Name: Solutions

Student ID:

ECE 366 EXAM 1
October 13, 2006

- No textbooks, notes or HW solutions.
- One page of hand-written notes.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [30] State whether the following statements are true or false. Please give a brief explanation to justify your answers.

a) The system with the input output relationship $y(t) = x(\alpha t)$ is non-causal for all values of $\alpha, \alpha \neq 0$.

<u>False</u>. It is causal for $\alpha = 1$

For all other values of α, it is non-causal since the output may depend on future inputs.

b) The signal given by $x(t) = \cos(\pi t) + 2 + e^{j\frac{\pi}{4}t}$ is periodic with period equal to 8 seconds.

<u>True</u>

$$T_1 = \frac{2\pi}{\pi} = 2$$

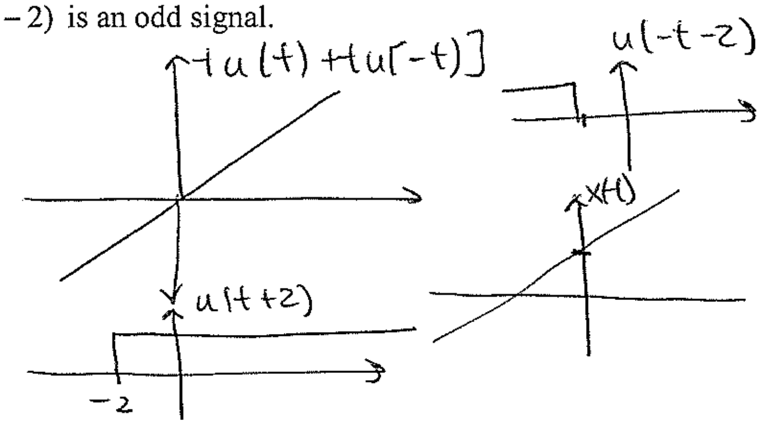
$$T_2 = \frac{2\pi}{\pi/4} = 8$$

Note: A constant is always periodic.

$$\frac{T_1}{T_2} = \frac{1}{4} \qquad \boxed{T_0 = 8}$$

c) $x(t) = tu(t) + tu(-t) + u(t+2) + u(-t-2)$ is an odd signal.

<u>False</u>.



d) It is known that the response of the system $\frac{dy(t)}{dt} + \alpha y(t) = x(t), \alpha \neq 0$ is given by $y(t) = (5 + 3e^{-2t})u(t)$. Therefore, α must be equal to -2.

$y_N(t) = 3e^{-2t} \Rightarrow s = -2$

$sY(s) + \alpha Y(s) = 0$ False

$s + \alpha = 0 \Rightarrow s = -2$

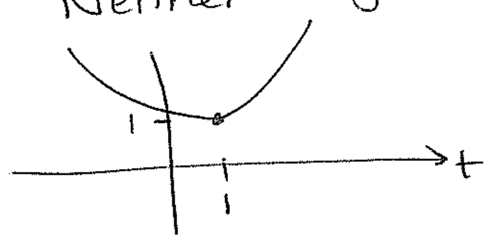
$\Rightarrow \boxed{\alpha = 2}$

e) $\int_{-\infty}^{t} e^{-2\tau}\delta(\tau - 2)d\tau = e^{-4}$

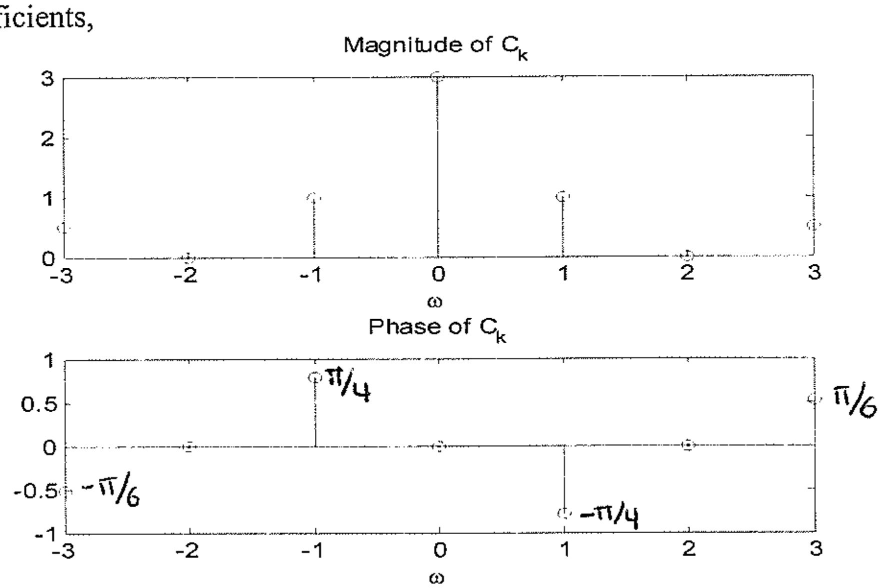
False. $e^{-4}u(t-2)$

f) $x(t) = e^{|1-t|}$ is a power signal.

False Neither grows infinitely.



2. [35] Given the frequency spectrum for a periodic signal, i.e. the Fourier series coefficients,



a) [5] Determine the fundamental frequency of this signal.

b) [10] Write the signal $x(t)$ with the given Fourier series coefficients. Simplify your answer in terms of sines/cosines.

c) [8] Determine whether this signal is even or odd, energy or power signal.

c) [12] If the signal found in part b) goes through a LTI system with transfer function $H(s) = \dfrac{2}{s+4}$, what is the steady-state response, y(t)?

