Practice Exam II

Part I- Short answer questions

1. [5] The transfer function of a system is \( H(s) = \frac{1}{s + 2} \). Which one of the following statements is true for this system?

   a) The system is unstable.
   b) The dc gain for this system is 0.5.
   c) The system behaves like a highpass filter.
   d) The impulse response of this system is \( h(t) = e^{2t} u(t) \).

2. For the signal \( x(t) = \text{tri}(t) \), what is \( \int_{-\infty}^{\infty} X(f)df \)?

   a) 0
   b) 1
   c) \( \frac{1}{2} \)
   d) 2

3. For the signal \( x(t) = \cos(200\pi t) + 25 \sin(500t) \), the Nyquist sampling rate is

   a) 1000 radians/sec
   b) 400 \( \pi \) radians/sec
   c) 250 radians/sec
   d) 100 \( \pi \) radians/sec

4. Let \( X(f) \) be the Fourier transform of a real signal \( x(t) \). Then it can be concluded that

   a) \( |X(f)| \) is odd and \( \angle X(f) \) is even.
   b) \( |X(f)| \) is even and \( \angle X(f) \) is odd.
   c) \( |X(f)| \) is even and \( \angle X(f) \) is even.
   d) \( |X(f)| \) is odd and \( \angle X(f) \) is odd.

5. Determine whether the following statements are true or false.

   a) The spectrum of a periodic signal is discrete.

   b) For a periodic signal, if \( \int_{-\infty}^{\infty} x(t)dt = 0 \), then \( X[0] = 0 \) in the Fourier series expansion.
c) The transfer function of the linear differential equation model, 
\[ \frac{d^3 y(t)}{dt^3} + 5 \frac{dy(t)}{dt} + y(t) = 2 \frac{dx(t)}{dt} + x(t-1) \] 
is 
\[ H(s) = \frac{2s + 1}{s^3 + 5s + 1}. \]

d) The periodic signal 
\[ x(t) = \sum_{k=-\infty}^{\infty} X[k]e^{j2\pi ft} \] 
is passed through a filter with 
\[ H(f) = \text{rect}\left(\frac{f}{3}\right), \] 
the Fourier transform of the output is 
\[ Y(f) = \sum_{k=1}^{\infty} X[k]\delta(f - k). \]

e) If the input to a LTI system is a sinusoid, then the steady-state output of the system is also a sinusoid at the same amplitude.

Part II- Partial Credit Section

1. For the system in the figure, sketch \( A(\omega), B(\omega), C(\omega), Y(\omega) \).

\[ X(\omega) = \text{tri}(\frac{\omega}{10}) \]
\[ H(\omega) = \begin{cases} 
1, & -50 \leq \omega \leq -40, 40 \leq \omega \leq 50 \\
0, & \text{otherwise} 
\end{cases} \]
\[ G(\omega) = \begin{cases} 
1, & -100 \leq \omega \leq 100 \\
0, & \text{otherwise} 
\end{cases} \]
2. For the following Fourier series coefficients;

Note: \[ \angle X[2] = 45^\circ \]
\[ \angle X[3] = 90^\circ \]

a) Find the power in the periodic signal.
b) Is this signal odd symmetric, even symmetric or neither?
c) Assuming that the fundamental frequency of this signal is 10Hz, what is \( x(t) \)?
d) If this signal is passed through an ideal lowpass filter with cutoff frequency 25Hz, what would be the steady-state response?