Name: Solutions

Student ID: ______

ECE 202 EXAM 3

April 13, 2007

- No textbooks, notes or HW solutions.
- Calculators are allowed.
- Exam is 50 minutes.
- To maximize your score on this exam, read the questions carefully and write legibly. For those problems that allow partial credit, show your work clearly.
- Good luck.

1. [25] Answer the following questions briefly.

a) For a circuit with impulse response $h(t) = e^{-2t}u(t)$, find the output to the input $x(t) = u(t)$ using the convolution integral.

$$y(t) = \int_0^t e^{-2(t-\tau)}\,d\tau = e^{-2t}\int_0^t e^{2\tau}\,d\tau$$

$$= e^{-2t}\frac{e^{2\tau}}{2}\Big|_0^t = e^{-2t}\left[\frac{e^{2t}}{2} - \frac{1}{2}\right] = \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\right]u(t)$$

b) For a RLC bandpass circuit, which element would you adjust (and by how much) to double the bandwidth without changing the center frequency?

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$BW = \frac{\omega_0}{Q_0} \rightarrow \text{double } BW \rightarrow \text{half } Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

$\boxed{\text{Double } R}$

c) For a circuit with transfer function, $T(s) = \dfrac{2000(s+200)}{(s+1000)(s-400)}$, determine whether the circuit is stable or not. Explain why.

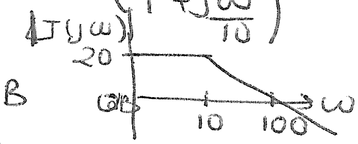
poles at -1000 and 400

Not stable since not all of the poles are in the LHP.

d) A voltage source of $v(t) = 10\cos(100t)$ is applied to a lowpass filter with a transfer function $T(s) = \dfrac{100}{s+10}$. Predict the output voltage $v_o(t)$.

Hint: Use Bode diagrams to estimate the gain and the phase at the input frequency.

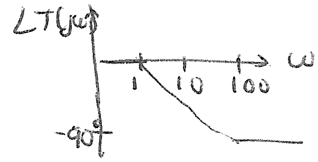
$$\frac{100}{j\omega+10} = \frac{100}{10}\cdot\frac{1}{1+j\frac{\omega}{10}} = (10)\left(\frac{1}{1+j\frac{\omega}{10}}\right)$$



$$20\log_{10}10 = 20\,\text{dB}$$

gain at $\omega = 100 \rightarrow 0\,\text{dB} = 1$

phase at $\omega = 100$ $\quad -90°$



$$V_o(t) = 10\cos(100t - 90°)$$

e) The first three terms in the Fourier series expansion of a periodic signal is $f(t) = \dfrac{8}{\pi^2}\cos(100\pi t) + \dfrac{8}{9\pi^2}\cos(300\pi t) + \dfrac{8}{25\pi^2}\cos(500\pi t) + \ldots$. Determine the fundamental frequency in Hz.

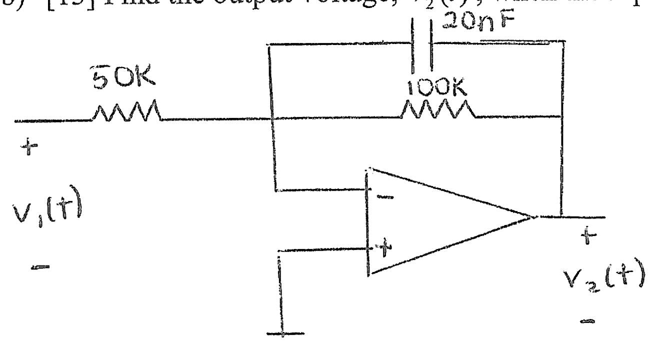
$$100\pi = 2\pi f_o$$

$$\boxed{f_o = 50\,\text{Hz}}$$

2. [25] Given the following inverting amplifier:

a) [12] Find the transfer function, $T(s)$.

b) [13] Find the output voltage, $v_2(t)$, when the input is $v_1(t) = u(t)$.



a) $T(s) = -\frac{Z_2}{Z_1} = -\frac{\left(100K \parallel \frac{1}{s\,20\times10^{-9}}\right)}{50K}$

$$= -\frac{\dfrac{\dfrac{100\times10^3}{s\,20\times10^{-9}}}{100\times10^3 + \dfrac{1}{20\times10^{-9}s}}}{50\times10^3} = -\frac{\dfrac{100\times10^3}{1+s\,2\times10^{-3}}}{50\times10^3}$$

$$= -\frac{2}{1+s\,2\times10^{-3}}$$

$$\boxed{= -\frac{1000}{s+500}}$$

b) $V_2(s) = \left(\frac{1}{s}\right)\left(\frac{-1000}{s+500}\right) = \frac{k_1}{s} + \frac{k_2}{s+500}$