Hint:

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

a) $\omega_0 = 1$ rad/sec from the figure.

b) $C_0 = 3$, $C_1 = e^{-j\pi/4}$ $\quad C_{-1} = e^{j\pi/4}$

$C_2 = 0$

$C_3 = 0.5 e^{j\pi/6}$ $\quad C_{-3} = 0.5 e^{-j\pi/6}$

$$x(t) = 3 + e^{-j\pi/4} e^{j\overset{1}{\omega_0} t} + e^{j\pi/4} e^{-jt} + 0.5 e^{j\pi/6} e^{j3t} + 0.5 e^{-j\pi/6} e^{-j3t}$$

$$x(t) = 3 + 2\cos(1 \cdot t - \pi/4) + \cos(3t + \pi/6)$$

Extra Page for Question 2:

c) Power, since it's periodic.

Neither odd/even (note that the cosines are shifted and are no longer even)

d) $y(t) = 3H(0) + 2|H(j)| \cos(t - \pi/4 + \angle H(j))$

$+ |H(3j)| \cos(3t + \pi/6 + \angle H(3j))$

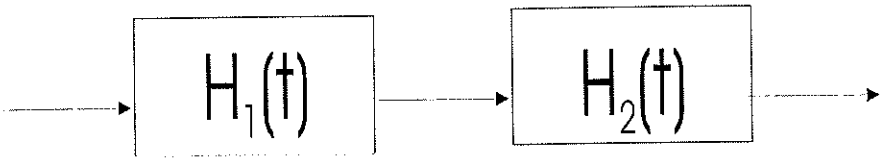
$H(0) = \frac{2}{4} = \frac{1}{2}$

$H(j) = \frac{2}{j+4} \Rightarrow |H(j)| = \frac{2}{\sqrt{17}}$ $\quad \angle H(j) = -\tan^{-1}\left(\frac{1}{4}\right)$

$H(3j) = \frac{2}{3j+4} \Rightarrow |H(3j)| = \frac{2}{\sqrt{25}} = 0.4$

$\angle H(3j) = -\tan^{-1}\left(\frac{3}{4}\right)$

3. [35] Consider two LTI systems connected in series with impulse response $h_1(t)$ and $h_2(t)$, respectively with $h_1(t) = t[u(t+2) - u(t-2)]$ and $h_2(t) = \delta(t-2)$.



a) [8] Determine whether each one of the systems, $h_1(t)$ and $h_2(t)$, are causal and stable. Justify your answers.
b) [10] Determine the impulse response of the composite system and state whether it is stable and causal.
c) [12] Assume that an input signal $x(t) = u(t)$ is applied to this composite system. What is the output, y(t)? Sketch y(t).
d) [5] Find a simple relationship between the output y(t) found in part c) and the impulse response of the composite system, i.e. your answer to part b).

a) $h_1(t) = t[u(t+2) - u(t-2)] \rightarrow$ not causal, $h_1(t) \neq 0$ for $t<0$

$\int |h_1(t)| dt = \int_{-2}^{2} |t| dt = 2\int_0^2 t\,dt = 2 \frac{t^2}{2}\Big|_0^2 = 4 < \infty$ stable

$h_2(t) = \delta(t-2) \rightarrow$ causal

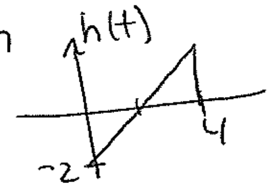
$\int h_2(t) dt = 1 < \infty$ stable.

b) $h(t) = h_1(t) * h_2(t)$

$= h_1(t) * \delta(t-2)$) property of convolution

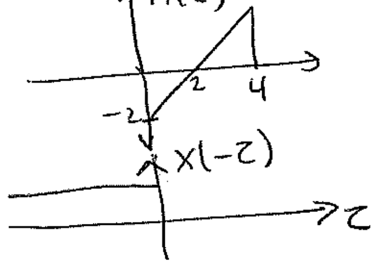
$= h_1(t-2)$

$= (t-2)[u(t) - u(t-4)] \rightarrow$



causal + stable

c) $x(t) = u(t)$ $\quad y(t) = u(t) * h(t)$



$t<0 \rightarrow$ no overlap $y(t) = 0$

$0 \leq t \leq 4 \rightarrow$ partial overlap

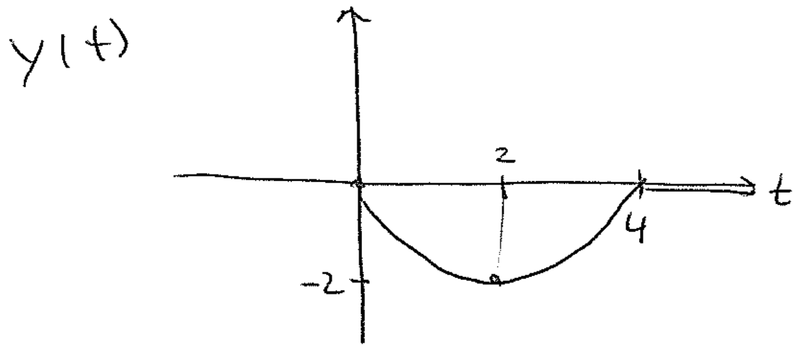
$y(t) = \int_0^t (\tau - 2) d\tau$

$= \frac{\tau^2}{2} - 2\tau\Big|_0^t = \frac{t^2}{2} - 2t$

$t > 4 \rightarrow$ full overlap

$y(t) = \int_0^4 (\tau - 2) d\tau = 0$

Extra Page for Question 3:



d) $y(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$

→ since y(t) is the step response of the system it is related to h(t) by an integral.