Extra Questions for Discrete-Time Signals and Systems

11/22

1. Determine which of the given signals are periodic and find the period.

a) $x[n] = \cos(\pi n)$

$\pi n = 2\pi F n$

$F = \frac{1}{2}$, periodic with $N=2$

b) $x[n] = \sin(3.15n)$

$3.15n = 2\pi F n$

$F = \frac{3.15}{2\pi}$, not periodic

c) $x[n] = e^{j\frac{0.3n}{\pi}}$

$2\pi F n = \frac{0.3n}{\pi}$

$F = \frac{0.3}{2\pi^2}$, not periodic

d) $x[n] = e^{j5\pi n/7}$

$2\pi F n = \frac{5\pi n}{7}$

$F = \frac{5}{14}$, periodic with $N=14$.

2. Determine if the system described by $y[n] = \ln(x[n])$ is a) Linear?

$a_1 x_1[n] + a_2 x_2[n] \rightarrow \ln(a_1 x_1[n] + a_2 x_2[n])$

$\neq a_1 \ln(x_1[n]) + a_2 \ln(x_2[n])$

$\therefore$ Not linear.

b) Time Invariant?

$\ln(x[n-1]) = y[n-1] \rightarrow$ time invariant

c) Causal?

$y[n]$ depends on present input, therefore causal.

d) Stable? $|x[n]| < M \rightarrow |\ln(x[n])| \rightarrow \infty$ for example $x[n] = 0$

$\rightarrow \ln(x[n]) \rightarrow \infty$

unbounded $\rightarrow$ not stable.

3) Given the LTI system $h[n]$ with input $x[n]$,

$$x[n] = \begin{cases} 2, & 1 \le n \le 3 \\ 0, & \text{otherwise} \end{cases}$$

$$h[n] = \begin{cases} 3, & -1 \le n \le 2 \\ 0, & \text{otherwise} \end{cases}$$

a) Solve for the system output at $n=5$, $y[5]$.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

$$y[5] = \sum_{k=-\infty}^{\infty} x[k]\, h[5-k]$$

$$y[5] = \sum_{k=1}^{3} 2\, h[5-k]$$

$$= 2\cancel{h[4]}^{\,0} + 2\cancel{h[3]}^{\,0} + 2\underbrace{h[2]}_{3}$$

$$= 6$$

b) Find the maximum value for the output $y[n]$.

$x[n] = 2\delta[n-1] + 2\delta[n-2] + 2\delta[n-3]$

$h[n] = 3\delta[n+1] + 3\delta[n] + 3\delta[n-1] + 3\delta[n-2]$

The convolution of $x[n] * h[n]$

$\delta[n] \rightarrow$	3	3	3	3		
$2\delta[n-1] \rightarrow$	0	6	6	6	6	
$2\delta[n-2] \rightarrow$	0	0	6	6	6	6
$2\delta[n-3] \rightarrow$	0	0	0	6	6	6
$x[n] \rightarrow$	0	6	12	18	18	12

Max. value of $y[n]$ is 18.

c) Find the values of n for which the output is maximum.

$n=2$ and 3.

(4) Find the response of the system described by the following difference equation.

$y[n] - 0.7y[n-1] = e^{-n}u[n] \qquad y[-1] = 0$

$\underline{n=0}$ $\quad y[0] = 1$

$\underline{n=1}$ $\quad y[1] - 0.7y[0] = e^{-1}$

$y[1] = 0.7 + e^{-1}$

$\underline{n=2}$ $\quad y[2] - 0.7y[1] = e^{-2}$

$y[2] = e^{-2} + (0.7)(0.7 + e^{-1})$

$= e^{-2} + 0.49 + 0.7e^{-1}$

$y[n] = e^{-n} + (0.7)e^{-(n-1)} + (0.7)^2 e^{-(n-2)} + \dots + (0.7)^n$

(5) Given an LTI system with the output given by

$$y[n] = \sum_{k=0}^{n} e^{-2k} x[n-k]$$

a) Find the impulse response of this system.

$$h[n] = \sum_{k=0}^{n} e^{-2k} \delta[n-k] = e^{-2n}u[n] * \delta[n] = e^{-2n}u[n]$$

$$= \delta[n] + e^{-2}\delta[n-1] + e^{-4}\delta[n-2] + \dots$$

$$+ e^{-2n}\delta[0] = e^{-2n}u[n]$$

b) Is the system causal?

Yes, $h[n] = 0$ for $n < 0$

c) Is the system stable?

$$\sum_{n=-\infty}^{\infty} |h[n]| = \sum_{n=0}^{\infty} |e^{-2n}| = \sum_{n=0}^{\infty} (e^{-2})^n = \frac{1}{1-e^{-2}} < \infty \quad \therefore \text{Stable.}$$

(6) Consider an LTI system with the input and output related by

$y[n] = 0.5\,(x[n+1] + x[n])$

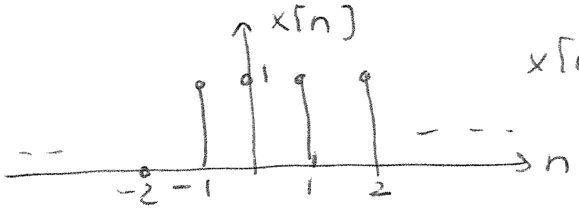
a) Find the system impulse response $h[n]$

$h[n] = 0.5\,(\delta[n+1] + \delta[n])$

b) Is this system causal? Why?

Not causal, since $h[n] \neq 0$ for $n < 0$
$(h[n] = 0.5$ for $n = -1)$

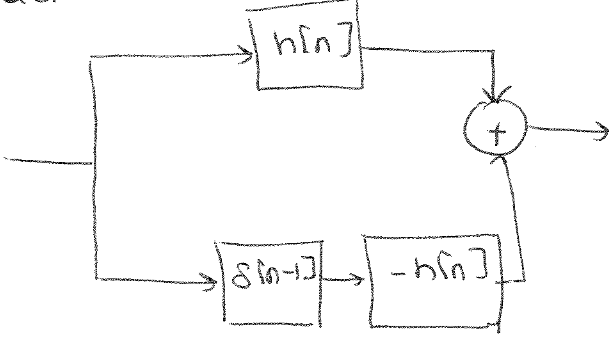
c) Determine the system response $y[n]$ for the following input.



$x[n] = u[n+1]$

$y[n] = h[n] * x[n]$
$= 0.5\,(\delta[n+1] + \delta[n]) * x[n]$
$y[n] = 0.5\,x[n+1] + 0.5\,x[n]$
$y[n] = 0.5\,u[n+2] + 0.5\,u[n+1]$

d) Consider the following system:



where $h[n]$ is found in (a).

Find impulse response of the total system.

$h[n] + \delta[n-1] * (-h[n])$
$= h[n] - h[n-1]$
$= 0.5(\delta[n+1] + \delta[n]) - 0.5(\delta[n] + \delta[n-1])$

$= 0.5(\delta[n+1] - 0.5\delta[n-1])$

e) What's the output of this system?

$y[n] = x[n] * h[n] - x[n] * h[n-1]$
$= 0.5u[n+2] + \cancel{0.5u[n+1]} - \cancel{0.5u[n+1]} - 0.5u[n]$
$= 0.5u[n+2] - 0.5u[n]$