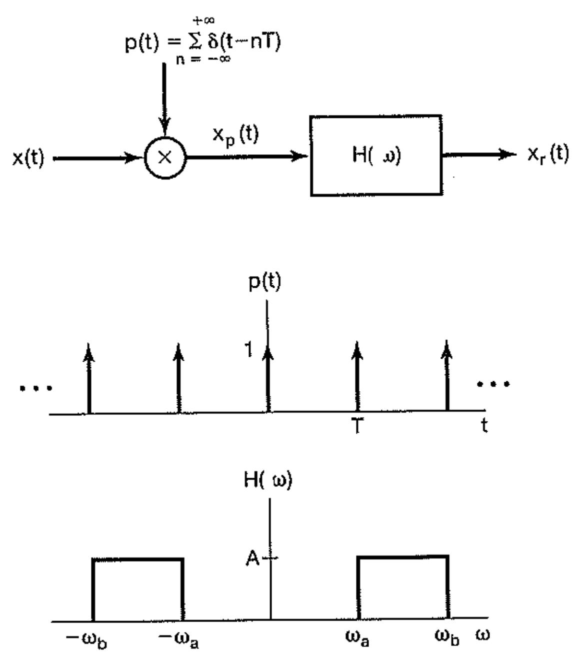
d) [6] Show that you can reconstruct the original signal from the sampled one. What are the values of A, ω_a, ω_b for the bandpass filter shown in the Figure?

e) [7] If the sampling rate T is now increased to 1 sec, can you still reconstruct the original signal? Sketch the spectrum for the sampled signal, and discuss whether you can recover the original signal.

f) [6] Show that the sampling frequency used in part (b) is less than the Nyquist sampling frequency. Explain why we can recover the original signal even though we sampled at a rate that is less than the Nyquist rate.



(a)



(b)

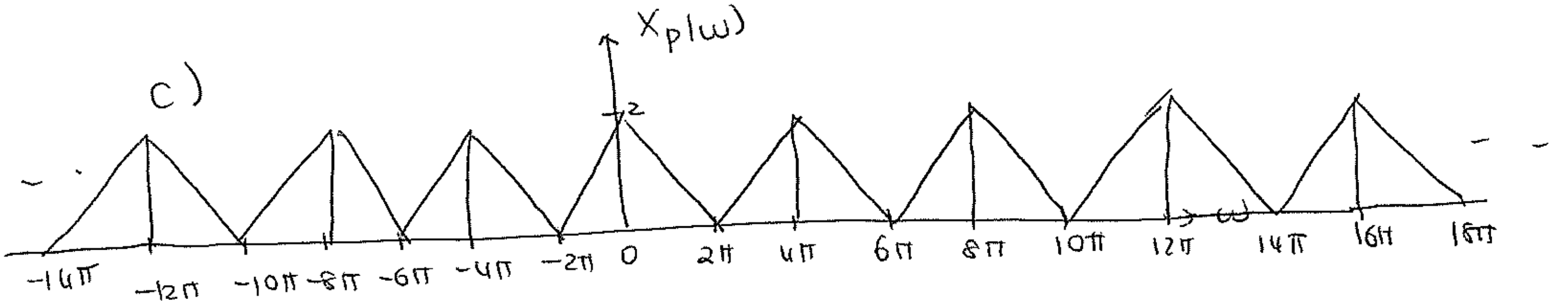
Extra Page for Question 2:

a) $\omega_s = 2(10\pi) = 20\pi$ rad/sec.

b) $X_p(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} X(\omega - k\omega_s)$

$\omega_s = \frac{2\pi}{0.5} = 4\pi$ $\qquad \boxed{2 \sum_{k=-\infty}^{\infty} X(\omega - k4\pi)}$

c)



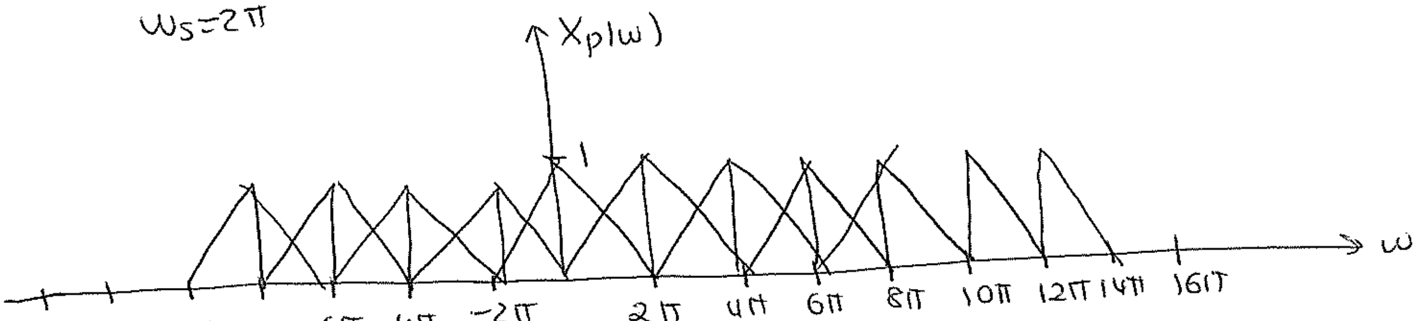
d) $A = 1/2$

$\omega_a = 8\pi$

$\omega_b = 10\pi$

e) $T = 1 \rightarrow X_p(\omega) = \sum_{k=-\infty}^{\infty} X(\omega - 2\pi k)$