$k_1 = -2$

$k_2 = 2$

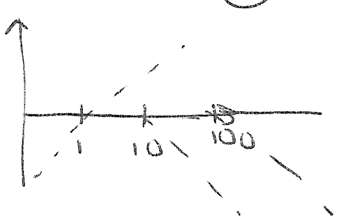
$V_2(t) = -2u(t) + 2e^{-500t}u(t)$ //

3. [25] Given the network function $T(s) = \dfrac{1000s}{(s+10)(s+100)}$, construct the Bode plot for $|T(j\omega)|$ and $\angle T(j\omega)$ with respect to frequency. The magnitude plot should be in terms of dBs and the phase plot in terms of degrees. Label your axis carefully. Determine whether this is a lowpass, highpass, bandpass or bandstop filter.

$$T(j\omega) = \frac{1000\, j\omega}{(j\omega+10)(j\omega+100)}$$

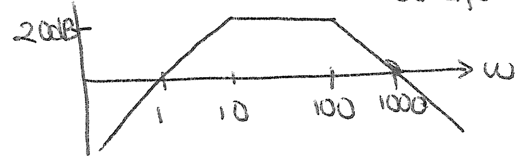
$$= \frac{1000}{(10)(100)}\, j\omega \left(\frac{1}{1+\frac{j\omega}{10}}\right)\left(\frac{1}{1+\frac{j\omega}{100}}\right)$$

$$= \underset{\text{I}}{(1)}\ \underset{\text{II}}{(j\omega)}\ \underset{\text{V}}{\left(\frac{1}{1+\frac{j\omega}{10}}\right)}\ \underset{\text{V}}{\left(\frac{1}{1+\frac{j\omega}{100}}\right)}$$

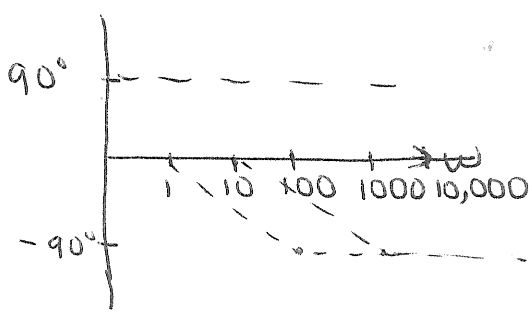


$\Rightarrow$ $|T(j\omega)|$

Bandpass

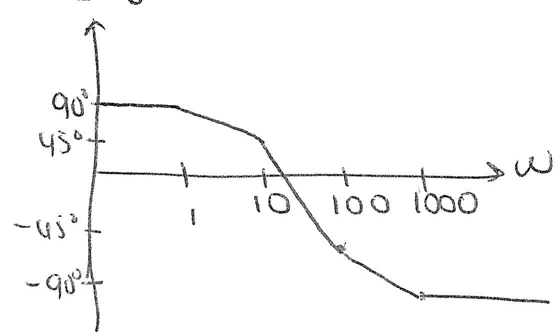


$\angle T(j\omega)$



$\angle T(j\omega)$

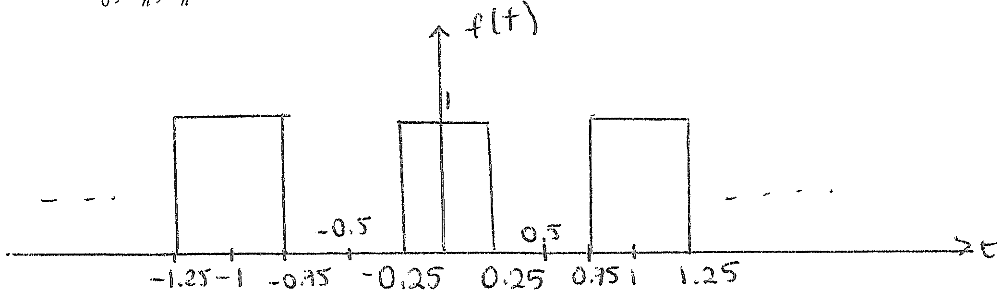
$\Rightarrow$



4. [25] For the following rectangular wave with amplitude 1 V and period 1 sec:

a) [5] Determine whether the signal has odd symmetry or even symmetry.

b) [20] Determine the Fourier series coefficients for this signal, i.e. find expressions for a_0, a_n, b_n.



a) Even symmetry.

b) $b_n = 0$ since even symmetric

$$a_0 = \int_{-0.25}^{0.75} 1\,dt = 0.5 \,//$$

$$f_0 = \frac{1}{T_0} = 1\,Hz$$

$$a_n = 2\int_{-0.25}^{0.25} \cos(2\pi n f_0 t)\,dt = \left. \frac{2\sin(2\pi n f_0 t)}{2\pi n f_0} \right|_{-0.75}^{0.75}$$

$-\sin(0.5\pi n)$

$$= 2\left[\frac{\sin(0.5\pi n) - \sin(-0.5\pi n)}{2\pi n}\right] = 2\left[\frac{2\sin(0.5\pi n)}{2\pi n}\right]$$

$$\boxed{a_n = \frac{2\sin(0.5\pi n)}{\pi n}}$$