$\omega_s = 2\pi$



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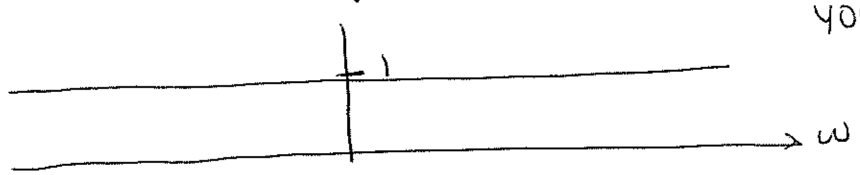


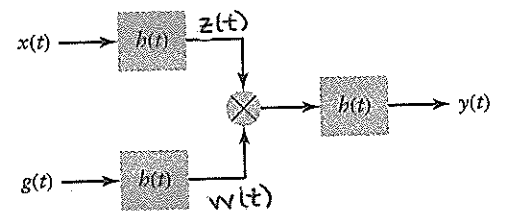
you cannot reconstruct.

f) Nyquist sampling freq. $20\pi > 4\pi$

we can still reconstruct the signal because it's a narrowband band pass signal. It does not occupy the whole 10π.

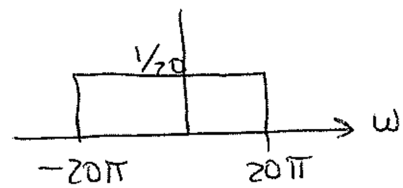
3. [40] Consider the system shown in the Figure below. Assume that $x(t) = \sin c(20\pi t), g(t) = \cos(8\pi t) + \cos(16\pi t)$ and $h(t) = \dfrac{\sin(10\pi t)}{\pi t}$.

a) [6] Compute and sketch $X(\omega)$.
b) [6] Compute and sketch $G(\omega)$.
c) [6] Compute and sketch $H(\omega)$.
d) [6] Compute and sketch $Z(\omega)$.
e) [6] Compute and sketch $W(\omega)$.
f) [10] Compute and sketch $Y(\omega)$.

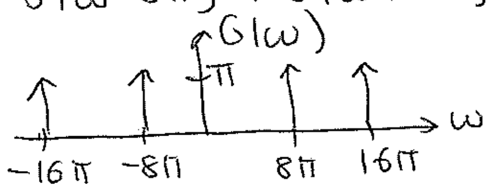


Hint: $\sin c(x) = \dfrac{\sin(x)}{x}$.

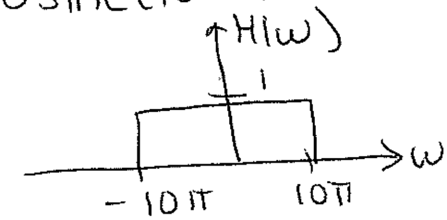
a) $X(\omega) = \dfrac{1}{20} \text{rect}\left(\dfrac{\omega}{40\pi}\right)$



b) $G(\omega) = \pi[\delta(\omega - 8\pi) + \delta(\omega + 8\pi)] + \pi[\delta(\omega - 16\pi + \delta(\omega + 16\pi)]$

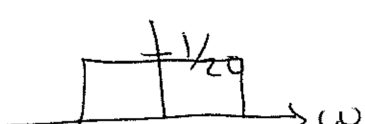


c) $h(t) = \dfrac{\sin(10\pi t)}{\pi t} = 10 \text{sinc}(10\pi t) \rightarrow \text{rect}\left(\dfrac{\omega}{20\pi}\right)$

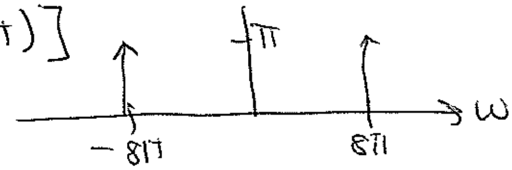


d) $Z(\omega) = X(\omega) H(\omega)$

$= \dfrac{1}{20} \text{rect}\left(\dfrac{\omega}{20\pi}\right)$



e) $W(\omega) = \pi[\delta(\omega - 8\pi) + \delta(\omega + 8\pi)]$
$\downarrow$
$G(\omega)H(\omega)$



f) $Y(\omega) = H(\omega) \cdot \dfrac{1}{2\pi} \{Z(\omega) * W(\omega)\}$ (multiplication in time is convolution in freq.)

$= \dfrac{1}{2\pi} H(\omega) \{\pi Z(\omega - 8\pi) + \pi Z(\omega + 8\pi)\}$

Extra Page for Question 3:

$$= \frac{1}{2}\, rect\left(\frac{\omega}{20\pi}\right)\left[\frac{1}{20}\, rect\left(\frac{\omega-8\pi}{20\pi}\right) + \frac{1}{20}\, rect\left(\frac{\omega+8\pi}{20\pi}\right)\right]$$

$$= \frac{1}{40}\, rect\left(\frac{\omega}{20\pi}\right)\left[rect\left(\frac{\omega-8\pi}{20\pi}\right) + rect\left(\frac{\omega+8\pi}{20\pi}\right)\right]$$